K-Player Bayesian Waterfilling Game for Fading Multiple Access Channels

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Abstract—We present a Bayesian game-theoretic approach for the distributed resource allocation problem in the context of K-user fading multiple access channels (MAC). We assume that users have incomplete information about the channel state information (CSI), i.e., each user knows his own channel state, but does not know the states of other users. All users (transmitters) are considered to be rational, selfish, and each one carries the objective of maximizing its own achievable data rate. In such a game-theoretic study, the central question is whether a Bayesian equilibrium (BE) exists. Based on the assumption of two channel states, we prove that there exists exactly one BE in this game.

I. INTRODUCTION

In recent years, there has been great interest in self-organizing wireless networks in which mobile devices allocate resource in a decentralized manner [1]. Tools of game theory [2] have been widely applied to study the resource allocation and power control problems in various types of networks, such as fading MAC [3], orthogonal frequency division multiplexing (OFDM) [4], multiple input and multiple output (MIMO) channels [5], [6], and interference channels [7], etc. Typically, the game-theoretic models used in these previous works assume that the information/knowledge about other devices is available to all devices. However, this assumption is hardly met in practice. In practical wireless communication scenarios, mobile devices can have local information but can barely access to global information on the network status.

In this paper, we focus our attentions on the fading MAC. The capacity region of fading MAC and the optimal resource allocation algorithms have been characterized and well studied in many pioneering works with assumptions of different information levels [8]-[11]. However, in order to achieve the full capacity region, it usually requires a central computing resource (a scheduler with comprehensive knowledge of the network information) to globally allocate the system resources. This process is centralized, it involves feedback and overhead communication whose load scales linearly with the number of transmitters and receivers in the network. In addition, with the fast evolution of wireless techniques, this centralized network infrastructure begins to expose its weakness in many aspects, e.g., slow reconfiguration against varying environment, increased computational complexity, etc.

A static non-cooperative game has been introduced in the context of fading MAC, known as "waterfilling game" [3].

From a simple two-user MAC setting, the authors show that the unique Nash equilibrium (NE) [12] of the corresponding waterfilling game is actually the maximum sum-rate point on the boundary of the MAC capacity region. However, their results rely on the fact that both transmitters have "complete information" about the CSI, and in particular, perfect CSI of all transmitters in the network. As we previously pointed out, this assumption is rarely possible in practice.

Thus, this power allocation game needs to be reconstructed with some realistic assumptions made on the knowledge level of mobile devices. Under this consideration, it is of great interest to investigate scenarios in which devices have "incomplete information" about their components, e.g., a device is ware of its own channel gain, but unaware of the channel gains of other devices. In game theory, a strategic game with incomplete information is called a "Bayesian game". Over the last ten years, Bayesian game-theoretic tools have been used to design distributed resource allocation strategies only in a few contexts, e.g., CDMA networks [13], [14], multi-carrier interference networks [15]. Our motivation is therefore to study how Bayesian games can be applied to the context of fading MAC. The goal of this paper is to investigate the outcome of our Bayesian waterfilling game, i.e., the existence and uniqueness of BE point(s).

The paper is organized in the following form: In Section II, we introduce the system model and describe briefly the background. In Section III, the K-user MAC is formulated as a Bayesian game. In Section IV, we characterize the BE set. Finally, we close with some concluding remarks in Section V.

II. SYSTEM MODEL AND BACKGROUND

A. System Model

We consider a time-slotted flat-fading MAC in a single-cell network, in which K users are simultaneously sending information to one base station. At time t, the signal received by the base station can be mathematically expressed as

$$y(t) = \sum_{k=1}^{K} \sqrt{g_k(t)} x_k(t) + z(t)$$

where $x_k(t)$ and $g_k(t)$ are the input signal and fading channel gain of user k, $z(t)$ is a zero-mean white Gaussian noise with

variance σ^2. The input signal $x_k(t)$ can be further written as

$$x_k(t) = \sqrt{p_k(t)} s_k(t)$$

where $p_k(t)$ and $s_k(t)$ are the transmitted power and data of user k at time t.

