Modelling the imperfections of analog circuits using Second-Order Statistics

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***Abstract*— The analog parts of mixed analog-digital systems are always subject to some imperfections. Considering Linear Time-Invariant (LTI) analog circuits, the real transfer function is then practically different from the desired nominal one and includes some deviations. The goal of this paper is to offer a model which digitally estimates the deviations from typical values supposing that the only available information is the sampled analog output (blind estimation). The model is independent from the type of the imperfections sources and is applicable when the input is a white noise (either Gaussian or non-Gaussian). The model has been applied to several RC and RLC circuits and the performance of the estimation is studied. The simulations show that the model can estimate the analog imperfections of $\pm 20\%$ with a precision $\pm 4\%$ in first- and second-order circuits which may be useful in correction purposes such as compensation.**

I. Introduction

Despite the fast development of the digital technology and signal processing methods, it is still at times required to have the analog circuits either through an analog system or along with a digital part at the hybrid systems. Both discrete and integrated Electronic components of analog circuits are always subject to some random deviations from the nominal values. Therefore, the electronic circuits of LTI systems are characterized by the transfer functions which include some uncertainties. The nominator and the denominator coefficients of the transfer function may be considered as random numbers. The average values of the coefficients are the typical values. The deviations from typical values are unknown. The analog imperfections associated with the fabrication process can be considered as time-independent factors. However, the analog imperfections include some time-varying contributions related to some phenomena such as the operative temperature. To lessen the consequences of the fabrication imperfections of analog circuits, some possibilities exist such as laser trimming in the case of integrated circuits at the production phase. Laser trimming is generally too expensive. Moreover, the time-varying imperfections can not be compensated during the fabrication phase. Therefore, digital compensation can be considered as a suitable solution particularly when mixed (analog-digital) circuits are dealt with.

There are some applications in which analog imperfections may deteriorate the performance or even make the realization impossible. Sigma-delta A/D converters belong to an exemplary domain of this subject. They are very sensitive to the nonlinearity of their internal multi-bit D/A converters [1]. Cascade architecture (MASH) has been proposed to handle the problems of high sensitivity and instability from which the sigma-delta modulators suffer. In return, a great sensitivity to analog circuit imperfections emerge when MASH is used [1]. In Switched-Capacitor (SC) circuits, these imperfections are mostly related to finite op-amp gains, capacitor ratio errors and settling times [2]. Another example is the parallel structure of Hybrid Filter Bank (HFB) A/D converters [3]. HFB-based A/D converters are subject to a very high sensitivity upon analog imperfections [4] [5]. To overcome the problem of analog imperfections in wide-band parallel HFB-based A/D converters, Velazques proposed a digital calibration method [6]. The calibration was established in the whole spectrum but the adaptive compensation of the comb filter was classified according to the different origins of the imperfections. Till now, most of digital compensation techniques dealt only with specific imperfections (for example only with capacitor ratio or with finite op-amp gain error). Accordingly, they are not generic methods and are applicable only for the supposed special cases. Besides, they sometimes utilize a reference signal that necessitates to use a subsystem which is completely dependent on the system [2] [7].

Thus, it is necessary to look for a general method so as to be able to estimate the real parameters of analog circuits using only the output of the systems. The estimation method has to be independent of the type and the origins of the errors. Then, one would be able to digitally compensate the analog imperfections of the electronic circuits. It will be very useful particularly for mixed analog-digital circuits. Digital estimation of analog imperfections would be used for compensating purposes. Accordingly, calibration phase could be omitted in fabrication process of electronic circuits. On the other hand, time-varying parameters (especially temperature-dependent factors) would be possible to be compensated in a real-time manner.

In this paper, a model is proposed for the blind estimation of analog imperfections. RC and RLC circuits have been considered for the simulations as the exemplary analog filters as these analog filters are used in the HFB-based A/D converters proposed by Petrescu et. al. [4] [5]. However, the proposed model is totally general and is applicable to other circuits and applications.

