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Abstract

In this paper, we consider a general bi-directional relay network with two sources and N relays when neither the source nodes nor the relays know the channel state information (CSI). A joint relay selection and analog network coding using differential modulation (RS-ANC-DM) is proposed. In the proposed scheme, the two sources employ differential modulations and transmit the differential modulated symbols to all relays at the same time. The signals received at the relay is a superposition of two transmitted symbols, which we call the analog network coded symbols. Then a single relay which has minimum sum SER is selected out of N relays to forward the ANC signals to both sources. To facilitate the selection process, in this paper we also propose a simple sub-optimal Min-Max criterion for relay selection, where a single relay which minimizes the maximum SER of two source nodes is selected. Simulation results show that the proposed Min-Max selection has almost the same performance as the optimal selection, but is much simpler. The performance of the proposed RS-ANC-DM scheme is analyzed, and a simple asymptotic SER expression is derived. The analytical results are verified through simulations.

Index Terms

Differential modulation, bi-directional relaying, analog network coding, amplify-and-forward protocol

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I. Introduction

In a bi-directional relay network, two source nodes exchange their messages through the aid of one or multiple relays. The transmission in bi-directional relay network can take place over either four, three or two time slots. In the four time slots transmission strategy, the relay helps to forward source S_1's message to source S_2 in the first two time slots and source S_2's message to source S_1 in the next two time slots. Four time slots transmission has been shown to be very inefficient. When the relay receives two sources' messages, it combines them before forwarding to the destination, which will save one time slot transmission. This three time slots transmission scheme is usually referred to as the digital network coding [1]–[3]. In this method, two source nodes transmit to the relay, separately. The relay decodes the received signals, performs binary network coding, and then broadcasts it back to both source nodes.

To further improve the spectral efficiency, the message exchange between two source nodes can actually take place in two time slots. In the first time slot, both source nodes transmit at the same time so that the relay receives a superimposed signal. The relay then amplifies the received signal and broadcasts it to both source nodes in the second time slots. This scheme is referred to as the analog network coding (ANC) [4]–[6]. Various transmission schemes and wireless network coding schemes in bi-directional relay networks have been analyzed and compared in [7]–[12].

Most of existing works in bi-directional relay communications consider the coherent detection at the destination and assume that perfect channel state information (CSI) are available at the sources and relays [1]–[12]. In some scenarios, e.g. the fast fading environment, the acquisition of accurate CSI may become difficult. In this case, the non-coherent or differential modulation would be a practical solution. In a differential bi-directional relay network, each source receives a superposition of differentially encoded signals from the other source, and it has no knowledge of CSI of both channels. All these problems present a great challenge for designing differential modulation schemes in two-way relay channels.

To solve this problem, in [13], a non-coherent receiver for two-way relaying was proposed for ANC based bi-directional relay networks. However, the schemes result in more than 3 dB performance loss compared to the coherent detection. To further improve the system performance, a differential ANC scheme was proposed in [14] and a simple linear detector was developed to recover the transmitted signals at two source nodes. Results have shown that it only has about 3 dB performance loss compared to its coherent counterpart.

Recently, it has been shown that the performance of wireless relay networks can be further enhanced by properly selecting the relays for transmission [15]–[17]. Consequently, it is beneficial to design an effective relay selection scheme for the bi-directional transmission scheme with multiple relays as well in order to achieve spatial diversity. In this paper, we propose a bi-directional joint relay selection and analog network coding using differential modulation (RS-ANC-DM) so that the CSI is not required at both sources and the selected relay. In the proposed BRS-DS-ANC scheme, two source nodes first differentially encoder their messages and then broadcast them to all the relays at the same time. The signals received at the relay is a superposition of two transmitted symbols. Then a single relay which minimizes the sum SER of two source nodes is selected out of all relays to forward the ANC signals to both sources. Each source node then performs the differential detection and subtract its own message to recover the message transmitted by the other source node.

The performance of optimal relay selection is very difficult to analyze. To facilitate the analysis and selection procedure, in this paper we also propose a simple sub-optimal Min-Max criterion for relay selection, where a single relay which minimizes the maximum BER of two source nodes is selected. Simulation results show that the proposed Min-Max selection has almost the same performance as the optimal selection, but is much simpler. The performance of the proposed BRS-ANC scheme is analyzed, and an asymptotic SER expression is derived. The analytical results are verified through simulations.

The rest of the paper is organized as follows: In Section II, we describe the proposed BRS-DS-ANC scheme. The performance of the RS-ANC-DM is analyzed Section III. Simulation results are provided in Section IV. In Section V, we draw the conclusions.

Notation: Boldface lower-case letters denote vectors, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose, and conjugate transpose, respectively. $\mathbb{E}$ is used for expectation, $\mathtt{Var}$ represents variance, $\|\mathbf{x}\|^2 = \mathbf{x}^H\mathbf{x}$, and $\Re(\cdot)$ denotes real part.

II. Joint Relay Selection and Analog Network Coding using Differential Modulation

We consider a general bi-directional relay network, consisting of two source nodes, denoted by S_1 and S_2, and N relay nodes, denoted by $R_1, \ldots, R_N$. We assume that all nodes are equipped with single antenna. In the proposed RS-ANC-DM scheme, each message exchange between two source nodes takes place in two phases, as shown in Fig. 1. In the first phase, both source nodes simultaneously send the differentially encoded information to all relays and the signal received at each relay is a superimposed signal. In the second phase, an optimal relay node is selected to forward the received signals to two source nodes and all other relay nodes keep idle.

