NEW FAST ALGORITHM FOR SIMULTANEOUS IDENTIFICATION AND OPTIMAL RECONSTRUCTION OF NON STATIONARY AR PROCESSES WITH MISSING OBSERVATIONS

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ABSTRACT

This paper deals with the problem of adaptive reconstruction and identification of AR processes with randomly missing observations. A new real time algorithm is proposed. It uses combined pseudo-linear RLS algorithm and Kalman filter. It offers an unbiased estimation of the AR parameters and an optimal reconstruction error in the least mean square sense. In addition, thanks to the pseudo-linear RLS identification, this algorithm can be used for the identification of non stationary AR signals. Moreover, simplifications of the algorithm reduces the calculation time, thus this algorithm can be used in real time applications.

1. INTRODUCTION

In many practical situations, periodically sampled signals with missing observations may be encountered. This is the case, for example, of errors in transmission, or of temporary unavailability of measurement. It is also the case, in coding of audio signals or images, for compression purposes.

Several methods have already been developed for the processing of AR signals with missing data. They are generally of two types:

- Off line methods that use all available data. They are developped for spectral estimation such as in [1], for identification purposes such as in [2, 3, 4], or for reconstruction such as in [5]. In [3], Jones used a Kalman filter to calculate the exact likelihood function for unequally spaced data. Model's parameters, that most fits the data, are then estimated by a non linear optimization. Isaksson [2], proposed an EM algorithm equivalent to a maximum likelihood algorithm but much faster. In [5], the estimates of the unknown samples are obtained by minimizing the sum of squares of the residual errors, it involves estimates of the autoregressive parameters. In [4], the identification of ARX models from incomplete data using least squares is studied.

- On line adaptive methods such as in [6, 7, 8, 9, 10]. In [8], an LMS-like algorithm for simultaneous reconstruction and identification is developed. In [9], the pseudo-linear RLS algorithm, an adaptation of the RLS algorithm to the case of signals with missing data, is derived. However these two algorithms converge toward biased parameters. In [10], we proposed an LMS-like algorithm based on the incomplete past predictor [11] for simultaneous optimal identification and reconstruction of AR processes subject to missing data. However, this algorithm is quite time consuming. In [6], the problem of recursive estimation of the output in missing-data situations is addressed. In [7], Isaksson derives a recursive EM algorithm for the identification of AR processes subject to missing data, based on the offline version of the one described in [2]. However, at each time, the inversion of a matrix is required to update the parameters. Therefore, it suffers from a high computational complexity.

In many applications, such as digital communications or systems tracking, on line processing is necessary. We are interested here in on line adaptive reconstruction and identification of autoregressive signals that can be non stationnary with randomly missing observations. The loss of samples process follows a Bernoulli law independent of the signal. In the following, we begin by presenting the RLS identification algorithm [12] and its adaptation to the case of missing observations [9, 6]. In a second part, the prediction of AR processes with missing observations using state space representation and Kalman filtering is discussed. A new adaptive algorithm for simultaneous reconstruction and identification, using combined RLS-like algorithm and Kalman filter, is proposed in the third part. This algorithm is simplified in order to become very fast. Finally, an example

illustrates the performances of the new recursive algorithm. It is compared to the previously proposed LMS-like algorithm based on the incomplete past predictor [10], and to the pseudo-linear RLS algorithm.

2. PRELIMINARIES

Let $\{x_n\}$ be an AR process of order L with parameters a_k. It satisfies the following difference equation:

$$x_n = a_1 x_{n-1} + \ldots + a_L x_{n-L} + \epsilon_n. \tag{1}$$

Where $\{\epsilon_n\}$ is the innovation process, a white noise of variance σ^2. The loss process is modeled by an i.i.d binary random variable $\{c_n\}$, $c_n = 1$ if x_n is available, otherwise $c_n = 0$. The probability to measure x_n is:

$$P\{c_n = 1\} = p = 1 - q. \tag{2}$$

Let $\{z_n\}$ be the reconstruction of the process $\{x_n\}$ subject to missing data. It is defined as:

$$z_n = \begin{cases} x_n & \text{if } x_n \text{ is available, i.e., } c_n = 1 \\ \hat{x}_n & \text{otherwise,} \end{cases} \tag{3}$$

where $\hat{x}_n$ is the prediction of x_n. In order to identify, in real time, the AR process subject to missing observations, we propose to use the pseudo-linear RLS identification algorithm.

