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A Recursive System Identification Method Based on Binary Measurements

Kian Jafari, Member, IEEE, Jerome Juillard, Member, IEEE and Eric Colinet

Abstract — An online approach to parameter estimation problems based on binary observations is presented in this paper. This recursive identification method relies on a least-mean squares approach which makes it possible to estimate the coefficients of a finite-impulse response system knowing only the system input and the sign of the system output. The impulse response is identified up to a positive multiplicative constant. The role of the regulative coefficient is investigated thanks to simulated data. The proposed method is compared with another online approach: it is shown that the proposed method is competitive with the other one in terms of estimation quality and of calculation complexity.

I. INTRODUCTION

Parameter estimation methods based on binary or quantized output observations have known an increasing interest in the past few years. The use of such methods is mostly motivated by the fact that low-resolution observations are usually much cheaper to obtain than high-resolution ones. Several applications based on binary sensors or on binary data are given in [1], [2].

The method presented in this paper is aimed at the test of micro-electronic devices, such as MEMS (Microelectromechanical systems). It is known that as characteristic dimensions become smaller, the dispersions afflicting electronic devices tend to become larger. Typical sources of dispersion are variations in the fabrication process or changes in the operating conditions, such as temperature, pressure or humidity [3], [4]. As a consequence, it is not usually possible to guarantee a priori that a given device will function properly. The tests that are run after fabrication, under different operating conditions, ensure that only suitable devices are commercialized. However, these tests are very costly: a typical figure is that one third of the cost of a micro-fabricated device is due to testing [5]. An alternative is to integrate self-test (and self-tuning) features, such as self-test approach should require as little data storage as possible and have a low computational complexity, so as to be easily implemented.

In this paper, we establish a recursive identification method based on binary observations which is well-adapted to the context of micro-electronic devices. This LMS (least-mean squares) parameter estimation method is based on the offline WLS (weighted-least squares) approach developed in [3], [4]. The convergence of this recursive algorithm is illustrated by simulation and the choice of an efficient regulative coefficient for convergence to the nominal system parameters is investigated. Our results are also compared with those obtained with the method developed by Wigren [6], [7], which is also well adapted to the context of microelectronics.

The structure of the article is the following. In section II, an overview of existing identification methods based on binary or quantized observations is given. In section III, the system and its model are introduced. In section IV, the LMS-like algorithm is derived. In section V, the role of the regulative coefficient of LMS algorithm is investigated by simulations. Furthermore, the proposed method is compared with an appropriate online method [7] in terms of estimation quality and of calculation complexity. Concluding remarks and perspectives are given in section VI.

II. STATE OF THE ART

Some important contributions in the field of parameter estimation based on binary (or quantized) data were presented in [1]–[4] and [6]–[13]. These methods can be divided in two categories, depending on whether they rely on the use of a dithering signal.

Although the methods using a dithering signal at the input of the quantizer [1], [8], [9], [12], [13] may be useful in several macro-scale applications [1], they do not scale down easily to the context of integrated micro-electronics: all these methods rely on the fact that the cumulative probability
III. FRAMEWORK AND NOTATIONS

The known input signal \( u_k \) is filtered by a discrete-time invariant linear system \( H(z^{-1}) \) to produce the (scalar) value of the system output \( y_k \) at time \( k \) (Fig. 1). \( H \) has a finite impulse response of length \( L \), i.e. the impulse response can be represented by a column vector \( \theta_0 = (\theta_k)^L_{k=1} \).

Let \( \hat{\theta}_k \) be the estimated vector of parameters (of length \( L \)) at time \( k \). The output and the estimated output can be expressed as:

\[
\begin{align*}
    y_k &= \theta_0^T \Phi_k, \\
    \hat{y}_k &= \hat{\theta}_k^T \Phi_k,
\end{align*}
\]

where \( \Phi_k = (u_l)_{l=1}^{L+1} \) is the (column) vector of inputs at time \( k \). The system output then goes through a 1-bit ADC so that only the sign of the system output is known. Thus, we define \( s_k = S(y_k) \) and \( \hat{s}_k = S(\hat{y}_k) \) where

\[
S(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{otherwise}. 
\end{cases}
\]

In [3] and [4], we showed that, given \( N \) observations, the problem of parameter estimation from binary measurements could be treated offline by minimizing a WLS criterion of the form:

\[
C_1^N(\hat{\theta}) = \frac{1}{4N} \sum_{k=1}^{N} \hat{y}_k^2 (s_k - \hat{s}_k)^2.
\]

It is also established that this criterion is continuously differentiable with respect to \( \hat{\theta} \) and is thus amenable to simple optimization algorithms (e.g. gradient) [3], [4].