A. *Differential Encoding and Decoding in Two-Way Relay Channels*

Let $c_i(t)$, $i = 1, 2$, denote the symbol to be transmitted by the source S_i at the time t. We consider a MPSK modulation and assume that $c_i(t)$ is chosen from a MPSK constellation of unity power $\mathcal{A}$. Source i first differentially encodes the information symbols $c_i(t)$

$$s_i(t) = s_i(t-1)c_i(t) \tag{1}$$

The differential encoded signals are then simultaneously transmitted by two source nodes with unit transmission power to all the relays. The signal received in the k-th relay at time t can be expressed as

$$y_{r,k}(t) = h_{1,k}s_1(t) + h_{2,k}s_2(t) + n_{r,k}(t), \tag{2}$$

where $h_{i,k}$, $i = 1, 2, k = 1, ..., N$, is the fading coefficient between S_i and R_k. In this paper, we consider a quasi-static fading channel for which the channels are constant within one frame, but change independently from one frame to another. $n_{r,k}(t)$ is a zero mean complex Gaussian random variable with two sided power spectral density of $N_0/2$ per dimension.

Upon receiving the signals, the relay R_k then processes the received signal and then forwards to two source nodes. Let $x_{r,k}(t)$ be the signal generated by the relay R_k and it is given by

$$x_{r,k}(t) = \beta_k y^*_{r,k}(t), \tag{3}$$

where β_k is an amplification factor, so that the signal transmitted by the relay satisfy the following power constraint

$$\mathbb{E}(|x_{r,k}(t)|^2) = 1. \tag{4}$$

We should note that unlike the traditional ANC schemes [4]–[6], the relay forwards the conjugate of the received signal. The reason of doing this is to facilitate the differential detection at the destination [14].

Substituting Eqs. (2) and (3) into (4), we can derive β_k,

$$\beta_k = \sqrt{\frac{1}{|h_{1,k}|^2 + |h_{2,k}|^2 + N_0}} \tag{5}$$

However, as the relay has no CSI, β_k has to be obtained in other way. Let $\mathbf{y}_{r,k} = [y_{r,k}(1), \ldots, y_{r,k}(L)]^T$, $\mathbf{s}_1 = [s_1(1), \ldots, s_1(L)]^T$, $\mathbf{s}_2 = [s_2(1), \ldots, s_2(L)]^T$, $\mathbf{n}_{r,k} = [n_{r,k}(1), \ldots, n_{r,k}(L)]^T$, where L is the frame length. Then we can rewrite the received signals in (2) in a vector format as follows

$$\mathbf{y}_{r,k} = h_{1,k}\mathbf{s}_1 + h_{2,k}\mathbf{s}_2 + \mathbf{n}_{r,k}, \tag{6}$$

and β_k can be then approximated by the k-th relay node as

$$\beta_k = \sqrt{\frac{\mathbb{E}\{\mathbf{y}_{r,k}^H \mathbf{y}_{r,k}\}}{L}} \approx \sqrt{\frac{\mathbf{y}_{r,k}^H \mathbf{y}_{r,k}}{L}}, \tag{7}$$

After deriving β_k, the relay R_k then forwards $x_{r,k}(t)$ to two source nodes. Since S_1 and S_2 are mathematically symmetrical, for simplicity, in the next we only discuss the decoding as well as the analysis for signals received by S_1. The signal received by S_1 at time t, denoted by $y_{i,k}(t)$, can be written as

$$\begin{aligned} y_{1,k}(t) &= \beta_k h_{1,k} y_{r,k}^*(t) + n_{i,k}(t) \\ &= \mu_k s_1^*(t) + \nu_k s_2^*(t) + w_{i,k}(t) \\ &= \mu_k s_1^*(t) + \nu_k s_2^*(t-1) c_2^*(t) + w_{1,k}(t), \end{aligned} \tag{8}$$

where $\mu_k \triangleq \beta_k |h_{1,k}|^2 > 0$, $\nu_k \triangleq \beta_k h_{1,k} h_{2,k}^*$, and $w_{1,k}(t) \triangleq \beta_k h_{1,k} n_{r,k}^*(t) + n_{1,k}(t)$.

Since $s_1(t)$ is known to the source S_1, to decode $c_2(t)$, S_1 needs to estimate μ_k and ν_k. Let $\mathbf{y}_{1,k} = [y_{1,k}(1), \ldots, y_{1,k}(L)]^T$ and $\mathbf{w}_{1,k} = [w_{1,k}(1), \ldots, w_{1,k}(L)]^T$. Then at high SNR, we can obtain the following approximation

$$\mu_k^2 + |\nu|_k^2 \approx \frac{\mathbf{y}_{1,k}^H \mathbf{y}_{1,k}}{L}. \tag{9}$$

Since the source node S_1 can retrieve its own information $s_1(t-1)$ and $c_1(t)$, we have

$$\begin{aligned} \widetilde{y}_{1,k}(t) &\triangleq c_1^*(t) y_{1,k}(t-1) - y_{1,k}(t) \\ &= \nu_k s_2^*(t-1) \left(c_1(t) - c_2(t)\right)^* + \widetilde{w}_{1,k}(t), \end{aligned} \tag{10}$$

where $\widetilde{w}_{1,k}(t) \triangleq c_1(t) w_{1,k}(t-1) + w_{1,k}(t)$. Then, $|\nu_k|^2$ can be approximated as [14]

$$|\nu_k|^2 \approx \frac{\widetilde{\mathbf{y}}_{1,k}^H \widetilde{\mathbf{y}}_{1,k}}{L\mathbb{E}\left[|c_1(t) - c_2(t)|^2\right]}, \tag{11}$$

where $\widetilde{\mathbf{y}}_{1,k} = [\widetilde{y}_{1,k}(1), \ldots, \widetilde{y}_{1,k}(L-1)]^T$, and $\mathbb{E}[|c_1(t) - c_2(t)|^2]$ is a constant which can be pre-calculated by two source nodes, which is given in Appendix A. As μ_k is positive, it can be derived from (9) and (11) as

$$\mu_k \approx (\Theta_k)_+ \tag{12}$$

where $\Theta_k \triangleq \frac{\mathbf{y}_{1,k}^H \mathbf{y}_{1,k}}{L} - |\nu_k|^2$, $(X)_+$ is equal to X when $X \geq 0$ and otherwise is equal to 0.

