Irregular pulsating polarization dynamics in gain-switched vertical-cavity surface-emitting lasers
Ángel Valle, Marc Sciamanna, Krassimir Panajotov

To cite this version:
Ángel Valle, Marc Sciamanna, Krassimir Panajotov. Irregular pulsating polarization dynamics in gain-switched vertical-cavity surface-emitting lasers. IEEE Journal of Quantum Electronics, Institute of Electrical and Electronics Engineers, 2008, 44 (2), pp.136-143. <10.1109/JQE.2007.910707>. <hal-00211181>

HAL Id: hal-00211181
https://hal-supelec.archives-ouvertes.fr/hal-00211181
Submitted on 21 Jan 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Irregular Pulsating Polarization Dynamics in Gain-Switched Vertical-Cavity Surface-Emitting Lasers

Angel Valle, Marc Sciamanna, and Krassimir Panajotov

Abstract—In this paper, we report on experimental and theoretical investigation on the nonlinear dynamics of the two orthogonal linearly polarized fundamental transverse modes of vertical-cavity surface-emitting lasers (VCSELs) under sinusoidal current modulation. Irregular pulses of the power of individual polarizations are measured with a period equal to twice the modulation period. In contrast with individual polarizations, total power displays irregular pulsations of the power of individual polarizations, displays an exponential decay for large values of that time. Those results are well reproduced by using a theoretical model that includes spontaneous emission fluctuations. However the previous qualitative features remain even in the absence of spontaneous emission noise. Our results therefore suggest that the irregular polarization dynamics have a deterministic origin and can be defined as deterministic chaos.

Index Terms—Current modulation, nonlinear dynamics, polarization switching, spontaneous emission noise, vertical-cavity surface-emitting laser (VCSEL).

I. INTRODUCTION

VERTICAL-CAVITY surface-emitting semiconductor lasers (VCSELs) present significant advantages over their edge-emitting counterparts, including low threshold current, low cost, circular output beam, and easy fabrication in two-dimensional arrays. Although VCSELs are intrinsically single-longitudinal mode devices, emission in multiple transverse and polarization modes is usually found [1]. The polarization is not well fixed and small changes of the injection current or the device temperature may result in a polarization switching (PS) between the two linearly polarized modes. While emission in several transverse modes is usually attributed to spatial-hole burning effects [1]–[4], a number of different physical mechanisms can be responsible for PS phenomenon in VCSELs. Therefore, different models of PS in VCSELs have been suggested, for example those taking into account spin relaxation mechanisms in semiconductor quantum wells (SFM model) [5], [6], thermal effects [7], or the relative modification of the net modal gain and losses with the injection current [8]–[10].

The study of chaotic behavior in semiconductor lasers is important nowadays from a fundamental point of view and for their potential use for data encryption applications [11]. Current-modulation of laser diodes is a simple and compact way of achieving the chaotic signals needed for those applications [12]. The dynamics of directly modulated semiconductor lasers has received a lot of attention, considering their potential to generate ultrafast sharp pulses but also their rich nonlinear behavior [13]–[22]. Most of existing studies relate to conventional edge-emitting semiconductor lasers [13]–[18]. Just a few reports of chaotic behavior can be found in the literature [15], [16] since only edge-emitting lasers with relatively small gain saturation and spontaneous emission noise parameters might undergo a period doubling route to chaos under current modulation [18]. Studies of nonlinear dynamics in directly modulated VCSELs remain scarce [19]–[22], while being of great interest both for fundamental and applied research [19]–[23]. Some theoretical and experimental work on the dynamics of directly modulated VCSELs with optical feedback has been recently done [24], [25]. The previous studies [19]–[22] are of theoretical nature and, to the best of our knowledge, no experimental study of the nonlinear dynamics of solitary gain-switched VCSELs has been performed yet. Nonlinear dynamics have been theoretically analyzed for linearly polarized single transverse mode [19], [20] and multimode VCSELs [20], [21]. Chaotic behavior appears in the multimode regime due to transverse mode competition [20], [21]. Single-transverse mode VCSELs have an extra degree of freedom with respect to the single-longitudinal mode edge-emitters, the polarization of the emitted light, that can enrich their nonlinear dynamical behavior under current modulation. A recent work has studied the effect the bias current sweep rate on the polarization switching of the directly modulated VCSEL [26]. The nonlinear dynamics of the two orthogonal polarizations of a directly modulated single-transverse mode VCSEL has been recently analyzed from a theoretical point of view [22]. Chaotic dynamics due to polarization competition has been found in a large range of laser and modulation parameters [22]. The chaotic dynamics of the polarization is found for much smaller amplitudes and frequencies of...
modulation than the chaotic dynamics of the transverse modes of the VCSEL. This occurs since the differences in gain/loss between the two polarizations of a given transverse mode are much smaller than gain/loss differences between different transverse modes [22] and then stronger competition between the orthogonal polarizations is obtained.

