Anti-aliasing Filter in Hybrid Filter Banks

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***Abstract*—** **Hybrid Filter Banks allow wide-band, high frequency conversion. All existing design methods suppose that the input signal is band-limited and that each sub-band signal is sampled at $1/M$ times the effective Nyquist frequency of the input signal $1/T$. To avoid aliasing in the sampling process, an analog anti-aliasing filter should be used in order to eliminate noise in frequency bands in which there is no signal (or a few signal) . In this paper, it is shown that this pre-filtering operation is critical and has to be done taking into account the respective power spectral densities of signal and noise due to the spectral aliasing with the sampling rate compressor. Results will be demonstrated for the design of a realistic 8 channel Hybrid Filter Bank.**

I. INTRODUCTION

In wireless systems and other domains, a trend of using higher data rates (up to 1 Gb/s for instance), high working frequencies and versatility can be noticed. Significant improvements have been achieved in the digital signal processing part of telecom systems, but the A/D conversion is still a bottleneck. To build very wide band ADCs, the parallelization of channels had been studied. Classical solutions such as time-interleaved ADCs are very sensitive to jitters, channel gain and phase mismatch errors... Hybrid Filter Banks (HFB) (Fig. 1) are very good candidates [6], [10], since they achieve an intrinsic parallel splitting without being subject to these drawbacks. Different methods of designing HFB have been proposed. Starting from a Digital Filter Bank prototype with perfect reconstruction and using a discrete to continuous time transform, Velasquez [10] transformed each digital filter of the analysis bank to a continuous-time correspondent. Olialei [4] also started with a digital prototype and found the continuous-time analysis filter by putting $TH_m(\mathrm{j}\Omega) = H_m(\mathrm{e}^{\mathrm{j}\omega})$ for $|\Omega T| < \pi$. In [11], [3], [7], [1], [8], starting from the knowledge of the frequency response of the analog filters, the digital synthesis filters are found after minimizing different error criteria. All the proposed design methods assume that the continuous time input signal $x(t)$ is bandlimited and that the analog M-band analysis filter bank is associated with M ADCs working at $1/M$ times the effective Nyquist frequency of the input signal to avoid aliasing in the sampling process [5]. In this paper, the effect of spectral aliasing in the sampling process is studied. It is shown that the analog pre-filtering used before the HFB in order to eliminate noise in frequency bands in which there is no signal (or a few signal) of interest should be particularly efficient to obtain a near perfect reconstruction. The results will be demonstrated for the design of a realistic 8 channel HFB structure.

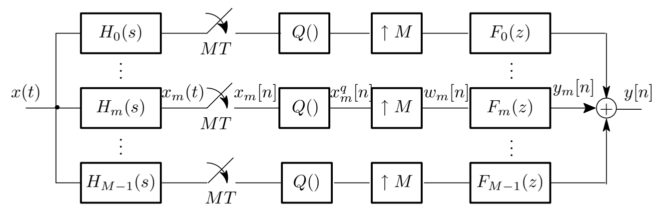

Fig. 1. General structure of an ADC using HFB with maximally-decimated architecture

II. HYBRID FILTER BANKS THEORY

Fig. 1 shows an example of an ADC using HFB. The input signal $x(t)$ is split into M subband signals $x_m(t)$, $m = 0, \dots, M-1$ via the M continuous-time analysis filters $H_0(s), H_1(s) \dots H_{M-1}(s)$. Each subband signal is down-sampled at $1/MT$, and quantized (block designed by $Q()$). Then, the digitized signals are up-sampled by M and the sampled version of the input signal is reconstructed via the discrete-time synthesis filters $F_0(z), F_1(z) \dots F_{M-1}(z)$.

The goal of the synthesis stage of the HFB is to reconstruct the original signal. Neglecting the effects of quantization, the synthesis stage of the HFB would have firstly to eliminate the aliasing terms due to down-sampling and secondly to reconstruct the original signal by compensating the distortion effects due to spectral aliasing with the sampling rate compressor and spectral imaging with the sample rate expander.