By subtracting $\mu_k s_1^*(t)$, (8) can be further written as

$$\begin{aligned} y'_{1,k}(t) &\triangleq y_{1,k}(t) - \mu_k s_1(t) \\ &= \nu_k s_2^*(t-1) c_2^*(t) + w_{1,k}(t) \\ &= \left(y'_{1,k}(t-1) - w_{1,k}(t-1)\right) c_2^*(t) + w_{1,k}(t). \end{aligned} \tag{13}$$

Finally, the following linear detector can be applied to recover $c_2(t)$

$$\widetilde{c}_2(t) = \arg\max_{c_2(t)\in\mathcal{A}} \text{Re}\left\{y'^*_{1,k}(t-1) y'_{1,k}(t) c_2(t)\right\}. \tag{14}$$

B. Relay Selection with Differential Modulation in Two-Way Relay Channels

In the proposed RS-ANC-DM scheme, only one best relay is selected out of N relays to forward the received ANC signals in the second phase transmission. We assume that at the beginning of each transmision, some pilot symbols are transmitted by two source nodes to assist in the relay selection. One source node (either source S_1 or S_2) will determine the one best relay according to a certain criterion and broadcast the index of the selected relay to all relays. Then only the selected relay is active in the second phase of transmission and the rest of relays will keep idle.

1) Optimal Single Relay Selection: For the optimal single relay selection, a single relay node, which minimizes the sum SERs of two source nodes, i.e. $\text{SER}_{1,k} + \text{SER}_{2,k}$, will be selected, where $\text{SER}_{1,k}$ and $\text{SER}_{2,k}$ represent the SERs at source nodes S_1 and S_2, respectively. The main challenges in relay selection for differential modulation as mentioned before is that the relay node is determined by only one source (either S_1 or S_2) without the knowledge of any CSI. For simplicity and without loss of generality, in the next we use S_1 to select the optimal relay node. Obviously, the main difficulty here is to estimate $\text{SER}_{2,k}$.

For M-PSK constellations, the conditional SER assoicated with the k-th relay at the source S_1 is given by [18]

$$\text{SER}_{1,k}(h_{1,k}, h_{2,k}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{g_{\text{psk}}\gamma_{d_1,k}}{\sin^2\theta}\right) d\theta, \tag{15}$$

where $\gamma_{d1,k}$ is the effective SNR at the source S_1 and $g_{\text{psk}} \triangleq \sin^2\frac{\pi}{M}$. As CSI is unknown to the receiver, the effective SNR $\gamma_{d1,k}$ has to be estimated without knowledge of CSI.

By ignoring the second order term, the corresponding SNR of the proposed differential detection scheme in (13) can be written as

$$\begin{aligned}\gamma_{d_1,k} &\approx \frac{|\nu_k|^2}{\mathtt{Var}\{2w_{1,k}(t)\}} \\ &\approx \frac{\beta_k^2|h_{1,k}|^2|h_{2,k}|^2}{2\beta_k^2 N_0|h_{1,k}|^2+2N_0} \\ &\approx \frac{\psi_r\psi_s|h_{1,k}|^2|h_{2,k}|^2}{\psi_r|h_{1,k}|^2+\psi_s|h_{2,k}|^2},\end{aligned} \tag{16}$$

where $\mathtt{Var}\{w_{1,k}(t)\} \approx \beta^2 N_0|h_{1,k}|^2 + N_0$, $\psi_s \triangleq \frac{1}{4N_0}$, and $\psi_r \triangleq \frac{1}{2N_0}$.

Recalling $\mu_k \triangleq \beta_k|h_{1,k}|^2$, $\nu_k \triangleq \beta_k h_{1,k}h_{2,k}^*$, and their corresponding estimates in (11) and (12), $\gamma_{d1,k}$ in (16) can be further calculated as

$$\gamma_{d_1,k} \approx \frac{|\mu_k|^4|\nu_k|^2}{2(2|\mu_k|^2+|\nu_k|^2)(|\mu_k|^2+|\nu_k|^2)N_0}, \tag{17}$$

Similar to (17), the SNR of the proposed differential detection scheme in the source S_2 can be written as

$$\begin{aligned}\gamma_{d_2,k} &\approx \frac{\psi_r\psi_s|h_{1,k}|^2|h_{2,k}|^2}{\psi_r|h_{2,k}|^2+\psi_s|h_{1,k}|^2} \\ &\approx \frac{|\mu_k|^4|\nu_k|^2}{2(2|\nu_k|^2+|\mu_k|^2)(|\mu_k|^2+|\nu_k|^2)N_0}.\end{aligned} \tag{18}$$

And its SER can be then calculated as

$$\mathrm{SER}_{2,k}(h_{1,k},h_{2,k}) = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} \exp\left(-\frac{g_{\mathrm{psk}}\gamma_{d_2,k}}{\sin^2\theta}\right)d\theta. \tag{19}$$

Among all relays, the destination will select one relay, denoted by $\mathcal{R}$, which has the minimum destination SER:

$$\mathcal{R} = \min_k\left\{\mathrm{SER}_{1,k}(h_{1,k},h_{2,k}) + \mathrm{SER}_{2,k}(h_{1,k},h_{2,k})\right\}, k \in 1,\ldots,N \tag{20}$$