In this paper we measure the dynamics of the two orthogonal polarizations of a single-transverse mode VCSEL subject to a sinusoidal current modulation to confirm the theoretical predictions of [22]. We have considered several amplitudes and several frequencies in the gigahertz range. For specific values of the modulation frequency we find an irregular pulsed behavior of the power of individual polarizations with a period equal to twice the modulation period. The total power also pulses with the same period but maintaining regularity in the height of the pulses. We define a residence time to characterize the variability of pulse streams. We show that the residence time distributions for individual linear polarizations display an exponential decay for large values of the time. All the previous results are also analyzed from a theoretical point of view by using the flip model [6], [22]. Good agreement between theoretical and experimental results is found. Special attention is given to the role played by spontaneous emission fluctuations in the results obtained in this work. We show that, although spontaneous emission noise inclusion is important to obtain a good comparison, the main qualitative results remain in the deterministic model. That indicates that spontaneous emission fluctuations are not essential for explaining our experimental observations. Therefore, the nonlinear dynamics of an essentially deterministic system has been observed.

Our paper is organised as follows. In Section II, we describe our experimental setup and results. In Section III, we present our theoretical model and results. A comparison with the experimental results is also performed in that section. Finally, in Section IV, a brief discussion and a summary of our results are presented.

II. EXPERIMENTAL RESULTS

A. Experimental Setup

Experimentally, the gain-switching of the VCSEL is achieved using the setup presented in Fig. 1. A quantum-well oxide-confined VCSEL supplied by the University of Ulm, emitting around 851 nm, is current modulated by using a Bias-T. The dc current applied to the VCSEL is controlled by a low-noise laser driver. A RF voltage is applied by using a RF-signal generator. The operating VCSEL temperature is also controlled and is set at 26 °C. The beam from the VCSEL is collimated by using a lens (COL). A nonpolarizing 50/50 beamsplitter (BS) splits the beam in two detection branches. In the first one a polarizer (P2) and a power meter (PM) are used to measure sequentially the power emitted by the VCSEL in the two orthogonal polarizations. The second branch is used to perform spectral measurements using either an optical spectrum analyzer (OSA) or a RF spectrum analyzer associated with a photodetector (D1). An optical isolator and a half wave plate were put between the fiber coupler and the P1 polarizer to avoid feedback effects in the VCSEL. Dynamical evolution of the power is recorded with a digital oscilloscope (OSC) associated with a photodetector (D2). Measurements of the polarized or total power can be performed by inserting or removing the polarizer (P1).

B. Experimental Results

The polarization-resolved light–current characteristic of the free-running VCSEL is presented in Fig. 2. If the bias current is increased the VCSEL first emits a fundamental linearly polarized vertical mode (y-LP mode) with a threshold current, $I_{th}$, of 1.2 mA. As the injection current is increased a polarization switching (PS) between the orthogonal fundamental modes is found at around 1.4 $I_{th}$. That switching is of type I (PS I) because it occurs from the high-frequency mode (y-LP mode) to the mode with a lower frequency (x-LP mode). The VCSEL exhibits a single-transverse mode operation for bias currents less than 2.7 $I_{th}$. For higher currents, excitation of the first higher order mode is observed.

After characterising the continuous-wave (CW) polarization behavior of the VCSEL a sinusoidal modulation of the current is applied. The modulation frequency is around 1 GHz. The dc bias current has been chosen slightly smaller than the current at which PS occurs. Amplitudes of the modulation are...
chosen large enough to achieve gain-switching operation (the VCSEL current always spends a fraction of the period below the threshold current) and small enough to avoid the excitation of the higher order transverse mode. Repetitive gain-switched pulses (one pulse each modulation period $T$) are obtained for small modulation frequencies. A regular stream of pulses with similar heights is obtained for the total power. The behavior of individual polarizations is different: again a pulse of a given polarization appears each modulation period but its height largely fluctuates from one pulse to another. Pulses in both polarizations fluctuate in such a way that the total power remains regularly pulsing with pulses of similar heights. That situation indicates a clear anticorrelation between both polarizations. For large enough amplitudes of modulation the laser current goes well below the threshold value in such a way that the laser power reaches the spontaneous emission noise levels. The polarization that is excited with larger power in each modulation period is mainly determined by the spontaneous emission noise events occurring just before the switch-on of the pulse since the gain/losses differences between polarizations are very small. Then for small modulation frequencies spontaneous emission noise is essential in determining the dynamics of the polarization in the pulsed VCSEL.