3. PSEUDO-LINEAR RLS ALGORITHM

The RLS algorithm [12] presents the advantages of the simplicity, the fast convergence and the fast adaptivity in the case of non stationary processes. It is applied to linear systems in terms of the parameters. For an AR process, the RLS identification algorithm equations can be written:

$$\Psi_{n+1} = \mathbf{x}_n = [x_n \ldots x_{n-L+1}]^\top, \tag{4a}$$
$$\hat{x}_{n+1} = \Psi_{n+1}^\top \hat{\mathbf{a}}_n, \tag{4b}$$
$$\gamma_{n+1} = \frac{G_n \Psi_{n+1}}{\lambda + \Psi_{n+1}^\top G_n \Psi_{n+1}}, \tag{4c}$$
$$\hat{\mathbf{a}}_{n+1} = \hat{\mathbf{a}}_n + \gamma_{n+1}(x_{n+1} - \hat{x}_{n+1}), \tag{4d}$$
$$G_{n+1} = \frac{1}{\lambda}(I_d - \gamma_{n+1}\Psi_{n+1}^\top) G_n \tag{4e}$$

where $\hat{\mathbf{a}}_{n+1}$ are the estimated parameters at time $n+1$, I_d the identity matrix and $\lambda \leq 1$ is a forgetting factor that helps to the fast adaptivity of the parameter estimation in the case of non stationary signals.

In the case of missing observations, the regression vector (4a) cannot be constructed with only available samples. Missing data are replaced by their predictions, i.e., $\hat{\Psi}_{n+1} = \mathbf{z}_n = [z_n \ldots z_{n-L+1}]^\top$. This leads to a pseudo-linear algorithm where the regression vector depends on the model's parameters.

Moreover, the prediction error cannot be calculated at the instants where the data are missing. The quadratic cost function to minimize is now the mean of the reconstruction error at the instants where the data is available. It can be written as:

$$J = (\mathbf{x} - \hat{\mathbf{x}})^\top Q^\top Q (\mathbf{x} - \hat{\mathbf{x}}), \tag{5}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ are N-vectors containing respectively the signal and its prediction, and Q is an NxN diagonal matrix, with $Q = \begin{bmatrix} c_1 & & 0 \\ & \ddots & \\ 0 & & c_N \end{bmatrix}$. The prediction errors at the times of missing data, i.e. $c_n = 0$, are not taken into account in the quadratic cost function thanks to the weighting matrix Q.

Neglecting the dependence in the parameters of the regression vector, the pseudo-linear RLS identification algorithm is given by [9]:

$$\hat{\Psi}_{n+1} = \mathbf{z}_n = [z_n \ldots z_{n-L+1}]^\top, \tag{6a}$$
$$\hat{x}_{n+1} = f(\hat{\mathbf{a}}_n, \mathbf{z}_n), \tag{6b}$$
$$\gamma_{n+1} = \frac{c_{n+1} G_n \hat{\Psi}_{n+1}}{\lambda + \hat{\Psi}_{n+1}^\top G_n \hat{\Psi}_{n+1}}, \tag{6c}$$
$$\hat{\mathbf{a}}_{n+1} = \hat{\mathbf{a}}_n + \gamma_{n+1}(x_{n+1} - \hat{\Psi}_{n+1}^\top \hat{\mathbf{a}}_n), \tag{6d}$$
$$G_{n+1} = \frac{1}{\lambda}(I_d - \gamma_{n+1}\hat{\Psi}_{n+1}^\top) G_n \tag{6e}$$

where f represents the samples prediction method as a function of the model parameters and the past available and predicted samples.