2) Sub-Optimal Single Relay Selection: The optimal single relay selection scheme described in the above section is very difficult to analyze. In this section we propose a sub-optimal single relay selection scheme. It is well-known that the sum SERs of two source nodes ($\mathrm{SER}_{1,k}+\mathrm{SER}_{2,k}$) is typically dominated by the SER of the worst user. As a result, for low complexity, the relay node, which minimizes the maximum SER of two users, can be selected to achieve the near-optimal SER performance. We refer to such a selection criterion as the Min-Max selection criterion. Let $\mathcal{R}$ denote the selected relay. Then the Min-Max selection can be formulated as follows,

$$\mathcal{R} = \min_k \max\left\{\mathrm{SER}_{1,k}(h_{1,k},h_{2,k}), \mathrm{SER}_{2,k}(h_{1,k},h_{2,k})\right\}, k \in 1,\ldots,N, \tag{21}$$

which can be further formulated by using the effective SNRs

$$\mathcal{R} = \max_{k} \min\{\gamma_{d_1,k}, \gamma_{d_2,k}\}, k \in 1, \ldots, N, \tag{22}$$

where the calculation of $\gamma_{d_1,k}$ and $\gamma_{d_2,k}$ can be obtained from (17) and (18), respectively.

III. Performance Analysis of RS-ANC-DM scheme Based on Min-Max Selection Criterion

In this section, we derive the analytical average SER of the proposed differential bi-directional relay selection schemes. As mentioned before, the optimal relay selection scheme is very difficult to analyze. As it will be shown later, the Min-Max selection scheme proposed in section II has almost the same performance as the optimal selection scheme. Therefore, in this section, we will analyze the RS-ANC-DM scheme based on the Min-Max selection criterion.

A. Sub-Optimal Relay Selection

For the Min-Max selection criterion, the effective SNR of the selected relay $\mathcal{R}$ can be expressed as follows,

$$\gamma_{\mathcal{R}} = \max \min\{\gamma_{d_1,k}, \gamma_{d_2,k}\}, k \in 1, \ldots, N. \tag{23}$$

Now let us first calculate the PDF of $\gamma_{\mathcal{R}}$. As $\gamma_{d_1,k}$ and $\gamma_{d_2,k}$ are identically distributed, they have the same PDF and CDF, denoted by $f_{\gamma_k}(x)$ and $F_{\gamma_k}(x)$, respectively.

Define $\gamma_k^{\min} \triangleq \min\{\gamma_{d_1,k}, \gamma_{d_2,k}\}$. Let $f_{\gamma_k^{\min}}(x)$ and $F_{\gamma_k^{\min}}(x)$ represent its PDF and CDF, respectively. Then the PDF of $\mathcal{R}$ can be calculated by using order statistics as [19]

$$f_{\gamma_{\mathcal{R}}}(x) = N f_{\gamma_k^{\min}}(x) F_{\gamma_k^{\min}}^{N-1}(x) = 2N f_{\gamma_k}(x)(1 - F_{\gamma_k}(x))[1 - (1 - F_{\gamma_k}(x))^2]^{N-1}, \tag{24}$$

where $f_{\gamma_k^{\min}}(x) = 2f_{\gamma_k}(x)(1 - F_{\gamma_k}(x))$, $F_{\gamma_k^{\min}}(x) = 1 - (1 - F_{\gamma_k}(x))^2$, and $f_{\gamma_k}(x)$ can be found in [20]

$$f_{\gamma_k}(x) = \frac{2x \exp\left(-x(\psi_r^{-1} + \psi_s^{-1})\right)}{\psi_r \psi_s} \left[\frac{\psi_r + \psi_s}{\sqrt{\psi_r \psi_s}} \times K_1\left(\frac{2x}{\sqrt{\psi_r \psi_s}}\right) + 2K_0\left(\frac{2x}{\sqrt{\psi_r \psi_s}}\right)\right] U(x), \tag{25}$$

where $K_0(\cdot)$ and $K_1(\cdot)$ denote the zeroth-order and first-order modified Bessel functions of the second kind, respectively, and $U(\cdot)$ is the unit step function. At high SNR, when z approaches zeros, the $K_1(z)$ function converges to $1/z$ [21], and the value of the $K_0(\cdot)$ function is comparatively small, which could be ignored for asymptotic analysis. Hence, at high SNR, $f_{\gamma_k}(x)$ in (25) can be reduced as

$$f_{\gamma_k}(x) \approx \frac{\psi}{2} \exp\left(-\frac{\psi}{2}x\right), \tag{26}$$

where $\psi \triangleq 2(\psi_r^{-1} + \psi_s^{-1})$. Its corresponding CDF can be written as

$$F_{\gamma_k}(x) \approx 1 - \exp\left(-\frac{\psi}{2}x\right). \tag{27}$$

The PDF of $\gamma_{\mathcal{R}}$ can then be approximately calculated as

$$f_{\gamma_{\mathcal{R}}}(x) \approx N\psi \exp\left(-\psi x\right)\left[1 - \exp(-\psi x)\right]^{N-1}. \tag{28}$$

The CDF of $\gamma_{\mathcal{R}}$ can be approximately written as

$$F_{\gamma_{\mathcal{R}}}(x) \approx \left[1 - \exp(-\psi x)\right]^{N} \approx (\psi x)^{N}, \tag{29}$$

where $\lim_{x \to \infty} 1 - \exp(-x) = x$.

The SER conditioned on the instantaneous received SNR is approximately [22]

$$\mathrm{SER}(\gamma_{\mathcal{R}}|h_1, h_2) \approx Q\left(\sqrt{c\gamma_{\mathcal{R}}}\right), \tag{30}$$

where $Q(\cdot)$ is the Gaussian-Q function, $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-t^2/2)\mathrm{d}t$, and c is a constant determined by the modulation format, e.g. $c = 2$ for BPSK constellation.