More interesting nonlinear dynamics is found by increasing the modulation frequency. We show in Fig. 3 the temporal traces of the total and polarized powers when the modulation frequency $f_m = 1/T = 2.88$ GHz, the dc current $I_{dc}$ is 1.26 $I_{th}$, and the current amplitude $\Delta I = 0.89 I_{th}$. The influence of a higher order transverse mode is very weak because the optical spectrum under those modulation conditions show that the suppression ratio was 30 dB. The right column of that figure is composed of zooms of the left part to better appreciate the details of the pulses. A regular stream of pulses is found for the total power. Those pulses have similar heights and appear each two periods of modulation [see Fig. 3(d)]. Small shoulders appear that are reminiscences of the period-1 solution. When the amplitude of the modulation decreases those shoulders become larger until similar heights are obtained for all the pulses in such a way that only one pulse appears each modulation cycle (period-1 solution). Pulses in individual polarizations also appear with that periodicity but their heights are very irregular. Fig. 3(b) and (c) shows that there are wide temporal regions in which pulses in one polarization are very small (see for instance, Fig. 3(c) for times between 3120 and 3170 modulation periods). For those regions pulses in the orthogonal polarization have similar heights. This fact is deduced from the regularity of the total power and can not be deduced by directly comparing Fig. 3(b) and (c) because the traces shown in Fig. 3(a)–(c) have been obtained for different runs of the experiment. Due to technical constraints we can not record the time traces of the two orthogonal linear polarizations simultaneously.

One way of characterizing the irregularity of the streams of pulses is by measuring the duration of the regions where one polarization pulsates at 2 T with large power and the other pulsates with the same periodicity but with small power. We estimate that duration by using the residence time $\tau$ that we define as follows. First, we fix a power level that we choose as one half of the maximum power of the complete temporal series. Then $\tau$ is given by the time between consecutive crossings, from below to above, of that level. The process for obtaining the residence times, together with some values of that random variable, $\tau_r$, are illustrated in Fig. 3(e). We have acquired data over a 0.4-ms window.

**Fig. 3.** Experimental time traces of the intensities of the total, $x$-polarized and $y$-polarized powers. Results plotted in (d)–(f) correspond to zooms of (a)–(c), respectively, where the latter result from three independent experimental runs. The current modulation is such that $f_m = 2.88$ GHz, $I_{dc} = 1.30 I_{th}$, and $\Delta I = 0.89 I_{th}$.
with a 50-ps sampling time. The probability density functions of the residence times for the individual polarizations and the total power are shown in Fig. 4. The distributions for the two polarizations are very similar: they both present a multipeaked structure with a long exponential envelope. Distributions have been plotted in a logarithmic scale to show that exponential decay of the pulse envelope. A modulation seems to be superimposed on that exponential decay. Peaks appear at multiples of 2 T since that is precisely the periodicity of the pulses. We also show in Fig. 4 zooms of the regions of small \( \tau \). The probability density functions of the residence times for the individual polarizations and the total power, as a result of the anticorrelation between both polarizations. Similar qualitative behaviors were found in a \( \pm 0.2 \)-GHz range. The 2.88-GHz modulation frequency value has been chosen since the peak that indicates period-doubling in the RF spectrum of the total power was maximum at that frequency.

### III. Theoretical Results

#### A. Theoretical Model

In this work we consider a model for the polarization of a single-transverse mode VCSEL [6], [22] that takes into account the combined effect of the VCSEL anisotropies, the linewidth enhancement factor and the spin-flip relaxation processes within a framework known as the SFM model [5]. The equations for the left and right circularly polarized components of the slowly varying optical field, \( E_{\pm} \), the total population difference between conduction and valence bands, \( N \), and the difference between the two distinct subpopulation inversion densities which couple separately to the emission of left and right circularly polarized light, \( n \), are the following:

\[
\dot{E}_\pm = \kappa (1 + i\alpha) [(N \pm n) - 1] E_\pm - (\gamma_0 + i\gamma_p) E_\mp + \sqrt{\beta_{sp}(N \pm n)} \xi_\pm
\]

