Transverse Mode Switching and Locking in Vertical-Cavity Surface-Emitting Lasers Subject to Orthogonal Optical injection
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Abstract—In this paper, we report on theoretical and experimental investigation on polarization and transverse mode behavior of vertical-cavity surface-emitting lasers (VCSELs) under orthogonal optical injection as a function of the injection strength and of the detuning between the injection frequency and the free-running frequency of the solitary laser. As the injection strength increases the VCSEL switches to the master laser polarization. We find that the injection power necessary to obtain such polarization switching is minimum at two different values of the frequency detuning: the first one corresponds to the frequency splitting between the two linearly polarized fundamental transverse modes, and the second one appears at a larger positive frequency detuning, close to the frequency difference between the first-order and the fundamental transverse modes of the solitary VCSEL. We show theoretically that both the depth and the frequency corresponding to the second minimum increase when the relative losses between the two transverse modes decrease. Bistability of the polarization switching is obtained for the whole frequency detuning range. Such a bistability is found for the fundamental mode only or for both transverse modes, depending on the value of the detuning. The theoretical and experimental optical spectra are in good agreement showing that the first-order transverse mode appears locked to the external injection.

Index Terms—Injection locking, nonlinear dynamics, polarization switching, transverse modes, vertical-cavity surface-emitting laser (VCSEL).

I. INTRODUCTION

Optical injection in semiconductor lasers is a research topic that has attracted much interest since the early 1980s [1], [2]. When injection locking is achieved, several improvements of the injected semiconductor laser characteristics are obtained like laser spectral narrowing [3], suppression of laser noise [4], reduction of frequency chirp under modulation [5], and improvement of the laser intrinsic frequency response [6]. Recent applications of optical injection include clock recovery by injection locking of passively mode-locked semiconductor lasers [7], waveform reshaping based on the transition between unlocked and locked laser state [8], optical generation of millimeter waves with low phase noise for the use in radio-over-fiber (RoF) systems [9], [10], optical injection phase-locked loop (OPLL) for synchronization of lasers [11], the development of light sources with reduced nonlinear distortion and improved spurious-free dynamic range (SFDR) for the use in analog fiber optic links [12]. Semiconductor lasers can be divided in two main categories depending on the dimensions and the geometry of the active cavity: edge-emitting lasers (EEL) and vertical-cavity surface-emitting lasers (VCSELs). VCSELs present significant advantages over their edge-emitting counterparts, including low threshold current, low cost, circular output beam, and easy fabrication in two-dimensional arrays. Although VCSELs are intrinsically single-longitudinal mode devices, emission in multiple transverse and polarization modes is usually found [13]. The polarization is not well fixed and small changes of the injection current or the device temperature may result in a polarization switching (PS) between the two linearly polarized modes. While emission in several transverse modes is usually attributed to (SHB) effects [13]–[15], a number of different physical mechanisms can be responsible for PS phenomenon in VCSELs. Therefore, different models of PS in VCSELs have been suggested, for example those taking into account spin relaxation mechanisms in semiconductor quantum wells spin flip model (SFM) [16], [17], thermal effects [18], or the relative modification of the net modal gain and losses with the injection current [19]–[21].

VCSELs are also attractive for use in injection locking because of its compactness, low power consumption, and circular output beam [22], [23]. Besides its fundamental research interest, optical injection in VCSELs can be used in all-optical and reconfigurable optical switches [24], to significatively increase the resonance frequency in VCSELs [22], to achieve transverse mode selection [25], or to perform several optical signal processing functions [26]. Stable injection locking within a large frequency detuning range has been observed when both the VCSEL and the injected light polarizations were parallel [27]. In that configuration, optical injection can be used to obtain chaotic instabilities in VCSELs [28]. A different configuration
was used in the seminal experimental work by Pan et al. [29]: linearly polarized light in the vertical polarization was injected from an external laser source in a VCSEL, which current was fixed in such a way that it emitted in a single mode with horizontal polarization. We shall call this configuration “orthogonal optical injection.” It was found that PS can be achieved through injection locking for certain values of wavelength detunings and injected power. A recent experimental study [30], [31] performed over a wider frequency detuning range (from ~82 to 89 GHz) revealed different regions of qualitatively different dynamical behaviors: a region of frequency locking, a region of PS with bistability, wave mixing, subharmonic resonance, time-periodic, and chaotic dynamics. Those different behaviors were summarized in a mapping of the bifurcation boundaries in the plane of the injection parameters: injection power $P_{\text{inj}}$ and frequency detuning between the external “master” laser (ML) and the injected “slave” laser (SL), $\Delta \nu = \nu_{\text{ML}} - \nu_{\text{SL}}$. That diagram was characterized by the existence of a minimum injection power needed to obtain PS, that appeared at a detuning corresponding to the frequency splitting between the two linearly polarized fundamental modes of the VCSEL. Similar results have been theoretically obtained by using a model for the two linear polarizations of the fundamental transverse mode of a VCSEL [32], [33].