Albertos et al. [6] were interested in the identification and output prediction of SISO linear systems in the case of dual-rate scarce sampling. To predict a sample, they used, in (4b), the regression vector completed by the prediction of the missing data (6a). The predicted sample is then, $\hat{x}_{n+1} = \hat{\Psi}_{n+1}^\top \hat{\mathbf{a}}_n$. This prediction approach was also used in [1, 2, 5, 8]. It leads to biased estimates using the least squares estimation. The bias was calculated by Wallin et al. [4]. Indeed, they studied the problem of identification of ARX models with missing observations using the least squares. They showed that least squares estimate of the parameters using any prediction of the data is unbiased if the following holds:

$$E\{\hat{\Psi}^\top V\} = 0. \tag{7}$$

where $\hat{\Psi}$ is the matrix formed by all the regressors $\hat{\Psi}_n$ and $V = \mathbf{z} - \hat{\Psi}\mathbf{a}$ is the equation error. Thus, for unbiased estimation of the parameters, a suitable predictor must be used. An optimal predictor in the least mean squares sense verifies (7).

Bondon [11] proposed an expression for the optimal predictor of an AR process with incomplete past, however this predictor is not recursive. Other prediction approaches use state space representation of the signal and a Kalman filter for recursive prediction [2, 6]. In [2], a state space representation with stochastic disturbances in the observation is proposed. In [6], the effect of unmeasured outputs is modelled by a disturbance of infinite variance on the observation. The drawback of this approach is the computational complexity. In addition, the model used is not appropriate to an AR process. In [13], a Kalman filter for the state estimation of jump linear systems where the discrete transitions are modeled as a Markov chain is presented. In the following, a particular form of the previous Kalman filter is presented, it is used for the case of recursive optimal prediction of AR processes subject to random missing observations.

4. PREDICTION USING A KALMAN FILTER

4.1. State-space representation

Let the observation process $\{y_k\}$ be defined as:

$$y_n = c_n x_n \;=\; \begin{cases} x_n & \text{if } x_n \text{ is available,} \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

Thus, y_n can be regarded as the measurement of x_n subject to missing data.

For the AR process $\{x_n\}$ with missing observations presented in section 2, we propose the following state space representation:

$$\begin{cases} \mathbf{x}_{n+1} &= A\,\mathbf{x}_n + \epsilon_n \left[1\;0\ldots0\right]^\top \\ y_{n+1} &= \mathbf{c}_{n+1}^\top\,\mathbf{x}_{n+1} \end{cases} \tag{9}$$

where $A = \left[\begin{array}{ccc|c} a_1 & \ldots & \ldots & a_L \\ \hline 1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & 1 & 0 \end{array}\right]$ is a LxL matrix and

$\mathbf{x}_n = \begin{bmatrix} x_n \\ \vdots \\ x_{n-L+1} \end{bmatrix}$, $\mathbf{c}_n = \begin{bmatrix} c_n \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ are L-vectors.

4.2. Kalman Filter

The predicted and filtered estimate will be denoted by $\hat{\mathbf{x}}_{n+1|n}$ and $\hat{\mathbf{x}}_{n+1|n+1}$, respectively. $P_{n+1|n}$ and $P_{n+1|n+1}$ are the a priori and a posteriori prediction error covariance matrix, respectively. K_{n+1} is the Kalman filter gain.

The Kalman filter equations can be found in, e.g. [12]. In our case, there is no disturbance in the observation, the Kalman filter equations resume to:

$$P_{n+1|n} = AP_{n|n}A^\top + R_\epsilon, \tag{10}$$

where, $R_\epsilon = \sigma_\epsilon^2 \left[1\;0\ldots0\right]^\top \left[1\;0\ldots0\right]$.