\[
\dot{N} = -\gamma [N - I + (N + n)|E_+|^2 + (N - n)|E_-|^2]
\]

\[
\dot{n} = -\gamma_\alpha n - \gamma [(N + n)|E_+|^2 - (N - n)|E_-|^2]
\]

where \( I \) is the normalized injection current (its value at threshold is \( I_{th} = 1 \)), modulated according to \( I = I_{dc} + \Delta I \sin(2\pi f_m t) \). \( \kappa \) is the electric field decay rate that includes the internal and facet losses, \( \alpha \) is the alpha factor or linewidth enhancement factor that describes phase-amplitude coupling mechanisms in semiconductor lasers and \( \gamma \) is the carrier relaxation rate. \( \gamma_\alpha = \gamma + 2\gamma_j \), where \( \gamma_j \) is a coupling rate between the two circularly polarized radiation channels, which models different microscopic relaxation mechanisms that equilibrate the spin of carriers [5]. \( \gamma_\alpha (\gamma_\alpha) \) is the magnitude of linear birefringence (dichroism), per intracavity round-trip
time. $\beta_{\text{sp}}$ is the spontaneous emission rate and $\xi_{\pm}$ are Gaussian white noises of zero mean value and delta-correlated in time.

B. Theoretical Results

We first calculate the polarization-resolved light–current characteristic of the free-running VCSEL. We solve the (1)–(3) by using the numerical algorithm reported in [27] with a 0.01 ps integration time step. The laser parameters have been chosen for obtaining a similar light–current characteristic to the experimental one presented in Fig. 2. A PS I switching is obtained at a current of 1.3 when $\kappa = 300$ ns$^{-1}$, $\alpha = 3$, $\gamma = 1$ ns$^{-1}$, $\gamma_n = 50$ ns$^{-1}$, $\gamma_a = 0.1$ ns$^{-1}$, $\gamma_p = 10$ ns$^{-1}$. We then calculate the time traces of the total and polarized powers for a sinusoidal modulation of the current with a frequency equal to the experimental one ($f_m = 2.88$ GHz). The distributions of the residence times are also calculated for a variety of $I_{\text{dc}}$, $\Delta I$ and $\beta_{\text{sp}}$ for finding the better match with the experimental results. The best agreement is found when $I_{\text{dc}} = 1.3$ and $\Delta I = 0.7$, that are near to the experimental values used in the previous section, and $\beta_{\text{sp}} = 10^{-4}$. The temporal traces of the total and polarized powers for those parameters are shown in Fig. 6. Nonlinear dynamics similar to the experimentally reported in the previous section is found. Irregular pulses of the power of individual polarizations are obtained with a period equal to twice the modulation period while the total power displays a more regular pulsing with the same periodicity. We note that the pulsing in the total power is not as regular as the experimental one. We also note that peaks in the simulation are narrower than the experimental ones. We explain it in terms of the absence of gain saturation terms in the spin-flip-model. In contrast with the experimental results reported in Fig. 3, the anticorrelation between both polarizations is now clearly seen by comparing Fig. 6(b) and (c) [or Fig. 6(e) and (f)] because all the traces shown in Fig. 6 have been obtained for the same simulation run.

In the previous section we characterized the variability of the streams of pulses by measuring the residence time, $\tau$, distribution. The theoretical probability density functions of $\tau$ for the individual polarizations and the total power are shown in Fig. 7. These distributions have been calculated by using data obtained over a 0.4-ms temporal window, equal to the experimentally used. The residence time distributions for the two polarizations present the same qualitative features as the experimental ones of Fig. 4, namely, the multiprobed structure at multiples of $2T$ and the long exponential tail. Also, the distribution of $\tau$ for the total power is concentrated at low values of $\tau$ because of the regular behavior of the total power. The residence time distributions change when changing the modulation parameters. The increase of $\Delta I$, for a fixed value of $I_{\text{dc}}$, change the exponential tail of the distribution corresponding to the $y$-polarization in a significant way (the maximum value changes from 32 to 96 modulation periods when $\Delta I$ changes from 0.6 to 0.8, for $I_{\text{dc}} = 1.3$). The tails corresponding to the $x$-polarization and the total power remain unchanged. The increase of $I_{K}$, for a fixed value of $\Delta I$, change the exponentials tails of the RTDs.

![Fig. 6. Theoretical time traces of the intensities of the total, x-polarized and y-polarized powers obtained from a single simulation run. Results plotted in Fig. 6(d)–(f) and correspond to zooms of Fig. 6(a)–(c), respectively. The current modulation is such that $f_m = 2.88$ GHz, $I_{\text{dc}} = 1.3$, and $\Delta I = 0.7$.](image-url)
corresponding to both polarizations in such a way that the maximum residence times decrease when increasing $I_{dc}$.