We assume that the channel gains $g_1, \ldots, g_K$ are deterministic constants during the period of each transmission block (which is assumed to be larger than a time slot interval). Therefore, within each time slot t, this is simply a Gaussian multi-user channel [16]. Now, instead of considering the whole capacity region, we are interested in the single-user achievable rate (assuming that the base station uses low complexity single-user decoder [16]), i.e.,

$$R_k = \log\left(1 + \frac{g_k p_k}{\sigma^2 + \sum_{j=1, j \neq k}^K g_j p_j}\right) \qquad (1)$$

Intuitively, the object of each user is to maximize this rate, which represents the amount of transmitted information.

B. Complete and Incomplete Information

Before introducing the problem, it is necessary to clarify the meanings of complete information and incomplete information. In game theory, the notion of complete information means that all players know completely the structure of the game, which includes:

- Player set (how many devices involved? what are they?)
- Other players' actions (what are their behaviors?)
- Other players' payoffs (what are their object functions?)

If some players do not completely know the game structure, we call it a game with incomplete information. For example, "chess" is a game with complete information, because the chess moves are known to all players; "poker" is a game with incomplete information, because players are uncertain about the card states of other players, therefore, the exact object functions of other players are unknown.

In our context, take an example of two-user MAC, complete information means that the channel gains g_1, g_2 are available at both transmitters $\mathrm{Tx}_1, \mathrm{Tx}_2$; incomplete information means that the channel gain g_k is only available at Tx_k, $k = 1, 2$. In this paper, we assume that the player set and action set (see their definitions in Section III) are known to all players.

C. Background: Waterfilling Game

An important related work is the "waterfilling game" [3], where multi-user resource allocation problem in fading MAC is studied in detail as a non-cooperative game with complete information. In this game, users are assumed to compete with transmission rates as payoff and transmit powers as moves.

Here, we briefly describe the problem, which is also helpful to understand our Bayesian game formulation. Under the assumption of complete information, each user k's transmit power strategy can be considered as a function of all users' channel gains, i.e., $p_k(g_1, \ldots, g_K)$. In the case of two-user MAC, for a fixed strategy $p_2(g_1, g_2)$, finding the optimal

strategy $p_1(g_1, g_2)$ for user 1 requires solving the following optimization problem:

$$\begin{aligned} \max_{p_k} \quad & \mathbb{E}_{g_1, g_2}\left[\log\left(1 + \frac{g_1 p_1(g_1, g_2)}{\sigma^2 + g_2 p_2(g_1, g_2)}\right)\right] \\ s.t. \quad & \mathbb{E}_{g_1, g_2}[p_1(g_1, g_2)] \leq P_1^{\max} \\ & p_1(g_1, g_2) \geq 0 \end{aligned} \qquad (2)$$

and similarly for user 2. Here, $P_1^{\max}$ is the average power constraint of user 1. Note that the solution of (2) depends on user 2's strategy $p_2(\cdot)$, which user 1 does not know, and reciprocally for user 2. However, the static non-cooperative game model given in [3] enables both users to adjust their strategies adaptively to their guesses of the strategy of the other user. This process converges to a unique NE:

$$\begin{cases} p_1^\star(g_1, g_2) & = \left(\lambda_1 - \frac{\sigma^2}{g_1}\right)^+, \text{ when } g_1 \geq \frac{\lambda_2}{\lambda_1} g_2 \\ p_2^\star(g_1, g_2) & = \left(\lambda_2 - \frac{\sigma^2}{g_2}\right)^+, \text{ when } g_2 \geq \frac{\lambda_1}{\lambda_2} g_1 \end{cases} \qquad (3)$$