The average SER can be then derived by averaging over the Rayleigh fading channels by

$$\mathrm{SER}(\gamma_{\mathcal{R}}) = \mathbb{E}\left[\mathrm{SER}(\gamma_{\mathcal{R}}|h_1, h_2)\right] = \mathbb{E}\left[Q\left(\sqrt{c\gamma_{\mathcal{R}}}\right)\right]. \tag{31}$$

By introducing a new random variable (RV) with standard Normal distribution $X \sim \mathcal{N}(0, 1)$, the average SER can be rewritten as [23]

$$\begin{aligned}
\mathrm{SER}(\gamma_{\mathcal{R}}) &= P\left\{X > \sqrt{c\gamma_{\mathcal{R}}}\right\} \\
&= P\left\{\gamma_{\mathcal{R}} < \frac{X^2}{c}\right\} \\
&= \mathbb{E}\left[F_{\gamma_{\mathcal{R}}}\left(\frac{X^2}{c}\right)\right] \\
&= \frac{1}{\sqrt{2\pi}}\left(\frac{\psi}{c}\right)^{N}\int_0^{\infty} x^{2N} \exp\left(-\frac{x^2}{2}\right)\mathrm{d}x.
\end{aligned} \tag{32}$$

Based on the fact that $\int_0^{\infty} t^{2n} \exp(-kt^2)\mathrm{d}t = \frac{(2n-1)!!}{2(2k)^n}\sqrt{\frac{\pi}{k}}$ [24], we can finally obtain

$$\mathrm{SER}(\gamma_{\mathcal{R}}) = \frac{(2N-1)!!}{2}\left(\frac{\psi}{c}\right)^{N}, \tag{33}$$

where $(2n-1)!! = \prod_{k=1}^{n} 2k - 1 = \frac{(2n-1)!}{n!2^n}$.

It clearly indicates in (33) that a diversity order of N can be achieved for the proposed RS-ANC-DM scheme in a bi-directional relay network with two sources and N relays.

IV. SIMULATION RESULTS

In this section, we provide simulation results for the proposed RS-ANC-DM scheme. We also include the corresponding coherent detection results for comparison. All simulations are performed with a BPSK modulation over the Rayleigh fading channels, and the frame length is $L = 100$. In order to calculate ν_k in (11), we use the normal constellation and the constellation rotation approaches introduced in Appendix A. For simplicity, we assume that S_1, S_2, and R_k ($k = 1, \ldots, N$) have the same noise variance.

A. Simulated Results

From Fig. 2 to Fig. 4, we present the simulated SER performance for the proposed RS-ANC-DM schemes. The performance of the corresponding coherent detection are plotted as well for better comparison. In Fig. 2, we compare the optimal relay selection method and the sub-optimal Min-Max relay selection method. It can be observed from the figure that the proposed Min-Max selection approach has almost the same SER as the optimal one. In particular, when the number of relay nodes increases, we almost cannot observe any difference between these two methods, which indicates that the Min-Max relay selection achieves near optimal single relay selection performance.

In Fig. 3, we compare the RS-ANC-DM scheme with its coherent detection counterpart. It can be noted that the differential scheme suffers about 3 dB performance loss compared to the corresponding coherent scheme. We can also see from Fig. 3 that the SER performance is significantly improved when the number of the relay increases. In Fig. 4, we include the Genie-aided result by assuming that μ is perfectly known by the source such that traditional differential decoding can be performed. It shows from the results that there is almost no performance loss using the estimation method in (12) which clearly justifies the robustness of the proposed differential decoder.

Fig. 5 compares the simulated SER performance for our proposed RS-ANC-DM and the non-coherent schemes [13] in bi-directional relaying without using constellation rotation, where $N = 1, 2, 4, 8$. It can be observed that our proposed scheme has much better performance than the detector in [13]. The main reason is that the non-coherent detection approach employed in [13] statistically averages off the impact of channel fading coefficients by ignoring the instantaneous channel state information and thus causes much performance loss. Comparatively, our proposed differential detection is a symbol by symbol based detection and is thus be able to adapt to the variation of the channel.

B. Analytical Results

In Fig. 6, we compare the analytical and simulated SER performance of the proposed differential modulation scheme.I n order to obtain fine estimation in (11), the signal constellation used by S_1 is rotated by $\pi/2$ relative to that by S_2. From the figure, it can be observed that at high SNR, the analytical SER derived by (33) is converged to the simulated result using optimal relay selection. This verifies the derived analytical expressions.

C. Constellation Rotation

In Fig 7, we examine the SER results of the proposed differential modulation scheme in comparison with the one without using constellation rotation, as shown in Appendix A, where the signal constellation used by S_1 is rotated by $\pi/2$ relative to that by S_2. It can be observed that the new result has very similar with the curve without rotating constellations. This indicates that using constellation rotation may not obtain any gains given large frame length.

V. Conclusions

In this paper, we proposed a joint relay selection and analog network coding using differential modulation over two-way relay channels when neither sources nor the relay has access to the channel state information. A simple Min-Max relay selection method is proposed and it has been shown that it achieves almost the same performance as the optimal single relay selection scheme. An asymptotic SER expression is derived. It is shown that the proposed RS-ANC-DM scheme can achieve the full diversity order of N for the system with N relays. Results are verified through simulations.

Appendix A
The Calculation of $\mathbb{E}[|c_1(t) - c_2(t)|^2]$ in (11)

From (11), it shows that the average power of $c_1(t) - c_2(t)$ needs to be calculated. When M-PSK constellations are applied, the number of symbols produced in the new constellations by $c_1(t)-c_2(t)$ is finite. Hence, it is easy to derive the average power of the new constellation sets. We refer to this as the normal constellation approach.