In this work, the output characteristics of an oxide-confined AlGaAs–As quantum-well VCSEL under orthogonal optical injection have been mapped as a function of the optical injection strength and the frequency detuning between MLs and SLs. This work extends the previous measurements [30] over larger frequency detunings (up to 180 GHz). We find that a new minimum of the injection power needed to obtain PS appears at a positive detuning of around 150 GHz. This frequency detuning is near the frequency difference between the fundamental and the first-order transverse modes of the solitary VCSEL. The first-order transverse mode is then playing a key role in the creation of this new minimum. This has motivated us to perform a theoretical study by using a model that includes polarized multitransverse mode operation. We show that the frequency detuning at which the second minimum appears is indeed mainly determined by the frequency splitting between the fundamental and the first-order transverse modes. We find that the depth and the frequency position of that minimum increase as the relative losses between the two transverse modes decrease, in agreement with our experimental results. Our theoretical and experimental results also show that a bistability in PS is obtained for the whole frequency detuning range. Such a bistability is obtained for the fundamental mode only or for both transverse modes, depending on the value of the detuning. The theoretical optical spectra reveal that the first-order transverse mode appears locked to the external injection, also in an agreement with our experimental results.

Our paper is organized as follows. In Section II, the theoretical model is presented. In Section III, we present our theoretical results corresponding to an orthogonal optical injection in multitransverse mode VCSELs. In Section IV-A, we describe our experimental setup. In Section IV-B, we present the experimental results showing PS and transverse mode competition induced by the orthogonal optical injection. Finally, in Section V, a discussion on the agreement between theoretical and experimental results and a summary are presented.