with $p_1^\star(g_1, g_2) = 0$ and $p_2^\star(g_1, g_2) = 0$ in all other cases. The dual variables λ_1 and λ_2 are given by

$$\begin{cases} \mathbb{E}_{g_1, g_2}\left[\left.\left(\lambda_1 - \frac{\sigma^2}{g_1}\right)^+ \right| g_1 \geq \frac{\lambda_2}{\lambda_1} g_2\right] = P_1^{\max} \\ \mathbb{E}_{g_1, g_2}\left[\left.\left(\lambda_2 - \frac{\sigma^2}{g_2}\right)^+ \right| g_2 \geq \frac{\lambda_1}{\lambda_2} g_1\right] = P_2^{\max} \end{cases} \qquad (4)$$

The authors show that the NE (3) corresponds to the maximum sum-rate point of the capacity region. This result is somewhat surprising, since NE is in general inefficient comparing to the Pareto optimality. Thus, an interesting conclusion is that the selfish behavior of the users leads them to jointly optimize the sum-rate of the channel.

III. BAYESIAN GAME MODEL

Before introducing our game model, we need to clarify a prior assumption, as follows,

Assumption III.1. *We assume that each user's channel gain g_k is i.i.d. from two discrete values: g_- and g_+ with probability ρ_- and ρ_+, respectively. WloG, we assume $g_- < g_+$.*

On the one hand, our assumption is closely related to the way how feedback information are signalled to the transmitters. In order to get the channel information g_k at the transmitter side, it requires the base station to estimate it and then feedback to user k *at a given precision*. Since in digital communications any information is represented by a finite number of bits (e.g., x bits), channels gains are mapped into a set that contains a finite number of states (2^x states).

On the other hand, this is a necessary assumption for analytical tractability, since in principle the functional strategic form of a player can be quite complex with both actions and states are continuous (or infinite). To avoid this problem, in [15] the authors successfully modelled a multi-carrier Gaussian interference channel as a Bayesian game with discrete (or finite) actions and continuous states. Inspired from [15], in this paper, we model the fading MAC as a Bayesian game under the assumption of continuous actions and discrete states.

Now, we model the K-user fading MAC as a Bayesian game. In such a communication system in which users have incomplete information, the natural object of each user is to maximize its *average* achievable rate, i.e.,

$$\max_{p_k} \quad \mathbb{E}_{\mathbf{g}}\left[\log\left(1 + \frac{g_k p_k(g_k)}{\sigma^2 + \sum_{j=1, j\neq k}^{K} g_j p_j(g_j)}\right)\right]$$
$$s.t. \quad \mathbb{E}_{g_k}\left[p_k(g_k)\right] \leq P_k^{\max} \qquad (5)$$
$$p_k(g_k) \geq 0$$

where $\mathbf{g} = \{g_1, \ldots, g_K\}$ is a set of channel gains, $P_k^{\max}$ is the average power constraint for user k. Note that under the assumption that each user has only incomplete information about the fading channel gains, user k's power strategy $p_k(\cdot)$ can only rely on its own channel gain g_k, written as $p_k(g_k)$.

For a given strategy $p_{-k} = \{p_1, \ldots, p_{k-1}, p_{k+1}, \ldots, p_K\}$, the single-user maximization problem (5) is a convex optimization problem [17]. Via Lagrangian duality, the solution is given by the following equation:

$$\mathbb{E}_{g_{-k}}\left[\frac{g_k}{\sigma^2 + g_k p_k(g_k) + \sum_{j\neq k} g_j p_j(g_j)}\right] = \lambda_k \qquad (6)$$

where $g_{-k} = \{g_1, \ldots, g_{k-1}, g_{k+1}, \ldots, g_K\}$, and the dual variable λ_k is chosen such that the power constraint in (5) is satisfied with equality. However, the solution of (6) also depends on $p_{-k}(\cdot)$ which user k does not know. Thus, in order to obtain the optimal power allocation, each user must adjusts its power level based on the guess of all other users' strategies. Given the following game model, each user is able to adjust its strategy according to the belief it has on the strategy of the other user.