Note that the value of $c_1(t) - c_2(t)$ can be equal to zero, which may affect the estimation accuracy in (11). In order to overcome this problem, we may properly choose a rotation angle for the symbol modulated in source S_2 by $c_2(t)e^{-j\theta}$, ensuring that $c_1(t) - c_2(t)$ in

(11) is nonzero. For a M-PSK constellation, the effective rotation angle is in the interval $[-\pi/M, \pi/M]$ from the symmetry of symbols. For a regular and symmetrical constellation, the rotation angle may be simply set as $\theta = \pi/M$. Similar approach may be used to generate the rotation angle for other types of constellations.

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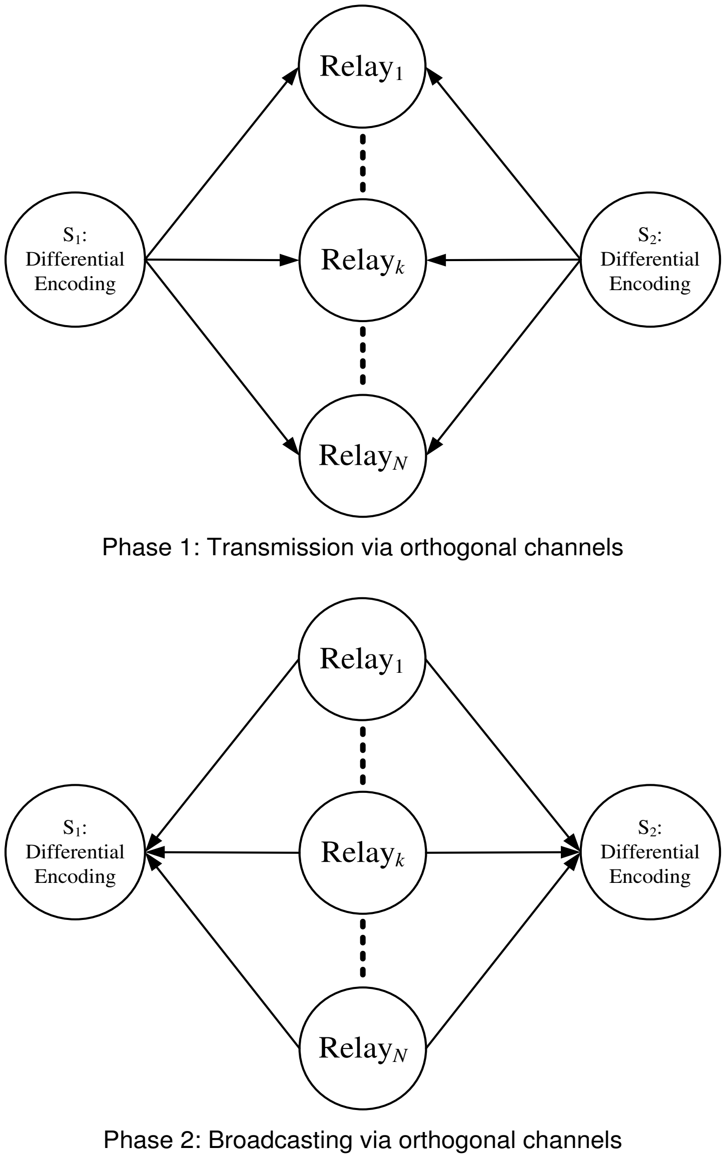

Fig. 1. Block diagram of the proposed BRS-DANC scheme.

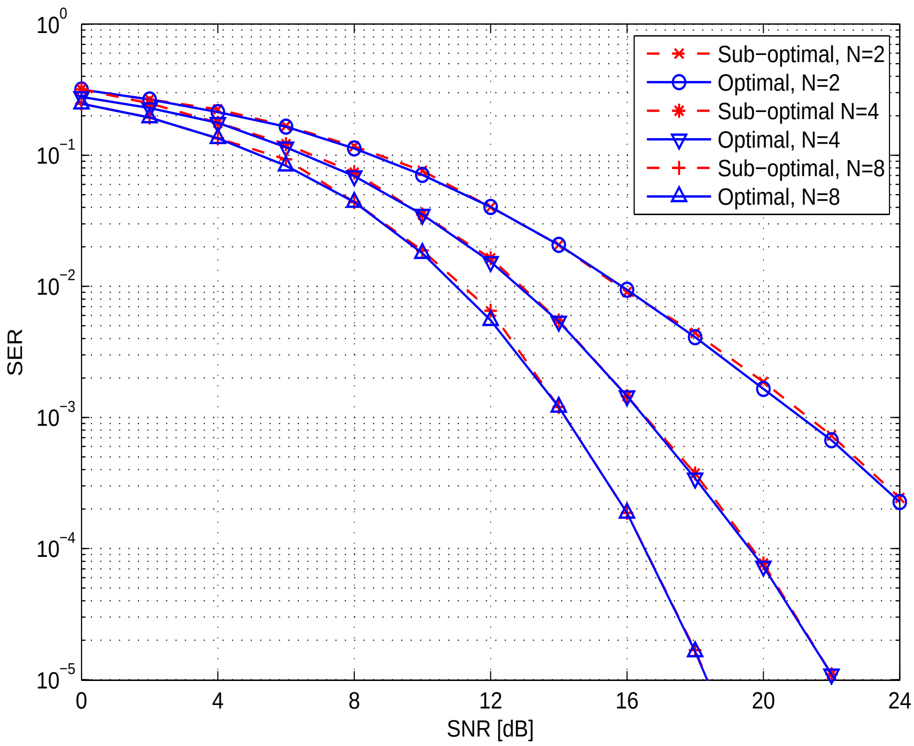

Fig. 2. Simulated SER performance by optimal and sub-optimal detections.