II. MODEL

In this work, we consider a model [34], [35] that takes into account two of the mechanisms that can define the polarization of a VCSEL. The first one is associated with the combined effect of the VCSEL anisotropies, the linewidth enhancement factor and the spin-flip relaxation processes within a framework known as the SFM model [16]. The second mechanism is related to the effect of having different electrical field profiles for each linear polarization due to the birefringence of the device [36]. We consider cylindrically symmetric weak index-guided devices with the structure illustrated in the inset of Fig. 1(a). The radius of the core region and the length of the cavity are denoted as $a$ and $L$, respectively. Subscripts $x$ and $y$ will be used to denote the polarization direction. Birefringence is taken into account by assuming that the core refractive index in the $x$ direction, $n_{\text{core}}(x)$, is larger than in the $y$ direction, $n_{\text{core}}(y)$, hence, the $x$ polarized mode emission frequency is lower than that of the $y$ polarized mode, while the cladding refractive index, $n_{\text{clad}}$, is the same in both directions. We will consider a small value of the index step (0.011) in such a way that the appropriate transverse modes of the structure are the $LP_{01}$ modes. That index step is greater than the contribution due to the carrier-induced refractive index [37] and then the evolution obtained with our model, based on a modal expansion, coincides with the one obtained with a full spatiotemporal model [37]. Here we treat the case of VCSELs that can operate in the fundamental ($LP_{01}$) and in the first-order ($LP_{11}$) transverse modes. Subscripts 0,1 will be used to denote
the LP\textsubscript{01} and LP\textsubscript{11} modes, respectively. In the basis of the linearly polarized modes and considering radial symmetry of the cavity, the optical field can be written as [34, 35]
\[ \hat{E}(r, t) = \left( E_{0x}(r)\psi_{i1}(r) + iE_{0y}(r)\psi_{i2}(r) \right) e^{i\omega t} + \sum_{i,j} \left( E_{ij}(r)\psi_{ij}(r) \right) e^{i\omega_{ij} t} + c.c. \] (1)
where \( \psi_{ij} \) are the modal profiles of the LP\textsubscript{01} and LP\textsubscript{11} modes, respectively, obtained by solving the Helmholtz equation [34]; \( E_{0x} \) and \( E_{0y} \) are the modal amplitudes of these modes; the subindex \( j \) stands for the linear polarization state of the given mode; \( k \) is the electric field decay rate that includes the internal and facet losses [2]; and \( \eta \) is the alpha factor or linewidth enhancement factor that describes phase-amplitude coupling mechanisms in semiconductor lasers [2]. The equations describing the polarization and transverse mode behavior of the VCSEL with an injected optical field, written appropriately in the cylindrical basis, read [35]
\[ \dot{E}_{0x} = i\eta + (E_{0x}g_{0x} - 1) + iE_{0y}g_{0y} \right) e^{i\omega_{0x} t} + \sum_{i} \left( E_{ij}g_{ij} \psi_{ij} \right) e^{i\omega_{ij} t} \] (2)
Note that the modal gains for the \( x \) and \( y \) polarizations are different due to the different optical mode profiles. However, we neglect the material gain difference since the frequency splitting is very small compared to the width of the gain curve. The injection terms are characterized by the \( k_{ij} \) injection strengths, the VCSEL round-trip time, \( \tau_{in} = 2L/\omega_{in} \), where \( \omega_{in} \) is the group velocity, and the detuning \( \Delta \omega = \omega_{inj} - \omega_{1} \), where \( \omega_{inj} \) is the frequency of the ML and \( \omega_{1} = (\omega_{x} + \omega_{y})/2 \), is the central frequency between the two polarizations of the fundamental mode. The injection strength \( k_{ij} \) is given by
\[ k_{ij} = \left( \frac{1}{\sqrt{R}} - \frac{1}{\sqrt{R}} \right) \sqrt{\eta_{inj}} \sqrt{P_{inj,i,j}} \] (3)
where \( R \) is the output-mirror reflectivity, \( \eta_{inj} \) is the coupling efficiency of the injected light to the optical field in the laser cavity, and \( P_{inj,i,j} \) is the power injected in the \( j \)-polarization of the \( j \)-transverse mode [2]. The rest of the parameters that appear in the equations are specified in the Table I. The frequency splitting between the orthogonal polarizations of the LP\textsubscript{01} mode, \( 2\gamma_{00}/(2\pi) \), between the orthogonal polarizations of the LP\textsubscript{11} mode, \( 2\gamma_{11}/(2\pi) \), and between the two transverse modes with the same polarization, \( \gamma_{ij}^{00}/(2\pi) \), are obtained from the calculation of the waveguide modes via the Helmholtz equation. We have chosen the values of \( n_{core,x} \), \( n_{core,y} \), and \( n_{clad} \) in such a way that \( 2\gamma_{00}/(2\pi) = 2 \text{GHz} \) and \( \gamma_{ij}^{00}/(2\pi) = 193 \text{GHz} \).
by the terms $\xi_x$ taken as complex Gaussian white noise sources of zero mean and delta-correlated in time. In the noise terms, the carrier distribution is integrated over the active region

$$
\mathcal{N} = \frac{a}{a^2} \int_0^a N(r,t) r dr, \quad \mathcal{P} = \frac{a}{a^2} \int_0^a n(r,t) r dr.
$$

In the following section, we will present the results obtained by integrating numerically the previous set of equations. Time and space integration steps of 0.01 ps and 0.12 $\mu$m, respectively, have been used. The boundary conditions for the carrier distribution are taken as $N(\infty, t) = 0$, $n(\infty, t) = 0$. The initial conditions correspond to the below threshold stationary solution, i.e., the VCSEL emits in the fundamental mode until $I = 0.1 I_{th}$, where $I_{th}$ is the threshold current.

III. THEORETICAL RESULTS

We first present some results corresponding to the solitary VCSEL operation. The transverse and polarization mode-resolved light–current characteristics of the solitary VCSEL is shown in Fig. 1(a). The VCSEL begins to emit in the fundamental mode with the smallest frequency, $L_{P_{01},x}$. A type II PS, i.e., from the lower to the higher frequency, within the fundamental mode is obtained at around 2.5 times the threshold current. The VCSEL then emits in the $L_{P_{01},y}$ mode until the higher order mode with orthogonal polarization appears at around 4.7 $I_{th}$. The value of the relative losses of the $L_{P_{11}}$ with respect to the $L_{P_{01}}$ mode has been chosen ($\kappa_x = 1.2$) in such a way that the $L_{P_{11}}$ mode appears near the experimental value found for the solitary VCSEL (see Section IV). In Fig. 1(b), we also show the light–current characteristics obtained when decreasing the current in the same way. The VCSEL switches back to the $x$ direction at 1.83 $I_{th}$, hence demonstrating a hysteresis zone.