The K-player MAC Bayesian waterfilling game can be completely characterized as:

$$\mathcal{G}_{MAC} \triangleq \langle \mathcal{K}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{U} \rangle$$

- **Player** set: $\mathcal{K} = \{1, \ldots, K\}$.
- **Type** set: $\mathcal{T} = \mathcal{T}_1 \times \ldots \times \mathcal{T}_K$ ('$\times$' is the Cartesian product) where $\mathcal{T}_k = \{g_-, g_+\}$, a player's type is defined as its channel gain, i.e., $g_k \in \mathcal{T}_k$.
- **Action** set: $\mathcal{P} = \mathcal{P}_1 \times \ldots \times \mathcal{P}_K$ where $\mathcal{P}_k = [0, P_k^{\max}]$, a player's action is defined as its transmit power, i.e., $p_k \in \mathcal{P}_k$.
- **Probability** set: $\mathcal{Q} = \mathcal{Q}_1 \times \ldots \times \mathcal{Q}_K$ where $\mathcal{Q}_k = \{\rho_-, \rho_+\}$, we have $\rho_+ = \Pr(g_k = g_+)$ and $\rho_- = \Pr(g_k = g_-)$.
- **Payoff** set: $\mathcal{U} = \{u_1, \ldots, u_K\}$ where u_k is chosen as player k's achievable rate (1)

$$u_k(p_1, \ldots, p_K) = \log\left(1 + \frac{g_k p_k(g_k)}{\sigma^2 + \sum_{j=1, j\neq k}^{K} g_j p_j(g_j)}\right) \quad (7)$$

In games of incomplete information, a player's type represents any kind of private information that is relevant to its decision making. In our context, the fading channel gain g_k is naturally considered as the type of user k's, since its decision (in terms of transmit power) can only rely on g_k.

IV. BAYESIAN EQUILIBRIUM

What we can expect of the outcome from a Bayesian game? Generally speaking, the process of rational players' behaviors usually results in BE, which represents a "stable" result of learning and evolution of all participants. Therefore, it is important to characterize its set, since it concerns the performance analysis of a distributed system.

A. Definition of Bayesian Equilibrium

Let $\{\hat{p}_k(\cdot), p_{-k}(\cdot)\}$ denote the strategy profile where all players play $p(\cdot)$ except player k who plays $\hat{p}_k(\cdot)$, we can then describe player k's payoff as:

$$u_k(\hat{p}_k, p_{-k}) = u_k(p_1, \ldots, p_{k-1}, \hat{p}_k, p_{k+1}, \ldots, p_K)$$

Definition IV.1. *(Bayesian equilibrium)*
The strategy profile $p^\star(\cdot) = \{p_k^\star(\cdot)\}_{k\in\mathcal{K}}$ is a (pure strategy) Bayesian equilibrium, if for all $k \in \mathcal{K}$, and for all $p_k(\cdot) \in \mathcal{P}_k$ and $p_{-k}(\cdot) \in \mathcal{P}_{-k}$

$$\bar{u}_k(p_k^\star, p_{-k}^\star) \geq \bar{u}_k(p_k, p_{-k}^\star)$$

where we define $\bar{u}_k \triangleq \mathbb{E}_{\mathbf{g}}[u_k]$.

From this definition, it is clear that at the BE no player can benefit by changing its strategy while the other players keep theirs unchanged. It is worth to mention that the action set of each player is independent of the type set, i.e., the actions available to user k is the same for every its type.

B. Characterization of Bayesian Equilibrium Set

It is well known that, in general, an equilibrium point does not necessarily exist [2]. Therefore, our primary interest is to investigate the *existence* and *uniqueness* of BE in $\mathcal{G}_{MAC}$. Here, we directly give our main result:

Theorem IV.2. *There exists a unique Bayesian equilibrium in the K-user MAC game $\mathcal{G}_{MAC}$.*

Proof: It is easy to prove the existence part, since the strategy space p_k is convex, compact and nonempty for each k; the payoff function u_k is continuous in both p_k and p_{-k}; and u_k is concave in p_k for any p_{-k} [2].