Our goal is to develop a recursive estimation method to estimate \( \theta_0 \) based on observations of the binary output \( s_k \) knowing \( u_k \). Note that \( \hat{\theta} = \lambda \theta_0 \) where \( \lambda > 0 \), is a minimizer of \( G_1^N \), i.e. the sign of \( y_k \) is not changed by multiplying it with a positive constant. Without loss of generality, we assume in the rest of the paper that \( ||\theta_0|| = 1 \).

IV. PROPOSED LMS APPROACH

Starting from a general LMS algorithm, we derive a practical LMS-like method (that is called LIMBO, for LMS-BIMBO, in the rest of paper) in order to estimate the system parameter online from binary observations (Fig. 2). A general LMS algorithm can be written:

\[
\hat{\theta}_{k+1} = \hat{\theta}_k - \alpha_k \frac{\partial E_k^2}{\partial \theta_k},
\]
where \( \alpha_k \) must satisfy some conditions to guarantee the stability and convergence of the algorithm. In classical problems, where the output is available, the instantaneous error is defined as:

\[
E_k = (y_k - \hat{y}_k).
\]

In the framework of system identification based on binary observations, we only have access to \( s_k \) and the unweighted instantaneous error,

\[
E_k = (s_k - \hat{s}_k),
\]

is not differentiable with respect to \( \hat{\theta}_k \). However, the following form of instantaneous error can be derived from (2) which is differentiable with respect to \( \hat{\theta}_k \) ([3], [4]) as opposed to (3):

\[
E_k = |s_k - \hat{s}_k| \hat{y}_k.
\]

As a consequence, we propose the following algorithm:

\[
\hat{\theta}_1 = [1 \ 0 \ 0 \ ... \ 0 \ 0],
\]

\[
\hat{\Psi}_{k+1} = \hat{\theta}_k - \alpha_k \frac{\partial E_k^2}{\partial \hat{\theta}_k},
\]

\[
\hat{\theta}_{k+1} = \frac{\hat{\Psi}_{k+1}}{\| \hat{\Psi}_{k+1} \|}.
\]

Equation (5c) ensures that the norm of \( \hat{\theta}_k \) is equal to unity. From (1) and (4), (5b) can be explicited as:

\[
\hat{\Psi}_{k+1} = \hat{\theta}_k - 2\alpha_k (s_k - \hat{s}_k)^2 \hat{y}_k \Phi_k.
\]

The quality of the online estimation \( \hat{\theta}_k \) is defined as:

\[
1 - v_k,
\]

where \( v_k \) is the cosine of the angle made by \( \hat{\theta}_k \) and \( \theta_0 \). Since both vectors are normalized, we have

\[
v_k = \theta_0^T \hat{\theta}_k,
\]

and \( (1 - v_k) \rightarrow 0 \iff v_k \rightarrow 1 \iff \hat{\theta}_k \rightarrow \theta_0 \).

V. RESULTS AND DISCUSSION

In this section, the results obtained with LIMBO are compared with those obtained with Wigren’s approach [7] which is an efficient online method to parameter estimation problems using quantized output measurements [3]. Note that when used in the context of binary observations, Wigren’s method [7] reduces, in its simplest form, to:

\[
\hat{\theta}_1 = [1 \ 0 \ 0 \ ... \ 0 \ 0],
\]

\[
\hat{\Psi}_{k+1} = \hat{\theta}_k - 2\alpha_k (s_k - \hat{s}_k) \frac{\partial \hat{s}_k}{\partial \hat{\theta}_k},
\]

\[
\hat{\theta}_{k+1} = \frac{\hat{\Psi}_{k+1}}{\| \hat{\Psi}_{k+1} \|},
\]

where

\[
\hat{s}_k = S(\bar{y}_k),
\]

\[
\bar{y}_k = \hat{\theta}_k^T \Phi_k,
\]

\[
\hat{s}_k \approx \frac{2}{\pi} \arctan \left( \frac{\bar{y}_k}{y_0} \right),
\]

and \( y_0 \) is an arbitrary constant [7]. The quantity \( \partial \hat{s}_k / \partial \hat{\theta}_k \) is the so-called ”pseudo-gradient” of \( \hat{s}_k \). The quality of the
A regulative coefficient is chosen, Wigren’s approach applied
the quality. The reason for this difference is that when a constant
estimation can be defined as 1 − wk where,

\[ w_k = \theta_0^T \tilde{\theta}_k. \]

Furthermore the role of the regulative coefficient (αk) is
addressed for two methods. We also discuss the efficiency
and the complexity of each method and their ease of imple-
mentation in the field of micro-devices.