We now consider the effect of the orthogonal optical injection on the transverse modes and polarization behavior of the VCSEL. We first set the current at a value, $I = 1.7 I_{th}$, slightly smaller than the lowest current of the hysteresis region of the solitary VCSEL. Orthogonal optical injection is then modeled by choosing $k_{0x} = 0$ and $k_{0y} > 0$ since at that value of the current the VCSEL emits in the $x$ polarization of the $L_{P_{01}}$ mode. We show in Fig. 2 the boundaries of PS in the plane of injection parameters (the detuning and the injected power). The injected power has been normalized to the output power of the solitary VCSEL and has been taken in logarithmic scale. For each value of the frequency detuning we have performed a sweeping along the horizontal axis, that is, increasing and then decreasing the injected power. That sweeping is performed in the following way: we first let the solitary VCSEL to reach the steady state and then we change the injected power step by step. We consider 100 steps between the minimum and the maximum value of the injected power. We calculate the average of each polarized transverse mode over the last nanosecond of each step (of 2-ns duration). We consider that a polarization switch-on (switch-off) is obtained when the averaged total $y$-polarized power becomes larger (smaller) than the averaged total $x$-polarized power. The polarization switch-on (switch-off) points for increasing (decreasing) the power of the injected light are represented by solid (dotted) lines in Fig. 2, respectively. In Fig. 2(a), the results for the case of $I = 1.7 I_{th}$ and $\kappa_x = 1.2$ show that there are two minima in the injected power required for switching. Those minima appear at around 0 and 155 GHz for both the cases of increasing and decreasing the injection power. The detuning at which the second minimum appears is near the frequency difference between the $L_{P_{01}}$ and the $L_{P_{11}}$ transverse modes of the solitary VCSEL. The injection power for the PS-off is smaller than the one required for the PS-on and then a region of bistable operation of the device is obtained. Bistable operation is obtained over wider ranges of injection powers when the detuning is near the 0 GHz minimum.

The effect of the transverse mode dichroism on the mapping of PS is analyzed in Fig. 2(b). In that figure, we compare the results obtained with the previously described VCSEL ($\kappa_x = 1.2$) with another VCSEL in which the higher order modes are much more suppressed ($\kappa_y = 1.4$); in fact, the first higher order mode appears at $I = 17.5 I_{th}$ when $\kappa_y = 1.4$. The injected power required for PS keeps on having two minima. The PS boundaries do not change in a significant way if the detuning is smaller than 35 GHz. For higher values of the detuning, there are several changes: the minimum injected power required for switching increases and the detuning at which this minimum appears decreases to 127 GHz. The effect of the injection current.

![Graph](image)
is also analyzed in Fig. 2(c). In that figure, we have decreased the injection current applied to the VCSEL of Fig. 2(a) to a value that is clearly below the hysteresis region, \( I = 1.4 I_{th} \). No appreciable effects are observed for the switch-on curve when changing the injection current. However, the switch-off curve changes in such a way that the width of the hysteresis region decreases when decreasing the injection current.