In order to prove the uniqueness part, we should rely on a sufficient condition [18]: a non-cooperative game has a unique equilibrium, if the non-negative weighted sum of the payoffs is *diagonally strictly concave*. We firstly give the definition:

Definition IV.3. *(Diagonally strictly concave)*
Function $f(\mathbf{x}, \mathbf{r}) = \sum_{i=1}^{n} r_i \varphi_i(\mathbf{x})$ is called diagonally strictly concave for any vector $\mathbf{x} \in \mathbb{R}^{n\times 1}$ and fixed vector $\mathbf{r} \in \mathbb{R}_{++}^{n\times 1}$, if for any two different vectors $\mathbf{x}^0, \mathbf{x}^1$, we have

$$\Omega(\mathbf{x}^0, \mathbf{x}^1, \mathbf{r}) \triangleq (\mathbf{x}^1 - \mathbf{x}^0)^{\mathrm{T}} \delta(\mathbf{x}^0, \mathbf{r}) + (\mathbf{x}^0 - \mathbf{x}^1)^{\mathrm{T}} \delta(\mathbf{x}^1, \mathbf{r}) > 0 \quad (8)$$

where $\delta(\mathbf{x}, \mathbf{r})$ is called pseudo-gradient of $f(\mathbf{x}, \mathbf{r})$, defined as

$$\delta(\mathbf{x}, \mathbf{r}) \triangleq \begin{bmatrix} r_1 \frac{\partial \varphi_1}{\partial x_1} \\ \vdots \\ r_n \frac{\partial \varphi_n}{\partial x_n} \end{bmatrix}. \quad (9)$$

Lemma IV.4. *The weighted non-negative sum of the average payoffs $\bar{u}_k$ in $\mathcal{G}_{MAC}$ is diagonally strictly concave.*

Proof: Write the sum of the average payoffs as:

$$f^u(\mathbf{p}, \mathbf{r}) \triangleq \sum_{k=1}^{K} r_k \bar{u}_k(\mathbf{p}), \qquad (10)$$

where $\mathbf{p} = [p_1 \ldots p_K]^{\mathrm{T}}$ is the transmit power vector, $\mathbf{r} = [r_1 \ldots r_K]^{\mathrm{T}}$ is a non-negative vector assigning weights $r_1, \ldots, r_K$ to the average payoffs $\bar{u}_1, \ldots, \bar{u}_K$, respectively. Similar to (9), we let $\delta^u(\mathbf{p}, \mathbf{r}) \triangleq [r_1 \frac{\partial \bar{u}_1}{\partial p_1} \ldots r_K \frac{\partial \bar{u}_K}{\partial p_K}]^{\mathrm{T}}$ be the pseudo-gradient of $f^u(\mathbf{p}, \mathbf{r})$. Now, we define

$$p_k \triangleq p_k(g_-), \ k \in \mathcal{K}$$

the transmit power of player k when its channel gain is g_-. Since we have shown from Lagrangian that at the equilibrium the power constraint is satisfied with equality, i.e., $\sum_{g_k} p_k(g_k) = P_k^{\max}$, we have $P_k^{\max} - p_k = p_k(g_+) \ k \in \mathcal{K}$, as the transmit power when its channel gain is g_+. Therefore, it is easy to find that the average payoff $\bar{u}_k$ can be actually transformed into a weighted sum-log function, as follows

$$\bar{u}_k(p_k) = \sum_i \omega_i \log\left[1 + \frac{\alpha_k^i + \beta_k^i p_k}{\sigma^2 + \sum_{j \neq k}\left(\alpha_j^i + \beta_j^i p_j\right)}\right]$$

where i represents the index for different jointly probability events, ω_i represents the corresponding probability for index i. Note that the following conditions hold for all i, k

$$\alpha_k^i + \beta_k^i p_k \geq 0, \ \alpha_k^i > 0, \ \beta_k^i \neq 0, \ \sigma^2 > 0$$

