Reduced-order modelling of the Reynolds equation for flexible structures

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The accurate modelling of damping is essential to capture the dynamic behaviour of a MEMS device. Our interest lies in squeeze-film damping which models the behaviour of a fluid in small gaps between a fixed surface and a structure moving perpendicular to this surface (Fig.1). The lateral dimensions of the surfaces are large compared to the gap and the system is considered isothermal. Squeeze film damping is then governed by the Reynolds equation [1]:

$$\nabla(G^{.3}P\nabla P) = 12\mu\frac{\partial GP}{\partial t}, \qquad (1)$$

where $G(x,y,t)$ is the distance between the moving and fixed surface, $P(x,y,t)$ is the pressure, and μ is the viscosity. This equation is nonlinear and is often coupled to the equation governing the structural behaviour. For small excitation frequencies or amplitudes it behaves as a nonlinear damper. For larger amplitudes or frequencies, the gas has no time to flow away and the pressure builds up creating a stiffening effect coupled to the damping effect. Most existing reduced-order models of the Reynolds equation solve the linearized Reynolds equation based on the hypothesis of small pressure variations, rigidity of the moving plate [2] or/and of small displacements [3],[4],[5]:

$$\nabla^2 p - \frac{12\mu.}{G_0^2.P_0}\frac{\partial p}{\partial t} = \frac{12\mu}{G_0^3.}\frac{\partial G}{\partial t}, \qquad (2)$$

where G_0 is the gap corresponding to the operating point, P_0 the ambient pressure, and p the pressure variation. The methods considering flexible structures use a modal projection method [3], [5], the modes being the mechanical modes, to extract modal frequency-dependent damping and stiffening coefficients. To extend the case to large displacements, Mehner [5] gives an analytical expression of these coefficients as a function of mechanical modal coordinates established by fitting simulation data to different bias voltages. In [6] and [7], the nonlinear Reynolds equation is projected on pressure mode shapes. In [6], Hung extracts mode shapes from simulation data via proper orthogonal decomposition, which requires a heavy complete finite element simulation. In [7], Rewienski and White construct a projection base by concatenation of Krylov bases extracted from finite difference models linearized around different operating points chosen along a training trajectory.

The reduced-order model presented here is valid for flexible structures and large displacements, the only hypothesis made being small pressure variations. In order to obtain this reduced order model, (1) is transformed through a change of variables so that the spatial differential operator no longer depends on time. A reduced-order model of this transformed equation can then be obtained by projection on the eigenmodes of the spatial operator, in this case the Laplacian. This is clearly an advantage compared to the heavy construction cost implied by either the complexity of the finite element simulations [6], the number of them [5] or the necessity of a training trajectory [7].

In the paper, we propose to establish the reduced-order model and demonstrate its validity in the case of the forced excitation of a beam, by comparison with a finite difference model (Fig. 2). The validity of this approach and its extension to fully-coupled reduced-order modelling are also discussed.

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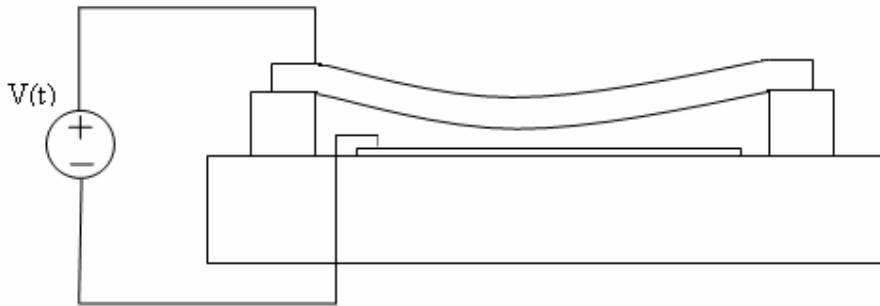
Fig.1 - Microswitch. When a voltage is applied to the beam, the electrostatic forces cause the structure to pull-in. Damping, which influences the switching time, plays a key role in such devices.
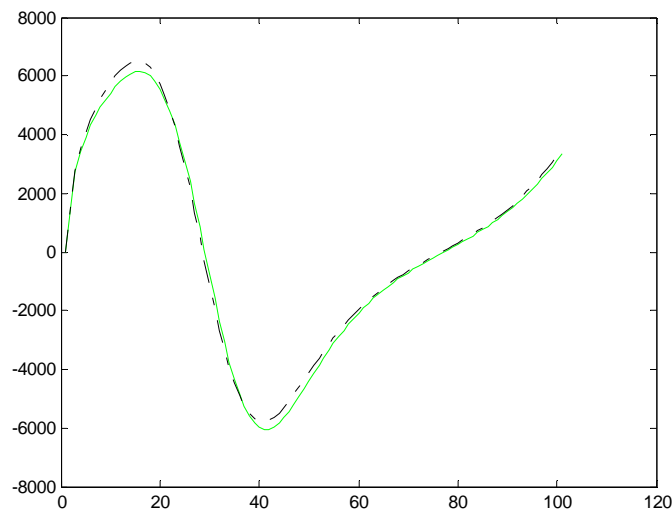

Fig. 2 - Pressure variation at the midpoint of a beam, as it is excited with a periodic force. The maximum amplitude of the displacements of the beam is equal to half the thickness of the fluid film. The dashed line corresponds to the results obtained with a finite difference model (55×25 nodes), whereas the continuous line corresponds to the proposed approach (5×3 modes)

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Keywords

Reduced-order modelling, squeeze-film damping, large displacements, modal projection techniques