The interpretation of some of the previous results can be obtained with the help of Figs. 3–5. In Fig. 3, the averaged power of the different transverse modes is plotted as a function of the injected power for three representative values of the frequency detuning: 13, 62, and 113 GHz. The upper (lower) part of the figure corresponds to \(-x\) (\(-y\)) polarized light. Those averaged powers are normalized to the power of the solitary VCSEL. The results corresponding to a frequency detuning of 13 GHz, slightly larger than the one of the first minimum, are shown in Fig. 3(a) and (b). The solitary VCSEL is mainly emitting in the \( \text{LP}_{01,x} \) mode. We first analyze the behavior obtained when increasing the injection power. When the injection power is less than \(-23\) dB, no appreciable changes are observed in the averaged modal powers. However, some changes are apparent from the optical spectra. Different optical spectra corresponding to representative values of the injection power when the detuning is 13 GHz are shown in Fig. 4. Fig. 4(a) shows that a contribution to the \(-x\)-polarized spectrum at the injection frequency appears for small values of the injection power. That contribution increases when the injection power increases. However the \(-x\)-polarized spectrum is very similar to the corresponding solitary spectrum. When the injection power increases beyond \(-23\) dB, the \( \text{LP}_{01,x} \) averaged power begins to decrease while the one of the \( \text{LP}_{01,y} \) begins to increase, as seen in Fig. 3(a) and (b). Fig. 4(b) shows that the increase of the \( \text{LP}_{01,y} \) power appears mainly at the injection frequency. Wave mixing is also observed by the peaks in the \(-x\)- and \(-y\)-polarization appearing at multiples of the frequency difference \( \nu_M - \nu_{01,x} \) between the ML frequency and the VCSEL \( \text{LP}_{01,x} \) mode. PS within the fundamental mode appears at \(-8\) dB and is illustrated in Fig. 3(a) and (b) and Fig. 4(c). After the PS, the VCSEL emits at a several frequencies at multiples of \( \nu_M - \nu_{01,x} \). Fig. 3(b) also shows that further increase of the injected power leads to the excitation of the \( \text{LP}_{01,y} \) mode as well.

A typical optical spectrum in this regime is the one shown in Fig. 4(e) that corresponds to an injection power of 0 dB. The optical spectrum is such that only one peak of the \(-y\)-polarized light at the injection frequency appears, i.e., injection locking has been achieved. The previously described situation changes when decreasing the injection power, as it can be seen in Fig. 3(a) and (b): a wide bistability region appears for the \( \text{LP}_{01,x} \) and \( \text{LP}_{01,y} \) modes. However, no bistability is observed for the \( \text{LP}_{11,y} \) mode. A value of the injection power of \(-20\) dB, in the middle of the hysteresis region has been chosen in Fig. 4(b) and (d) to illustrate the bistability by using optical spectra.

We now describe results for a detuning of 62 GHz that corresponds to the local maximum of the injection power required for
the PS-on in the mapping in Fig. 2(a). We first analyze the behavior obtained when increasing the injection power. A monotonic decrease of the LP_{01,x} averaged power together with a monotonic increases of the LP_{01,y} and LP_{11,y} averaged powers are obtained, as it can be seen in Fig. 3(c) and (d). PS-on occurs at around 3.5-dB injection power. There is an appreciable contribution of the LP_{11,y} mode to that PS, in contrast to the 13-GHz detuning case. We also observe that there is a value of the injection power, around 8 dB, beyond which the main contribution to the optical power is given by the LP_{11,y} mode. The width of the bistable region of the LP_{11,y} mode is smaller than the one for the 13-GHz detuning case. However, bistability also appears for the LP_{11,y} mode. Typical optical spectra for the 62-GHz detuning case are illustrated in Fig. 5. Again, the component near the zero frequency (at the injection frequency) of the y-polarized spectrum decreases (increases) when increasing the injection power as it can be seen in Fig. 5(a) and (c). Optical spectra at 6.3 dB, beyond the PS point, are also illustrated in Fig. 5(b) and (d), for increasing and decreasing optical injection, respectively. Only one peak of the y-polarized light at the injection frequency appears in both figures. Comparison between those figures also illustrate the bistability regime because the x-polarized power has disappeared when decreasing the injection power.

The results obtained for a frequency detuning approaching the second minimum of Fig. 2(a) are shown in Fig. 3(e) and (f). Again, a PS appears at around 1 dB injection power but now the contribution of the LP_{01,y} power to that switching is small. In fact the switching is mainly performed between the LP_{01,x} and the LP_{11,y} modes. We also show that when the averaged power of LP_{01,x} vanishes the growth of the LP_{11,y} modes becomes much larger while there is a small range of injection power, around 3 dB, where the LP_{01,y} averaged power decreases. Fig. 3(e) and (f) also shows that the qualitative behavior obtained when decreasing the injection power is similar to the one obtained for the 62-GHz detuning case.
III. EXPERIMENTAL RESULTS