II. Digital estimation of analog imperfections

A. Problem definition and linearization

Considering system in figure 1, it is supposed that the Nyquist sampling rate has been respected and that the sampled output of the system $y[n]$ is the only available data. The problem is now to estimate the real spectral parameters of the circuit (coefficients of $H(s)$) using the only available data $y[n]$. Regarding to the problem of analog imperfections, the coefficients of the numerator and the denominator of $H(s)$ are the random variables which have the different distributions depending on the fabrication factors, the number and the type of the electronic elements and the structure of the circuit. The central values (expectation) of these parameters are often known but the real values are subject to a random additive error or deviation from the typical values. Analog imperfections cause a change in the coefficients of $H(s)$ but they have no effect on the order of the system. Accordingly, one can try to estimate the real coefficients in order to compensate the analog imperfections as the first and the most direct way. An algorithm is then required to directly estimate the relative imperfections through the output samples.

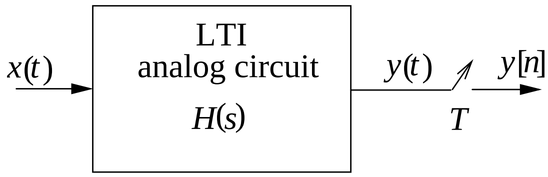

Fig. 1. An arbitrary LTI analog circuit with transfer function of $H(s)$. $y[n]$ represents the output after sampling.

It is supposed to have K unknown parameters $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$ through which $H(s)$ is described. These parameters may be either the coefficients of $H(s)$ or some functions of the coefficients (such as cut-off frequency or resonance frequency and quality factor for first-order and second-order analog circuits respectively). The transfer function of the analog circuit can be described as follows:

$$H(s) = g(\boldsymbol{\alpha}, s) \tag{1}$$

Each element α_i of the vector $\boldsymbol{\alpha}$ is supposed to be randomly distributed around a known central expected value $\alpha_{i_\circ}$:

$$\begin{aligned} \alpha_i &= \alpha_{i_\circ} + \triangle\alpha_i \\ &= \alpha_{i_\circ}(1 + \delta_{\alpha_i}) \end{aligned} \tag{2}$$

where

$$\delta_{\alpha_i} = \frac{\triangle\alpha_i}{\alpha_{i_\circ}} \qquad i = 1, \ldots, K$$

that $\triangle\alpha_i$ is a random variable which represents the total effects of analog imperfections on α_i. Therefore, respective distribution is not necessarily Gaussian even in the case of Gaussian fabrication errors. The objective is to estimate the relative imperfection δ_{α_i}. In general (even for first order RC filter), the imperfection parameters have a nonlinear contribution in the transfer function. Therefore, it is proposed to use the fist-order linear approximation using the Taylor development of the transfer function assuming $\delta_{\alpha_i} \ll 1$. Thus, it is concluded that:

$$\begin{aligned} H(s) &\cong g(\boldsymbol{\alpha}_\circ, s) + \sum_{i=1}^{K} \delta_{\alpha_i} \cdot (\alpha_i \frac{\partial g(\boldsymbol{\alpha}, s)}{\partial \alpha_i})\Big|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_\circ} \\ &= H_\circ(s) + \sum_{i=1}^{K} \delta_{\alpha_i} \cdot H_i(s) \end{aligned} \tag{3}$$

where $H_\circ(s)$ is supposed to be the transfer function of the circuit when there is no imperfection and the other transfer functions are defined as follows:

$$H_i(s) = \alpha_i \frac{\partial g(\boldsymbol{\alpha}, s)}{\partial \alpha_i}\Big|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}_\circ} \qquad i = 1, 2, \ldots, K \tag{4}$$

that $H_i(s)$ represents the sensitivity function associated to the parameter α_i. Equally, the following relationship can be established in the time domain as follows:

$$h(t) \cong h_\circ(t) + \sum_{i=1}^{K} \delta_{\alpha_i} \cdot h_i(t) \tag{5}$$

where each $h_i(t)$ is the impulse response associated with the respective transfer function $H_i(s)$. $h_i(t)$ is independent of the input and output of the system as well as of the imperfections. According to equation 4, it has no dependence on the imperfections neither on input/output signals.