We have also found different dynamical behaviors in which irregular pulses of the power of individual polarizations are obtained with a period equal to 3 T while the total power displays regular pulsing with a 3 T period. We show in Fig. 8 the residence time distributions obtained when changing $I_{dc}$, $\Delta I$, and $\beta_{sp}$. Multiple peaks appear at multiples of 3 T in the individual polarizations while the probability for $\tau$ corresponding to the total power is concentrated at the 3 T value. Again the slow exponential decay is clear for the $\tau$ that corresponds to the individual polarizations. The laser parameters have changed (a very different value of $f_m$ is considered) with respect to the case of Figs. 6 and 7. That can be the reason why that behavior has not been found experimentally.

IV. DISCUSSION AND CONCLUSION

Our theoretical results of the previous section have been obtained with a model that includes a random spontaneous emission term. It is desirable to know if the behavior that we have found, especially the slow exponential decay of the residence time distributions, is caused by the fluctuating nature of the spontaneous emission or if it is just a result inherent to the deterministic dynamics of the system. To answer that question we have repeated the simulations reported in Figs. 6 and 7 with $\beta_{sp} = 0$. In those deterministic simulations we have found the same qualitative dynamics as in the simulations with included spontaneous emission noise. The driving force for the fluctuations in the deterministic simulations is the complicated nonlinear dynamics of the system. We show in Fig. 9 the distributions of the residence times for the deterministic case. They remain qualitatively similar to those found when spontaneous emission noise is included (see Fig. 7). We only find changes in the extension of the exponential decay: the maximum residence times for the $x$ and $y$ polarizations are 80 and 150 T, respectively, in the deterministic simulations. That can be explained by taking a look in Fig. 10 at the temporal traces of the polarized intensities obtained with the simulation including noise ($\beta_{sp} = 10^{-4}$). In Fig. 10, the intensities are plotted in logarithmic scale to show the effect of spontaneous emission noise and the parameters are the same than those of Fig. 6. Fig. 10 shows that the minimum values of the individual polarizations are above the power level where the fluctuations due to spontaneous emission dominate the evolution (around $10^{-7}$).

To conclude, we have performed a theoretical and an experimental study of the nonlinear dynamics of the two orthogonal linearly polarized fundamental transverse modes of VCSELs under sinusoidal current modulation. For large modulation frequencies, around 2.9 GHz, we have found irregular pulses of the power of individual polarizations with a period equal to twice the modulation period while the total power displays regular
The parameters are such that the period equal to three times the modulation period while the total power displays regular pulsing at three times the modulation period. The analysis of the influence of the laser parameters on the dynamics reported in this work is interesting and will be the subject of future work. The variability of pulse streams has been characterized by using residence times probability density functions. We have shown that those distributions for individual linear polarizations display an exponential decay of pulse envelope for large values of the time. Those results are well reproduced by using a theoretical model that includes spontaneous emission fluctuations. However that qualitative features remain even in the absence of spontaneous emission noise. Our results therefore suggest that the irregular polarization dynamics have a deterministic origin and can be defined as deterministic chaos.

In our experimental work we have focused on very specific values of the system parameters: just one value of the modulation frequency, dc bias current and amplitude of the modulation for which the more complicated dynamics was observed. The theoretical work have tried to identify the parameters of the model that can explain that behavior. That irregular behavior is then a particular case of the chaotic deterministic dynamics due to polarization competition that was described in [22]. An experimental analysis of the dynamics for a large range of laser and modulation parameters would be also very interesting for comparing with all the predictions of [22] and it will be the subject of future work.

ACKNOWLEDGMENT

The authors would like to thank R. Michalzik and his coworkers at the Optoelectronics Department, University of Ulm, Ulm, Germany, for providing the VCSELs. The authors also acknowledge T. Iwanow and M. Arizala for their help in the experiments.

REFERENCES


Fig. 9. Theoretical probability density functions of the residence time for the (a) x-polarization, (b) y-polarization and (c) total power. The parameters are those of Fig. 6 but with $\beta_{ap} = 0$.

Fig. 10. Theoretical time traces of the x-polarized and y-polarized powers. The parameters are such that $f_m = 2.88 \text{ GHz}$, $I_{dc} = 1.3$, $\Delta f = 0.7$ and $\beta_{ap} = 10^{-4}$.