A. Experimental Setup

Experimentally, the orthogonal optical injection is achieved using the setup presented in Fig. 6. A quantum-well VCSEL that emits around 845 nm is used as a SL. Its temperature and bias current are controlled by a low-noise laser driver (CTR1). An external-cavity laser diode is used as an ML. The wavelength of the light emitted by the ML can be tuned within the range of 845–855 nm by another low-noise laser driver (CTR2). The injection beam from the ML is then focused on the SL using a lens (L) while the light emitted by the SL is collimated by another lens (COL). An isolator (ISO1) with 36–40 dB of attenuation achieves a unidirectional coupling between the ML and the SL. The strength of the injected beam is varied using a polarizer P1. A half-wave plate (HWP1) fixes the polarization of the injected light to be orthogonal to the polarization direction of the free-running VCSEL. A nonpolarizing 50/50 beam-splitter and a mirror M are used to align the SL and the ML with the detection branch. The polarization in which the measurements are performed is selected by a half wave plate HWP2. The second isolator ISO2 with 36–40 dB of attenuation together with two polarizers P2 and P3 prevents the VCSEL from feedback-induced instabilities that may be generated by the light reflected by the fiber-coupling device FC. A power meter PM is used to measure the power emitted by the ML or the SL whereas spectral measurements are performed using either an optical spectrum analyzer (OSA) or a Fabry-Pérot spectrometer associated with a photodetector D1 and amplifier (Ampl) coupled to a computer (PC).
B. Experimental Results

1) Light–Current Characteristics: The polarization-resolved light–current characteristics of the free-running VCSEL is presented in Fig. 7. If the bias current is increased [see Fig. 7(a)], the VCSEL first emits a fundamental linearly polarized vertical mode (y-LP vertical mode) with a threshold current of 1 mA. As the injection current is progressively increased, two successive PS between the orthogonal fundamental modes are found. The first switching (PS I) from the high-frequency mode (y-LP mode) to the mode with a lower frequency (x-LP mode) is observed for a bias current of 1.9 mA. The second switching point (PS II) is found at a bias current of 4.2 mA and corresponds to PS from the x-LP horizontal mode to the y-LP vertical mode. Before PS II point is reached, we observe an elliptically polarized state which corresponds to a bump in the LI curve. However, if the VCSEL bias current is decreased after PS II has been achieved [see Fig. 7(b)], such an elliptical state is not found. As also shown in Fig. 7(b), a large hysteresis region is associated to PS II. In fact, if we decrease the injection current after PS II has been achieved [see Fig. 7(b)], such an elliptical state is not found. As also shown in Fig. 7(b), a large hysteresis region is associated to PS II. In fact, if we decrease the injection current after PS II has been achieved, the VCSEL switches back to x-LP at a much lower current, i.e., 2.14 mA. It is worth noting that the VCSEL exhibits a monomode operation for bias currents less than 5 mA. For higher currents, excitation of the first-order transverse mode has been observed.

2) PS and Transverse Mode Competition: In the following section, the PS of the VCSEL with orthogonal optical injection is investigated experimentally in the plane of the frequency detuning versus injection power plane. The free-running VCSEL is biased at 2.105 mA [which is less than the lower limit of the hysteresis region associated to PS II—see Fig. 7(b)] and its temperature is stabilized at 20 °C. With these operating conditions, the free-running VCSEL emits a horizontal linearly polarized (LP) light. In Fig. 8, the mapping of the VCSEL subject to optical injection is presented for a very large positive detuning range, i.e., from 2 to 180 GHz. For a fixed detuning value, polarization-resolved dynamics as well as transverse mode competition are analyzed when the injection strength is scanned. If the injection strength is increased, and depending on the frequency detuning, different switching scenarios are resolved. A switching mechanism that involves the VCSEL fundamental orthogonal transverse modes, i.e., from the horizontal (x-LP) to the vertical (y-LP) mode, is observed for the whole frequency detuning range. The corresponding boundary is labeled by black triangles in Fig. 8. This boundary exhibits two minima for the switching power. A first minimum is located at a detuning of 2 GHz for which PS is achieved at 7.1 μW. A second minimum for the switching power is found for a detuning of 150 GHz and an injection power of 623.9 μW. It is worth mentioning that the second minimum is at much larger power than the one for a detuning of 2 GHz.