A. Frequency-domain analysis

The nonlinear part due to quantizers are hereby neglected. The Fourier transform of the output signal can be written:

$$Y(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{m=0}^{M-1} F_m(\mathrm{e}^{\mathrm{j}\omega})\, X_m(\mathrm{e}^{\mathrm{j}\omega M}), \tag{1}$$

with $\omega = \Omega T$ and:

$$X_m(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{MT}\sum_{k=-\infty}^{+\infty} H_m\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}k}{MT}\right) X\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}k}{MT}\right). \tag{2}$$

Noting $\widetilde{H}_m(\mathrm{j}\Omega)$ the $2\pi/T$ periodic extension of the analysis filter $H_m(\mathrm{j}\Omega)$ and $\widetilde{X}(\mathrm{j}\Omega)$ the $2\pi/T$ periodic extension of the input signal $X(\mathrm{j}\Omega)$, (1) can be rewritten as follows:

$$Y(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{m=0}^{M-1} T_m(\mathrm{e}^{\mathrm{j}\omega})\, \widetilde{X}\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}m}{MT}\right), \tag{3}$$

where:

$$T_m(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{MT}\sum_{k=0}^{M-1} \widetilde{H}_k\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}m}{MT}\right) F_k(\mathrm{e}^{\mathrm{j}\omega}). \tag{4}$$

$T_0(\mathrm{e}^{\mathrm{j}\omega})$ stands for the distortion function and $T_m(\mathrm{e}^{\mathrm{j}\omega})$, $m = 1, \ldots, M-1$ are the $(M-1)$ terms of the aliasing function.

B. Perfect reconstruction in band limited case

To make possible a perfect reconstruction for all input signals, the input signal $x(t)$ should be band-limited to $]l\pi/T\,,\,(l+2)\pi/T[$, $l \in \mathbb{Z}$ [5]. Therefore, if the input signal spectrum is null outside the frequency interval $]-\pi/T\,,\,\pi/T[$ for example, it is necessary to extend $H_m(\mathrm{j}\Omega)$ considering only its $\pm\pi/T$ duration. It is clear that there are only three replica of $\widetilde{X}_m(\mathrm{j}\Omega)$ which take part in $Y(\mathrm{e}^{\mathrm{j}\omega})$, $|\omega| < \pi$ through $\widetilde{X}_m(\mathrm{j}\Omega)$. These are $X_m(\mathrm{j}\Omega)$ itself and its replicas $X_m(\mathrm{j}\Omega \pm 2\pi/T)$. Therefore, for $|\omega| < \pi$ and $k = 0, \ldots, M-1$:

$$T_m(\mathrm{e}^{\mathrm{j}\omega}) = \frac{1}{MT}\sum_{k=0}^{M-1} H_k^s\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}m}{MT}\right) F_k(\mathrm{e}^{\mathrm{j}\omega}), \tag{5}$$

where:

$$H_k^s\left(\frac{\mathrm{j}\omega}{T}\right) = H_k\left(\frac{\mathrm{j}\omega}{T}\right) + H_k\left(\frac{\mathrm{j}\omega}{T} - \frac{2\pi\mathrm{j}}{T}\right) + H_k\left(\frac{\mathrm{j}\omega}{T} + \frac{2\pi\mathrm{j}}{T}\right) \tag{6}$$

For the case of perfect reconstruction for all input signals, the distortion function $T_0(\mathrm{e}^{\mathrm{j}\omega)}$ should be a pure delay all over the band and aliasing $T_m(\mathrm{e}^{\mathrm{j}\omega)}$ $(m = 1, \ldots, M-1)$ are undesirable terms. There will be M equations, which every equation is defined throughout a period of ω, for example $\pm\pi$:

$$\begin{aligned} T_0(\mathrm{e}^{\mathrm{j}\omega}) &= c\,\mathrm{e}^{-\mathrm{j}\omega\tau} && && |\omega| < \pi \\ T_m(\mathrm{e}^{\mathrm{j}\omega}) &= 0 && m = 1, \ldots, M-1 && |\omega| < \pi, \end{aligned} \tag{7}$$

where $\tau \in \mathbb{Z}^+$ is the filter bank's delay, $c \in \mathbb{R}$ is a scale factor.

C. Design method

Even if the input signal is band limited, analog filters cannot be band limited and the HFB design procedure leads to discontinuities of the synthesis frequency responses. Therefore, (7) cannot be exactly satisfied. Synthesis methods aim at finding an HFB that minimizes the distortion and aliasing. Several methods may be found in the literature. We chose the global frequency domain least square solving method [8] since it gives the best results in terms of distortion and aliasing.