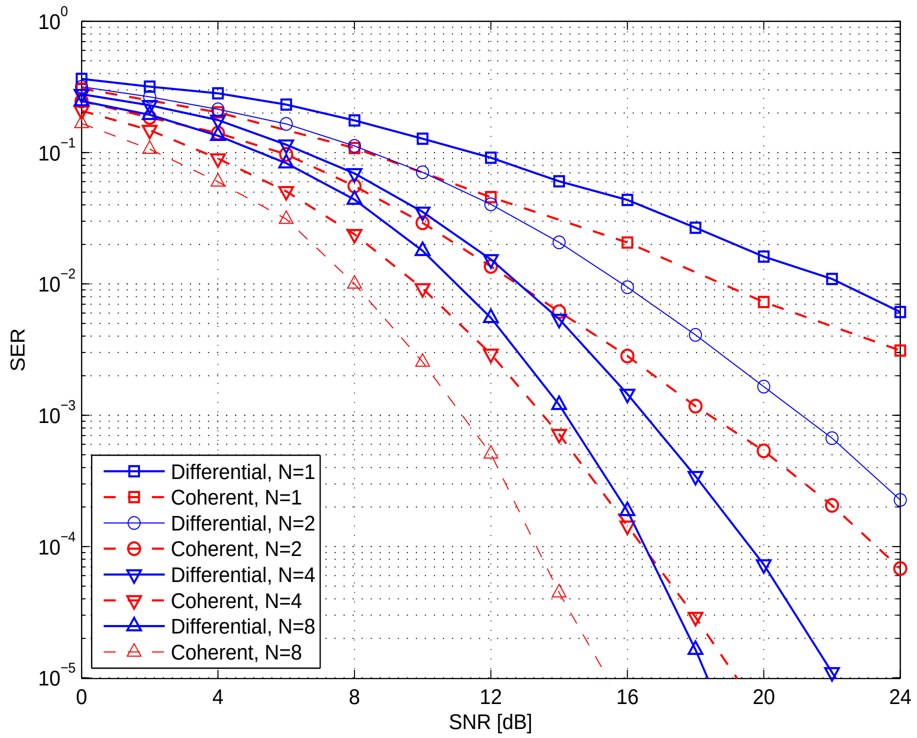

Fig. 3. Simulated SER performance by differential and coherent detections, where $N = 1, 2, 4, 8$.

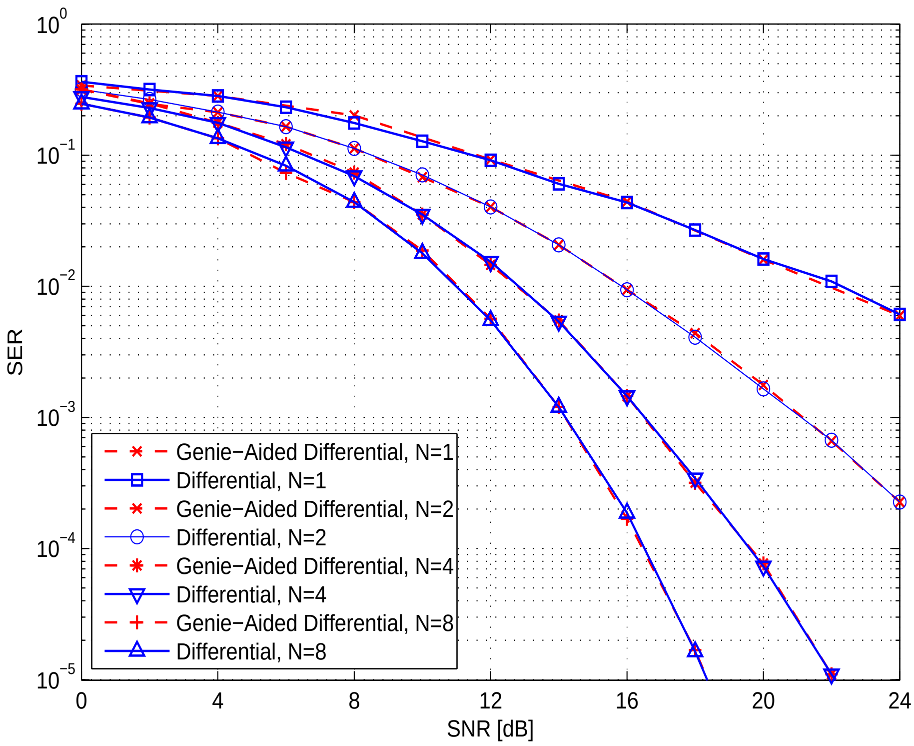

Fig. 4. Simulated SER performance by differential and Genie-aided detections, where $N = 1, 2, 4, 8$.