B. Second-Order Statistics equations

Figure 1 is considered. If the input is supposed to be a white noise (either Gaussian or non-Gaussian process), the following equation will always hold between the second-order moments of the input $x(t)$ and the analog output signal $y(t)$ [8]:

$$\sigma_y^2 = \sigma_x^2 \int (h(t))^2 dt \tag{6}$$

where σ_x^2 and σ_y^2 are the input and output variances respectively. Supposing that the filter $H(s)$ is band limited and using sufficiently high sampling rate, equation 6 can be approximated in discrete-time domain as following:

$$\sigma_y^2 \cong \sigma_x^2 \sum_n (h[n])^2 \tag{7}$$

And using equations 5, 6 and 7 along with linear approximation, the following relationship is obtained:

$$\begin{aligned} \frac{\sigma_y^2}{\sigma_x^2} &\cong \sum_n h_\circ[n]^2 + 2\delta_{\alpha_1} \sum_n h_1[n].h_\circ[n] \\ &+ \ldots + 2\delta_{\alpha_K} \sum_n h_K[n].h_\circ[n] \end{aligned} \tag{8}$$

where the relative imperfections and input variance are unknown. K additional equations have to be established for reaching the unknown relative imperfections. For this purpose, it is proposed to choose K auxiliary FIR filters which are applied separately to the system output, $y[n]$. For instance, figure 2 shows this process after applying i^{th} auxiliary FIR filter $f_i[n]$. Then, equation 8 would be possible to be rewritten for each new output signal $v_i[n]$ versus original input $x(t)$ as the convolution of two filters $h[n]$ and $f_i[n]$ is an LTI filter. Applying equation 8 to this new configuration, the following relationship yields:

$$\frac{\sigma^2_{v_i}}{\sigma^2_x} \cong \sum_n s_\circ[n]^2 + 2\delta_{\alpha_1}\sum_n s_1[n].s_\circ[n] + \cdots + 2\delta_{\alpha_K}\sum_n s_K[n].s_\circ[n] \tag{9}$$

that $s_j[n]$ is an intermediate impulse response defined as follows:

$$s_j[n] = h_j[n] \star f[n] \qquad j \in \{\circ, 1, 2, \cdots, K\}$$

where $\star$ represents the convolution operation.

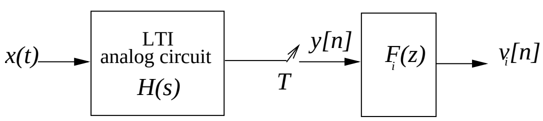

Fig. 2. An LTI analog circuit with transfer function of $H(s)$ to which another auxiliary FIR filter of $F_i(z)$ has been applied.

Some choices of auxiliary FIR filters have been tried in the simulations. An FIR filter $f[n]$ approximating the inverse of typical transfer function $H_\circ(s)$ shows a good performance when $K = 1$. For $K > 1$, it is proposed to have a quasi-orthogonality in frequency domain. This means that K mutually orthogonal FIR filters must be chosen. For example, k^{th} FIR filter $f_k[n]$ is a filter with pass band $[(k-1)\frac{\pi}{T}, k\frac{\pi}{T}]$ where T is the sampling period and $1 \le k \le K$.

III. Implementation of the Estimation Algorithm

A. Estimation algorithm

Considering equations 8 and 9, there will exist $K + 1$ equations as following:

$$\begin{cases} C_{\circ\circ}\delta_{\alpha_1} + \cdots + C_{\circ K}\delta_{\alpha_K} + (\sigma^2_y)\frac{1}{\sigma^2_x} = d_\circ \\ C_{11}\delta_{\alpha_1} + \cdots + C_{1K}\delta_{\alpha_K} + (\sigma^2_{v_1})\frac{1}{\sigma^2_x} = d_1 \\ \vdots\ \vdots\ \vdots \\ C_{K1}\delta_{\alpha_1} + \cdots + C_{KK}\delta_{\alpha_K} + (\sigma^2_{v_K})\frac{1}{\sigma^2_x} = d_K \end{cases} \tag{10}$$