The input signal is assumed band limited to $\pm\pi/T$. Starting with the knowledge of frequency responses of analog filters (for the sake of analog feasibility), the synthesis filter bank includes a set of M L-coefficients FIR filters to design.

Perfect reconstruction conditions (7) are then written for each of the N frequency points ω_n equally distributed in $\pi < \omega < \pi$. Noting $\mathbf{H}$ the $MN \times MN$ matrix of the frequency response of the analysis filters calculated at the selected frequency values and $\mathbf{F}$ the $MN \times 1$ matrix of the associated frequency response of the synthesis filters, (7) gives:

$$\mathbf{H}\,\mathbf{F} = \mathbf{t}, \tag{8}$$

with:

$$\mathbf{t} = c\left[\mathrm{e}^{-\mathrm{j}\omega_1\tau} \quad \ldots \quad \mathrm{e}^{-\mathrm{j}\omega_N\tau}\ \mathbf{0}_{(M-1)N}\right]^{\mathrm{T}}, \tag{9}$$

where $\mathbf{0}_{(M-1)N}$ is the $1 \times (M-1)N$ raw vector filled with zeros. A delay τ equal to half the FIR filter length ($\tau = L/2$) was used in the simulations.

If $\mathbf{f}$ represents the $ML \times 1$ FIR coefficients matrix of the synthesis filter bank, (8) can be written:

$$(\mathbf{H}\,\mathbf{D})\,\mathbf{f} = \mathbf{t}. \tag{10}$$

where $\mathbf{D}$ stands for the $MN \times ML$ matrix of the Discrete Fourier Transform coefficients.

If $N > L$, the linear system (10) is over determined. However, a least square solution can be found [9]:

$$\begin{pmatrix} \mathrm{Re}\left\{(\mathbf{H}\,\mathbf{D})\,\mathbf{f}\right\} \\ \mathrm{Im}\left\{(\mathbf{H}\,\mathbf{D})\,\mathbf{f}\right\} \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\left\{\mathbf{t}\right\} \\ \mathrm{Im}\left\{\mathbf{t}\right\} \end{pmatrix} \tag{11}$$

where$\mathrm{Re}\left\{\mathbf{A}\right\}$ and $\mathrm{Im}\left\{\mathbf{A}\right\}$ denote the real and imaginary parts of matrix $\mathbf{A}$.

In order to dramatically increases the filter bank performances and reach acceptable levels of aliasing, [8] chooses to slightly rise the sample frequency against the band of interest. An oversampling ratio of $\eta\%$ is used.

III. Anti-aliasing filtering

The band of interest is now supposed to be $\pm\pi/(1-\eta)T$. So, the input signal of interest $x(t)$ is assumed band-limited:

$$X(\mathrm{j}\Omega) = 0 \qquad |\Omega| > \frac{\pi}{(1-\eta)T} \tag{12}$$

This approximation can be relevant if an analog anti-aliasing filter is used before the HFB. This pre-filtering is used in order to eliminate noise in frequency bands in which there is no (or a few) signal of interest.

Nevertheless, the anti-aliasing filter cannot perfectly eliminate noise. The real input signal $y(t)$ of the HFB can then be written:

$$y(t) = x(t) + n(t) \tag{13}$$

where $n(t)$ stands for the additive noise not filtered.

The resolution of the HFB depends on the quantization noise and the aliasing level. Quantization noise has been studied in [2]. The worse between aliasing and quantization noise will limit the resolution.

The product of each aliasing function by the whole input signal spectrum $Y(\mathrm{j}\Omega)$ can be considered as an additive noise at the output of the HFB. The respective contributions of the signal of interest $x(t)$ and the noise $n(t)$ need to be taken into account at the output of the ADC converter.

Then, the aliasing terms due to $n(t)$ should be less (or of the same order) than the aliasing terms due to $x(t)$ to preserve the performance of the HFB. In other words, the signal to noise ratio at the input of the HFB is enforced by this requirement. Accordingly, the pre-filtering operation has to be done taking into account the respective power spectral densities of signal and noise.