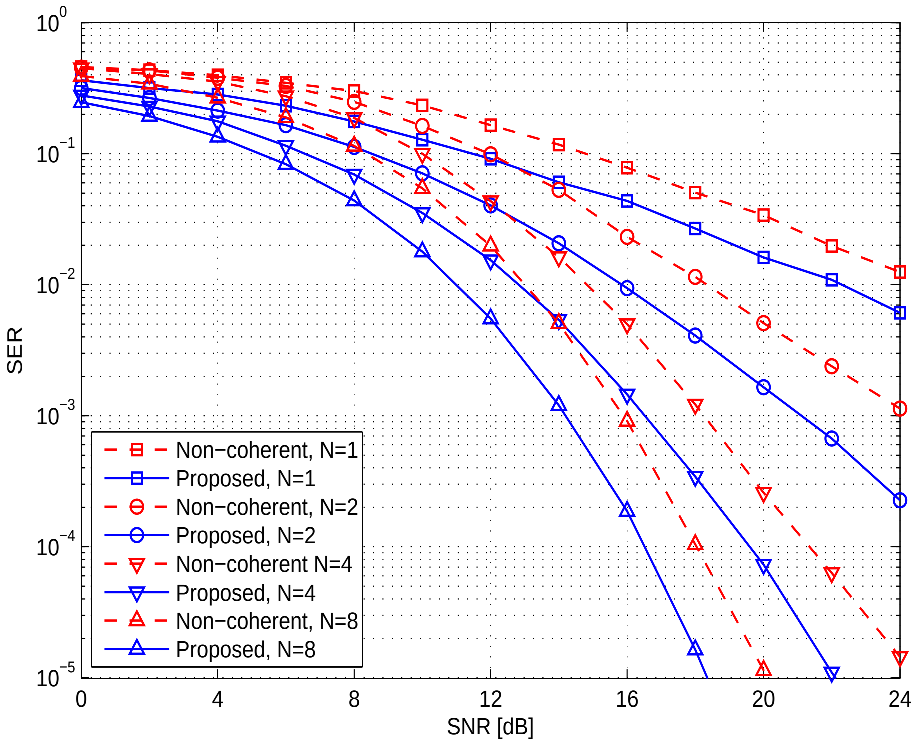

Fig. 5. Simulated SER performance comparisons by our proposed differential approach and the non-coherent scheme in [13], where $N = 1, 2, 4, 8$.

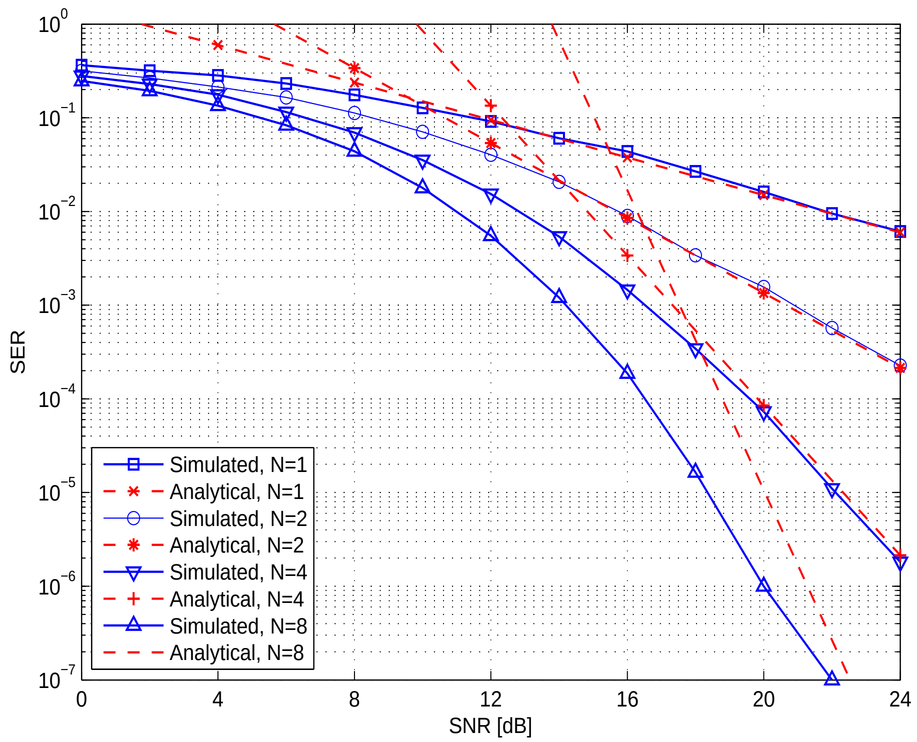

Fig. 6. Analytical and Simulated SER performance by the proposed differential scheme, where $N = 1, 2, 4, 8$.

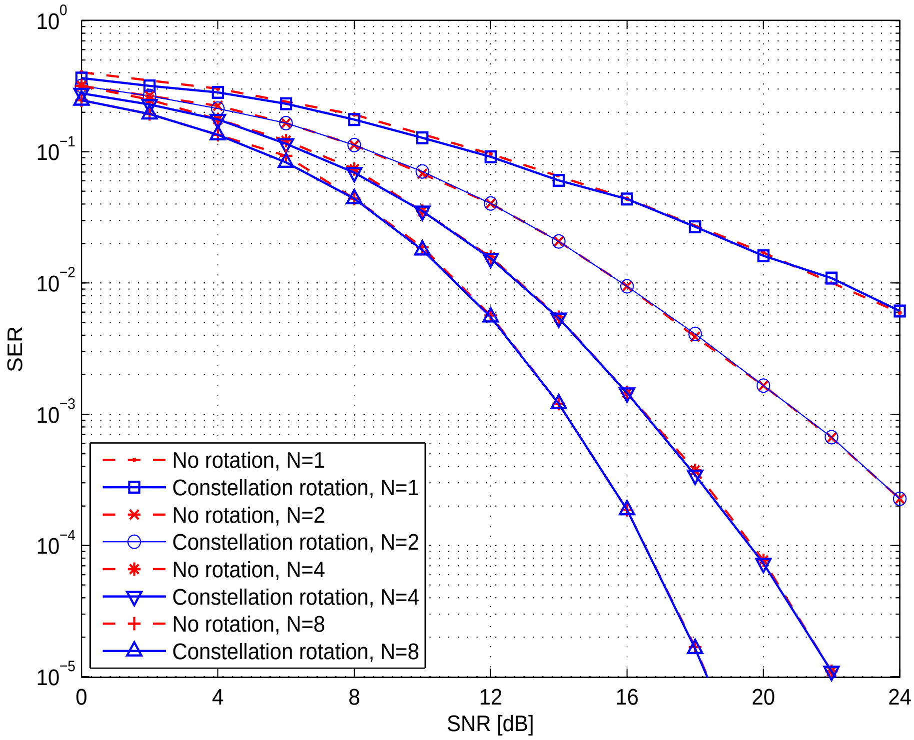

Fig. 7. Simulated SER performance by the proposed differential scheme with and without using constellation rotation, where $N = 1, 2, 4, 8$.