We analyze in more detail the transverse mode competition behavior for detunings ranging from 61 to 120 GHz (see Fig. 8). With increasing the injection power we first observe PS between the LP_{01,a} and the LP_{01,b} modes (denoted by black triangles). When increasing further the injection power we observe injection locking of the LP_{11,0} mode—it frequency locks to the one of the ML, together with suppression of the fundamental transverse mode LP_{01,y}. The corresponding injection locking boundary is denoted by black diamonds. Fig. 9 represents a sample of such a scenario for a detuning of 100 GHz by using the experimental optical spectra. The experimental spectra shown in this paper have been recorded in the vertical polarization direction only. As the injection strength is increased, the VCSEL is initially frequency pushed but still emits a horizontal x-LP mode Fig. 9(a). For a further increase in the injection strength, switching from horizontal x-LP to vertical y-LP fundamental mode is achieved, as shown by the sudden increase of the SL peak in vertical polarization direction in Fig. 9(b). By still increasing the injection strength, an abrupt injection locking of the
first-order transverse mode to ML with suppression of the fundamental mode is resolved [see Fig. 9(c)]. It is worth mentioning that for a much higher injection power, a relatively weak emission of the fundamental $y$-LP mode can be observed with still the injection locked LP$_{11y}$ mode dominating (Fig. 9(d)). Bistability is observed if the injection power is decreased after injection locking of the LP$_{11y}$ mode is achieved, i.e., the VCSEL unlocks at an injection strength less than the one necessary to induce the locking regime (see the boundary labeled with light gray squares in Fig. 8). The width of the bistable region associated to the locking of the LP$_{11y}$ mode decreases as we increase the detuning as indicated by the zone with a dark gray shadding in Fig. 8.

For frequency detunings larger than 120 GHz, injection locking of the LP$_{11y}$ mode accompanied by suppression of the fundamental transverse mode LP$_{01x}$ is not observed anymore (see Fig. 10). Fig. 10(a) represents the situation for which the VCSEL is under optical injection but the injection strength is not sufficient to induce PS. By increasing the injection level PS from $x$-LP to $y$-LP fundamental modes is achieved Fig. 10(b). A further increase in the injection strength leads to a strong competition between the LP$_{01y}$ and LP$_{11y}$ modes. The onset of such a mode competition is shown on the mapping in Fig. 8 by black circles which correspond to the observation of a progressive decrease of the intensity at the SL frequency and a relatively strong increase of power at the ML frequency [see Fig. 10(c)]. Again, at a much stronger injection, a weak increase of the intensity at the SL frequency, i.e., a recovery of the $y$-LP fundamental mode, has been observed [Fig. 10(d) and the inset]. As shown in Fig. 8, the transverse mode competition appears at much lower injection power for a detuning of 150 GHz, which corresponds to the second minimum of the switching power. For larger positive detunings up to around 165 GHz, the mode competition is still resolved but at progressively increasing injection levels. Above this detuning range and as we increase the injection power, PS between the fundamental modes is still observed but afterwards the VCSEL keeps emitting an unlocked $y$-LP fundamental mode.

V. DISCUSSION AND CONCLUSIONS

We now discuss the similarities and the differences between the experimental and theoretical results presented in this work. The experimental light–current curve shows a switching from the higher to the lower frequency [see Fig. 7(a) at a current around 2 mA] that does not appear in our theoretical results (see Fig. 1). That switching would also appear in our theoretical results if we would consider an additional physical mechanism with a significant influence on the polarization of the device, for instance, successive types I and II switchings have been observed when adding thermal [38] or polarization dependent gain/loss mechanisms [21] to SFM model. However, the absence of such switching in the theoretical results is not so important because the PS which we have been investigating is the second one, from the lower to the higher frequency polarization mode. In both, experiment and theory, this switching occurs within the fundamental mode of the device. Also in both cases small bumps in the $y$-polarization appear for currents smaller than the PS current due to the appearance of elliptically polarized states. Excitation of the first-order mode occurs at similar currents (at around five times the threshold current). A wide bistability region also appears in both cases in such a way that the switching from the $y$- to the $x$-polarization when decreasing the current is also similar (at around two times the threshold current). We then conclude that the essential qualitative features of the experimental light current characteristics are well described by our model.
to the theoretical ones. As shown in Section III, the detuning frequency at which the new minimum occurs depends on the relative losses between the two transverse modes. The theoretical relative losses in Fig. 11 correspond to a VCSEL that becomes multimode at 4.7 times threshold. This result is consistent with the experimental result in which the first-order mode appears at five times threshold. The experimental $P_{\text{inj}}$ required for PS at the second minimum at 150 GHz detuning, is 19.5 dB higher than the first one at 2 GHz detuning, while the corresponding theoretical quantity is 23.8 dB. Also, the experimental (theoretical) $P_{\text{inj}}$ required for PS at 150 GHz, is 16 (27) times smaller than the maximum one.