IV. Eight channel HFB simulation results

To observe the behavior of the HFB, an eight channel HFB has been considered. The analysis bank includes a low-pass RC filter and simple RLC circuits with equal bandwidths distributed through the band $|\Omega| < \pi/T$. The synthesis bank was design for a band of interest $|\Omega| < \pi/(1-\eta)T$, $\eta = 6\%$ using 128 length FIR filters . 128 points were chosen for the discrete frequency domain.

The simulation was carried out for a signal in the band of interest. Fig. 2 shows the related magnitude and phase of the distortion functions versus frequencies. Fig. 3 illustrates the corresponding magnitude of the aliasing functions. Distortion is below 0.21dB and aliasing is below -100dB. The product of the m^{th} aliasing function and $\widetilde{X}(\mathrm{j}\omega/T - 2\pi\mathrm{j}m/MT)$ appears at the output ($m = 1, \ldots, M-1$). Thus, the narrow band of oversampling for which the input spectrum is assumed null is frequency shift by $m\pi/MT$ and gaps (6% of total bandwidth) can be seen in Fig. 3 (and better in Fig. 7, see below).

The simulation was also fulfilled for a filtered white noise (band limited to $|\Omega| < \pi/T$). Fig. 4 and 5 show the associated distortion and aliasing functions (to better compare the performances, the magnitude of the first aliasing term $\hat{T}_1(\mathrm{e}^{\mathrm{j}\omega})$ associated with the two case of study is shown Fig. 6 and 7). One can see that noise has practically no effect on the distortion function. The gap observed in Fig. 3 cannot be seen. Aliasing is now only -18dB. Comparing Fig. 3 and 5, one can obviously notice that the main contribution on the aliasing functions are due to component frequencies in the band $|\Omega| > \pi/0.94T$. Hence, at the input of the HFB, the ratio of spectral densities in the band $|\Omega| > \pi/0.94T$ (noise) and $|\Omega| < \pi/0.94T$ (signal of interest) needs to be less than -80dB preserve the performances of the HFB in the band of interest. Therefore, this involves a stop band attenuation greater 80dB for the anti-aliasing filter .

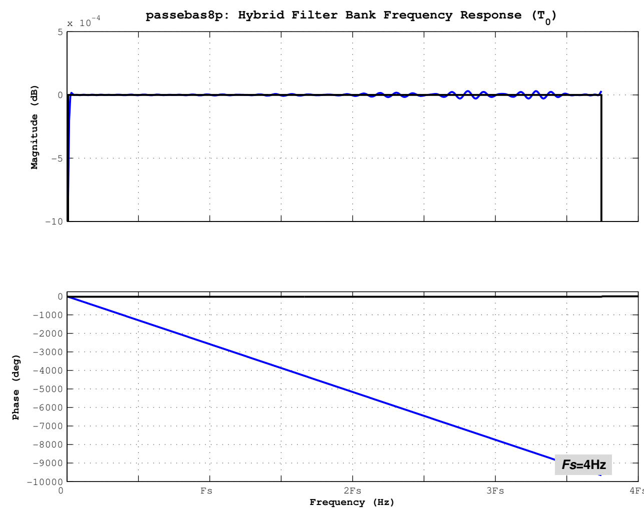

Fig. 2. Magnitude (dB) and phase (deg.) of the distortion function for the signal of interest (band $|\omega| < \pi/(1-\eta)T$).

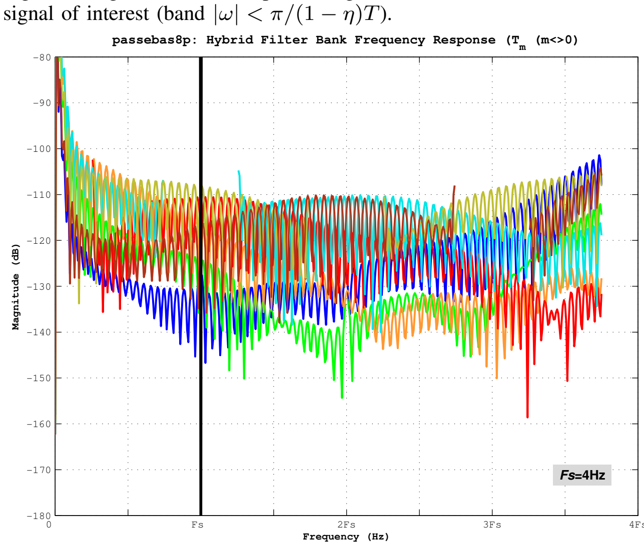

Fig. 3. Magnitude (dB) of the aliasing functions ($m = 1, \ldots, M-1$) for the signal of interest (band $|\omega| < \pi/(1-\eta)T$).