In (5) or (7), the regulative coefficient (αk) can be chosen
with different strategies (constant or adaptive step size [18]).
For instance, in [7], the convergence of the pseudo-gradient
LMS is proved with αk = 1/k. However, it should be noted
that this particular choice of regulative coefficient
(αk = 1/k) is not perfectly adjusted to the integration con-
text of micro-devices, because division, being one of the most
complicated and expensive arithmetic operations, is costly to
implement. This is the reason why we investigate the use of
a constant regulative coefficient in (5) and (7).

The methods are compared, without measurement noise,
on the same test case as in [7]:

\[ \theta_0 = [0.2 \text{ } -0.14 \text{ } 0.8 \text{ } -0.56], \]

\[ u_k \text{ is a Gaussian white noise with zero mean and unit standard deviation.} \]

The value of y0 in Wigren’s method is set by trial-and-error to
y0 = 1 when αk = 1/k and to y0 = 10^4αk while αk = α (using a constant regulative
coefficient), so that a good compromise between convergence
speed and estimation quality is reached. Typical results are
shown in Fig. 3.

When a constant regulative coefficient is used, we find
that α < 0.2 yield reasonable results for both methods, in
terms of convergence speed and estimation quality (table I),
with a notable advantage for LIMBO in terms of estimation
quality. The reason for this difference is that when a constant
regulative coefficient is chosen, Wigren’s approach applied
to binary observations stops converging after reaching a
certain accuracy (dependent on the choice of y0). On the
other hand, the LIMBO approach converges to the nominal
parameters within the limits of finite machine precision. This
is illustrated in Fig. 4.

When a monotonously decreasing regulative coefficient
is used (in the present case, when αk = 1/k), there is a
distinct advantage to using Wigren’s approach: (5) converges
very slowly, as is shown in Fig. 5. On the other hand, (7)
converges faster to the nominal parameters (Fig. 5). However,
the convergence speed and the estimation quality of LIMBO
when an appropriate constant regulative coefficient is used
are completely competitive to those obtained with Wigren’s
approach while αk = 1/k (Fig. 6).

Qualitatively similar results can be obtained for other
test cases: it means that the parameters of both methods
(y0 for Wigren’s approach, α for LIMBO) can always
be adjusted so that they converge to the nominal system
parameters with a similar speed, using αk = 1/k for (7)
and a constant regulative coefficient (α) for (5). However,
the computational cost of (5) is always smaller than that of
(7); only one division (5-c) is required in LIMBO with
a constant coefficient, whereas Wigren’s approach with a
monotonously decreasing coefficient requires at least two or
three divisions, depending on whether the pseudogradient
∂es_k/∂\theta_k is actually computed or tabulated.

\[ \begin{array}{|c|c|c|}
\hline
\alpha_k & 1 - w_{10^6} & 1 - w_{10^8} \\
\hline
\alpha = 1/k & 6.4e-4 & 2.5e-8 \\
\alpha = 0.01 & 7.04e-8 & 7.7e^{-6} \\
\alpha = 0.05 & 5.99e-9 & 1.74e-5 \\
\alpha = 0.1 & 3.54e-6 & 1.93e-5 \\
\alpha = 0.15 & 1.14e-6 & 1.77e-5 \\
\alpha = 0.2 & 3.5e-3 & 7.7e-5 \\
\hline
\end{array} \]

![Fig. 4. Comparison of LIMBO and Wigren’s method using a constant regulative coefficient (α = 0.05) for two methods.](image)

![Fig. 5. Comparison of LIMBO and Wigren’s method using αk = 1/k for two methods.](image)
VI. CONCLUSION AND FUTURE WORKS

In this paper, LIMBO, an LMS-like method for estimating system parameters based on binary observations, was derived from the offline WLS approach presented in [3], [4]. Simulations showed that the results obtained when using a constant regulative coefficient in LIMBO were comparable, in terms of convergence speed and of estimation quality, to those obtained with the approach presented in [7], with a lesser computational complexity. This makes LIMBO an inexpensive test method, requiring only little memory storage (as opposed to an offline method) and amenable to implementation on FPGAs or small processor cores as used in System-on-Chip applications.

Proof of the convergence of LIMBO to the nominal system parameters will be given in an oncoming paper.

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