Now, we can write the pseudo-gradient δ^u as

$$\delta^u(\mathbf{p}, \mathbf{r}) = \begin{bmatrix} r_1 \frac{\partial u_1}{\partial p_1} \\ \vdots \\ r_K \frac{\partial u_K}{\partial p_K} \end{bmatrix} = \begin{bmatrix} r_1 \sum_i \beta_1^i \phi_i^{-1}(\mathbf{p}) \\ \vdots \\ r_K \sum_i \beta_K^i \phi_i^{-1}(\mathbf{p}) \end{bmatrix}$$

where function $\phi_i(\mathbf{x})$ is defined as

$$\phi_i(\mathbf{x}) \triangleq \sigma^2 + \sum_{k=1}^{K}\left(\alpha_k^i + \beta_k^i x_k\right)$$

To check the diagonally strictly concave condition (8), we let $\mathbf{p}^0, \mathbf{p}^1$ be two different vectors, and define

$$\Omega^u(\mathbf{p}^0, \mathbf{p}^1, \mathbf{r}) \triangleq (\mathbf{p}^1 - \mathbf{p}^0)^{\mathrm{T}} \delta^u(\mathbf{p}^0, \mathbf{r}) + (\mathbf{p}^0 - \mathbf{p}^1)^{\mathrm{T}} \delta^u(\mathbf{p}^1, \mathbf{r})$$
$$= \sum_i \left[\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)\right] \zeta_i$$
$$= \sum_i \phi_i^{-1}(\mathbf{p^0}) \phi_i^{-1}(\mathbf{p^1}) \zeta_i^2$$

where $\zeta_i = \sum_{k=1}^{K} r_k \beta_k^i \left(p_k^1 - p_k^0\right)$. Define $\Delta p_k = p_k^1 - p_k^0$, since $\mathbf{p}^0, \mathbf{p}^1$ are two different vectors, we must have $\Delta \mathbf{p} = [\Delta p_1 \cdots \Delta p_K]^{\mathrm{T}} \neq \mathbf{0}$. Now, we can draw a conclusion from the equation above: $\Omega^u(\mathbf{p}^0, \mathbf{p}^1, \mathbf{r}) > 0$. This is because: (1) the first part $\phi_i^{-1}(\mathbf{p}^0)\phi_i^{-1}(\mathbf{p^1}) > 0$ for all i, since $\sigma^2 > 0$ and $\alpha_k^i + \beta_k^i p_k \geq 0$ for all i, k; (2) the second part $\zeta_i^2 \geq 0$ for all i, and there exists at least one nonzero term ζ_i^2,

due to $\Delta \mathbf{p} \neq 0$ and $r_k \neq 0, \beta_k^i \neq 0$ for all i, k. Therefore, the summation of all the products of the first and the second terms must be positive. From Definition IV.3, the sum-payoff function $f^u(\mathbf{p}, \mathbf{r})$ satisfies the condition of diagonally strictly concave. This completes the proof of this lemma. ∎

Since our sum-payoff function $f^u(\mathbf{p}, \mathbf{r})$ (10) is diagonally strictly concave, from the Theorem 2 in [18], we have the uniqueness of Nash equilibrium in our game $\mathcal{G}_{MAC}$. ∎

V. Conclusion

We presented a Bayesian game-theoretic framework for distributed resource allocation in fading MAC, where users are assumed to have only local information about the network channel gain states. By introducing the assumption of finite channel states, we successfully found a analytical way to characterize the BE set. We proved the existence and uniqueness of BE in our game. This result is important for predicting the system performance of a distributed wireless network.

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