V. Conclusion

As future A/D converters shall be versatile, they will have to deal with wide bandwidths, which is not likely to be possible with today's solutions. HFB A/D converters appear to be appropriate, especially because they allow wider bandwidth. A small oversampling allows to reach great performances through the band of interest. Nevertheless, noise out of this band can strongly decrease the performances of the HFB due to aliasing during sampling, even with high signal to noise ratio. The pre-filtering (anti-aliasing filter) operation is critical.

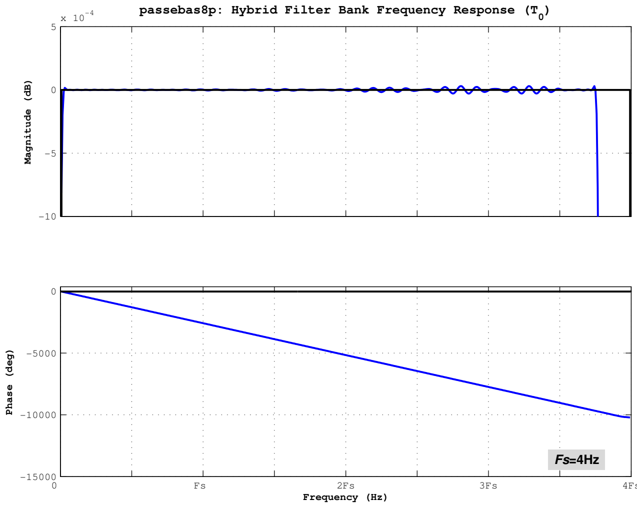

Fig. 4. Magnitude (dB) and phase (deg.) of the distortion function in the whole band (band $|\omega| < \pi/T$).

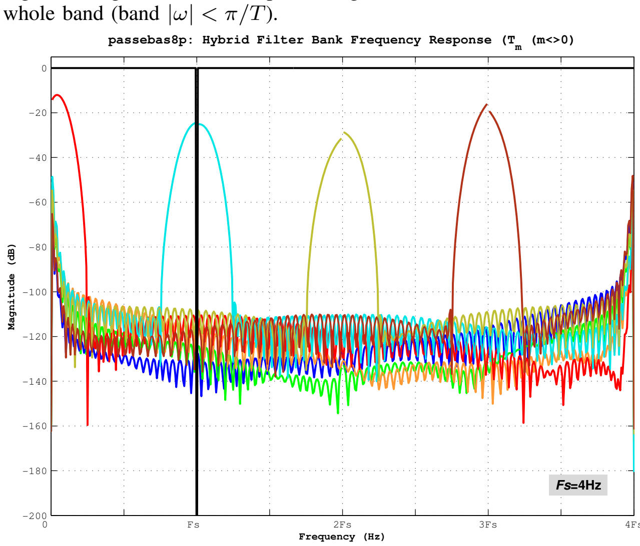

Fig. 5. Magnitude (dB) of the aliasing functions ($m = 1, \ldots, M-1$) for the whole band ($|\omega| < \pi/T$).

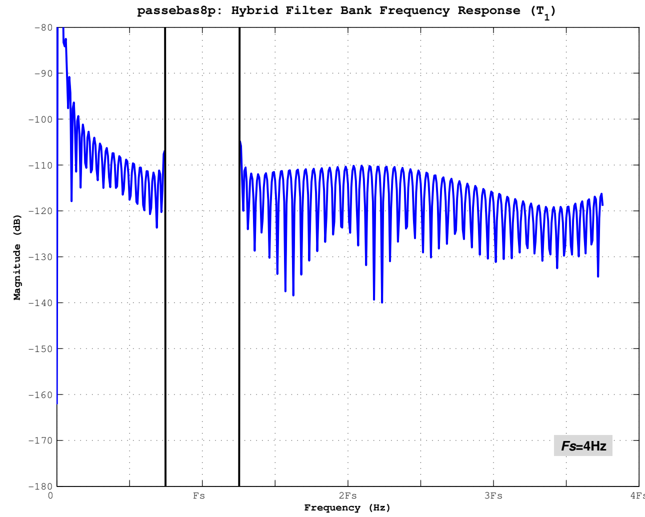

Fig. 6. Magnitude (dB) of the first aliasing functions $\hat{T}_1$ for signal of interest (band $|\omega| < \pi/(1-\eta)T$).