We find experimentally that, in the whole frequency detuning range we investigate, the PS involves the VCSEL fundamental orthogonal transverse modes, i.e., from the $LP_{01,x}$ to the $LP_{01,y}$ modes. However our theoretical results also unveil an additional possible scenario, in which a switching from the $LP_{01,x}$ to the $LP_{11,y}$ mode is observed when increasing the injection strength [see Fig. 3(e) and (f)]. Theoretically, we find that this second switching scenario appears for frequency detunings larger than 85 GHz and for a value of the injection strength larger than that leading to a switching between the orthogonal fundamental transverse modes [compare Fig. 3(a) and (b) with Fig. 3(e) and (f)]. We find that this second switching scenario appears at smaller frequency detunings when increasing the $\kappa_r$ parameter (65 GHz when $\kappa_r = 1.4$). In this way, a wider range of frequency detunings over which the $LP_{01,x}$ switches to the $LP_{01,y}$ modes would be obtained if the $\kappa_r$ parameter is decreased. Whether it is possible to observe this second switching scenario in experiment remains an interesting question for future investigations. We also find experimentally that the $LP_{11,y}$ mode appears locked to the injection. This is also the case for our theoretical results since our optical spectra

A comparison between the theoretical (Fig. 2) and the experimental (Fig. 8) mapping of the injection power required for PS shows that both theory and experiment feature a new minimum at a frequency detuning of around 150 GHz. A more detailed comparison is done in Fig. 11 where we have plotted the results from Fig. 2(a) in a linear horizontal axis. The injection power required for PS when increasing the injection power is plotted with black triangles. The experimental detuning frequencies at which the injection power is minimum or maximum are similar

![Fig. 10. Polarization mode competition when the detuning is fixed at 125 GHz. (a) VCSEL emits in horizontal before PS, $P_{\text{inj}} = 0.72$ mW. (b) After PS to vertical polarization, $P_{\text{inj}} = 2.23$ mW. (c) Injection locking of the $LP_{11,y}$ mode to the ML after a progressive decrease of power at the SL frequency $P_{\text{inj}} = 6.97$ mW. (d) Injection locking of the $LP_{11,y}$ mode with a relatively weak recovery of the VCSEL fundamental $LP_{01,y}$ mode $P_{\text{inj}} = 22.13$ mW.](image)

![Fig. 11. Injection power required for PS in a VCSEL subject to orthogonal optical injection. Switching from $x$- to $y$-polarization when increasing the injection power is shown with triangles. The injection power required for the $LP_{11,y}$ modal power to reach an appreciable value, 0.5, has been plotted if $P_{\text{inj}}$ is increased (diamonds) and if $P_{\text{inj}}$ is decreased (squares). Parameters correspond to those in Fig. 2(a).](image)
indicate locking of the LP_{11,0} mode can be achieved for all the considered frequency detuning range. Bistability in PS has been found in the experiment and theory and is demonstrated in Fig. 11 where the injection power required for the LP_{11,0} modal power to reach an appreciable value has been plotted when increasing (diamonds) and when decreasing (squares) the injection power. Theoretically, this bistability can be for the fundamental mode only or for both transverse modes. In such a way, our numerical results complement the experiment, which can not distinguish for the contribution to the PS and for the hysteresis of the two transverse modes separately.

To summarize, we have performed theoretical and experimental investigations of VCSELs subject to orthogonal optical injection as a function of the injection strength and of the frequency detuning between the master and the SLs. These investigations have extended previous experimental work over larger frequency detunings. Within this extended range, we have found a new minimum of the injection power needed to obtain PS that appears at a frequency detuning that is near the frequency difference between the fundamental and the first-order transverse modes of the solitary VCSEL. We have found that both the depth and the frequency position of this minimum increase when considering lasers that become multitransverse mode at lower injection currents. Our theoretical and experimental results have also shown that bistability in PS is obtained for all the frequency detuning range. Such a bistability is obtained for the fundamental mode only or for both transverse modes, depending on the value of the detuning. The theoretical and experimental optical spectra have shown that the first-order transverse mode appears locked to the external injection. Our theoretical model have captured most of the fundamental features of the experiment.

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REFERENCES
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