where the $(K + 1)$ unknown parameters are $\{\delta_{\alpha_1}, \delta_{\alpha_2}, \ldots, \delta_{\alpha_K}, \frac{1}{\sigma^2_x}\}$. All the coefficients C_{ij} and d_i are independent of the input and imperfections (refer to the previous subsection). Invoking the set of equations (10) and using Cramer method, the unknown relative imperfection δ_{α_i} is found as follows:

$$\delta_{\alpha_i} = \frac{b_\circ\sigma^2_y + \sum_{k=1}^K b_k\sigma^2_{v_k}}{a_\circ\sigma^2_y + \sum_{k=1}^K a_k\sigma^2_{v_k}} \tag{11}$$

where $B^{(i)} = [b_\circ, \cdots, b_K, a_\circ, \cdots, a_K]$ is the coefficients vector associated with the model of δ_{α_i}. To calculate the coefficients, some known imperfections are applied to the system and the coefficients are then approximated using the Least Squares (LS) method and the gradient algorithm. Thus, N known relative imperfections are selected and the system is simulated using a white noise at the input. For having an overdetermined problem, N is considered much higher than K ($N \gg K$). Therefore, the vector of coefficients $B^{(i)}$ associated with the relative imperfection δ_{α_i} can be approximated as follows:

$$B^{(i)} = arg\ \ min\ \ \|\delta^m_{\alpha_i} - \delta^r_{\alpha_i}\|^2 \tag{12}$$

that $\delta^m_{\alpha_i}$ and $\delta^r_{\alpha_i}$ represent the model and real values of the relative imperfection δ_{α_i}. This model can be separately established for each unknown imperfection $(\delta_{\alpha_i},\ 1 \le i \le K)$.

B. Simulation results

The algorithm described in the previous section has been applied to several first- and second-order circuits. Implementation of the procedure depends on the number of relative imperfections which are present in the problem. Therefore, the result of the blind estimation for relative imperfections are discussed depending on the number of unknown variables. An analog circuit may include only one unknown variable independent of its order. If there is only an unknown imperfection variable, one auxiliary filter will be required. For respective RC and RLC circuits, an approximative inverse FIR filter with three non-zero coefficients has been used. The impulse and frequency response of that auxiliary FIR filter has been shown in figure 3.

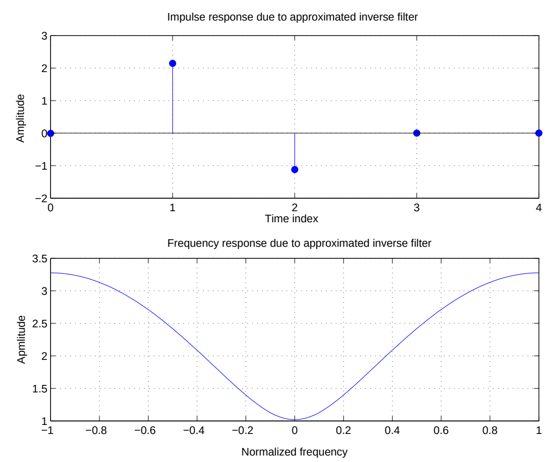

Fig. 3. Impulse response (above) and frequency spectrum (below) due to the estimated inverse filter of an RC circuit.

This FIR filter was obtained by blind equalization technique applied to an RC circuit [8]. The model is implemented for an RC circuit with the imperfections considered through its cut-off frequency. The estimation has been implemented for the imperfection range of $\pm 20\%$. Figure 4 shows the estimated deviation from typical values versus the real values in this case. The average precision of this estimation is $\pm 2.7\%$ (ratio of the standard deviation of the estimation errors on real values in percent). Figure 5 shows the result of the estimation associated with an RLC second-order circuit. In this case, the resonance frequency is subject to the analog imperfections. Estimation is