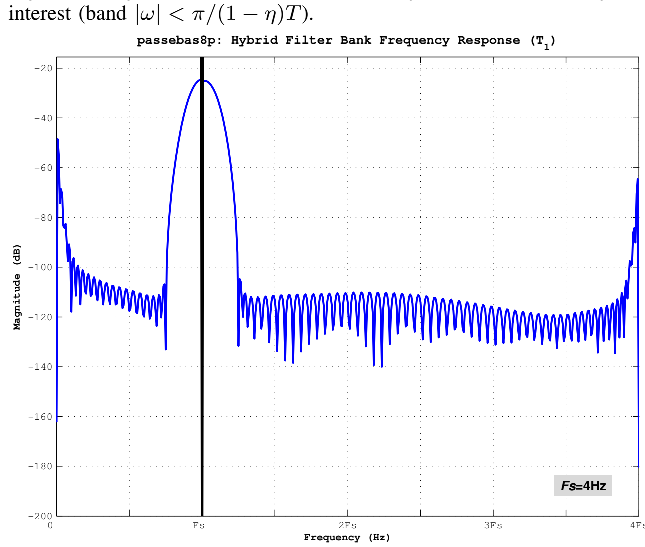

Fig. 7. Magnitude (dB) of the first aliasing functions $\hat{T}_1$ for the whole band ($|\omega| < \pi/T$).

REFERENCES

[1] C. Lelandais-Perrault, D. Poulton, and J. Oksman, *Synthesis of hybrid filter banks for a/d conversion with implementation constraints -direct approach-*, Proceedings of IEEE Midwest Symposium for Circuits and Systems, December 2003.

[2] P. Lowenborg, *Analysis and synthesis of asymmetric filter banks with application to analog-to-digital conversion*, Ph.D. thesis, Institute of Technology - Linkopings universitet, may 2001.

[3] P. Löwenborg, H. Johansson, and L. Wanhammar, *A two-channel hybrid analog and iir filter bank aproximating perfect magnitude reconstruction*, Proceedings of IEEE Nordic Signal Processing Symposium, vol. vol. 1, June 2000.

[4] O Oliaei, *Asymptotically perfect reconstruction in hybrid filter banks*, Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing, vol. vol. 3, May 1998, pp. 1829–1832.

[5] A. Papoulis, *Systems and transforms with applications in optics*, MacGraw Hill, 1968.

[6] A. Petraglia and S. K. Mitra, *High speed A/D conversion incorporating a QMF bank*, IEEE Transactions on Signal Processing **vol. 41** (1992), pp. 427–431.

[7] T. G. Petrescu and J. Oksman, *Synthesis of hybrid filter banks for a/d conversion with implementation constraints -optimized frequency response approach-*, Proceedings of IEEE Midwest Symposium fort Circuits and Systems, December 2003.

[8] T. G. Petrescu, J. Oksman, and P. Duhamel, *Synthesis of hybrid filter banks by global frequency domain least square solving*, IEEE International Symposium on Circuits and Systems (2005).

[9] G. Strang, *Linear algebra and its applications*, second edition ed., Academiv Press, Orlando, 1980.

[10] S. R. Velazquez, T. Q. Nguyen, and S. R. Broadstone, *Design of hybrid filter banks for analog/digital conversion*, IEEE Transactions on Signal Processing **vol. 46** (1998), no. 4, pp. 956–967.

[11] S. R. Velazquez, T. Q. Nguyen, S. R. Broadstone, and J. K. Roberge, *A hybrid filter bank approach to analog to digital conversion*, Proceedings of IEEE International Symposium on Time-Frequency and Time-Scale Analysis, October 1994, pp. pp. 116–119.