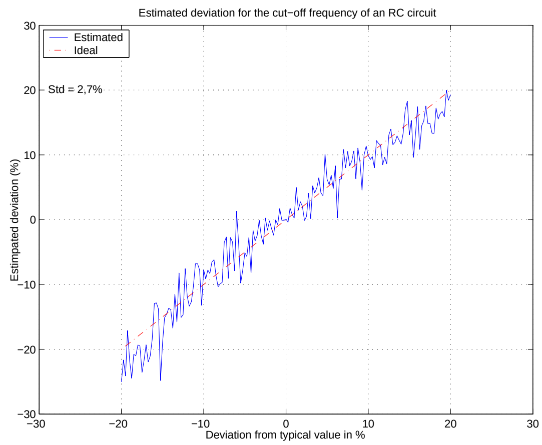

Fig. 4. Estimated deviation (solid) from typical value of cut-off frequency versus real values for an RC circuit. The dashed line represents the ideal response.

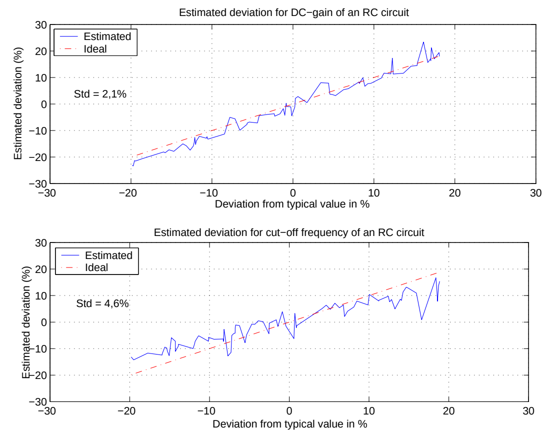

Fig. 6. Estimated deviation (solid) from typical value versus real values for DC-gain (above) and cut-off frequency (below) of an RC circuit. The dashed lines represent the ideal responses.

again regarded for a range of $\pm 20\%$. The same auxiliary filter has been again used. There is a standard deviation of $\pm 3.9\%$ for the errors of this estimation. It is seen that the quality of the estimation is lower in the case of second-order circuit.

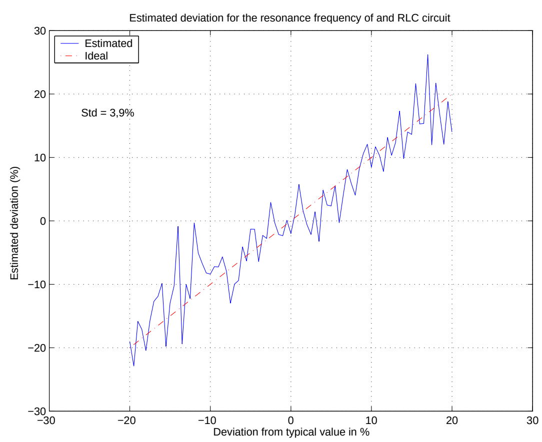

Fig. 5. Estimated deviation (solid) from typical value of resonance frequency versus real values for an RLC circuit. The dashed line represents the ideal response.

This method has also been used in the case of two unknown variables, considering an RC circuit including some imperfections applied on its DC-gain and cut-off frequency. The estimation is implemented for the imperfections in the range of $\pm 20\%$. Figure 6 demonstrates the result of the estimation. The values of standard deviation for the estimation errors are $\pm 2.1\%$ and $\pm 4.6\%$ associated to the parameters of DC-gain and cut-off frequency respectively. Considering a shorter range of estimation, the performance is developed.

IV. Conclusion

The estimation of analog circuit imperfections involved in analog or hybrid systems was studied in this paper. A model for approximating the imperfections was extracted using a linear estimation of the spectral imperfections due to analog systems. This model necessitates an analog input which is a white or i.i.d. (independent and identically distributed) stochastic process. Supposing some small errors at the coefficients of the transfer function due to analog system, the proposed model approximates these errors using only the sampled output of the system. So, an imperfection of 20% may be estimated with a significant precision which allows a further correction of the output signal. The model needs some auxiliary FIR digital filters. Orthogonality is the condition proposed for choosing the auxiliary FIR filters. The blind estimation method of the model is valid for both Gaussian and non-Gaussian distributed signals.

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