$$K_{n+1} = \begin{cases} \frac{P_{n+1|n}\mathbf{c}_{n+1}}{\mathbf{c}_{n+1}^\top P_{n+1|n}\mathbf{c}_{n+1}} & \text{if } x_{n+1} \text{ is available} \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

$$P_{n+1|n+1} = (I_d - K_{n+1}\mathbf{c}_{n+1}^\top)P_{n+1|n}, \tag{12}$$

$$\hat{\mathbf{x}}_{n+1|n+1} = \hat{\mathbf{x}}_{n+1|n} + K_{n+1}(y_{n+1} - \hat{y}_{n+1|n}), \tag{13}$$

where,

$$\hat{\mathbf{x}}_{n+1|n} = A\hat{\mathbf{x}}_{n|n}, \tag{14}$$

and

$$\hat{y}_{n+1|n} = \mathbf{c}_{n+1}^\top \hat{\mathbf{x}}_{n+1|n}. \tag{15}$$

Due to the state space representation chosen, the previous Kalman filter can be simplified. Indeed, if, at time $n+1$, x_{n+1} is available, i.e., $c_{n+1} = 1$, the term $P_{n+1|n}\mathbf{c}_{n+1}$ is equal to the first column of $P_{n+1|n}$ and $(\mathbf{c}_{n+1}^\top P_{n+1|n}\mathbf{c}_{n+1})^{-1}$ is a scalar equal to the first term of the matrix $P_{n+1|n}$. Consequently the Kalman gain K_{n+1} is calculated by dividing each element of the first column of $P_{n+1|n}$ by the first element of $P_{n+1|n}$, which requires L multiplications instead of $2L(L+1)$ multiplications. In addition, we deduce that the first element of K_{n+1}, $K_{n+1}(1)$, is always equal to 1 if the data is available. Recalling the filter's equation system (13), the first equation of it gives that when a sample is available:

$$\begin{aligned} \hat{x}_{n+1|n+1} &= \hat{x}_{n+1|n} + K_{n+1}(1)\left(y_{n+1} - \hat{y}_{n+1}\right) \\ &= y_{n+1} = x_{n+1}. \end{aligned}$$

An observed sample is then unchanged by the Kalman filter. Otherwise, if x_{n+1} is missing, i.e., $c_{n+1} = 0$, then $K_{n+1} = 0$ and the prediction error covariance matrix does not change a posteriori. This is verified by equation (12): for $K_{n+1} = 0$, we get $P_{n+1|n+1} = P_{n+1|n}$. The predicted value $\hat{\mathbf{x}}_{n+1|n}$ is not filtered at time $n+1$, and $\hat{\mathbf{x}}_{n+1|n+1} = \hat{\mathbf{x}}_{n+1|n}$. However, this value is corrected in the subsequent L steps due to the Kalman filtering of the state when a data is available, corrective terms are added to that prediction. It can be expressed as:

$$\hat{x}_{n+1|n+t} = \hat{x}_{n+1|n} + \sum_{i=1}^{t} K_{n+i}(i)\left(y_{n+i} - \hat{y}_{n+i|n+i-1}\right). \tag{16}$$

Since we are interested in real time reconstruction, the corrected prediction $\hat{x}_{n+1|n+t}$ can not be used for the reconstruction, it is only used for the prediction of subsequent samples. Thus $\{z_n\}$, the reconstruction of the process, is defined as:

$$z_n \;=\; \begin{cases} x_n & \text{if } x_n \text{ is available, i.e., } c_n = 1 \\ \hat{x}_{n|n-1} & \text{otherwise,} \end{cases} \tag{17}$$

5. COMBINED RLS-LIKE ALGORITHM AND KALMAN FILTER

For a real time identification and optimal reconstruction, we propose here to use the proposed Kalman filter as a predictor with the pseudo-linear RLS algorithm. These two algorithms are combined. The Kalman filter uses at each iteration the AR parameters, estimated using the pseudo-linear RLS algorithm, to predict the new state. Hence, at time $n+1$, the first line of the matrix A is replaced by $\hat{\mathbf{a}}_n^\top$, the vector of the parameters estimated at time n. The matrix is then named A_{n+1}. If a new sample x_{n+1} is available, the pseudo-linear RLS algorithm assigns the Kalman filter predictions, $\hat{y}_{n+1|n}$ and $\hat{\mathbf{x}}_{n|n}$, to $\hat{x}_{n+1}$ and $\hat{\Psi}_{n+1}$ respectively, to update the model parameters. The optimal reconstruction is $z_{n+1} = \hat{x}_{n+1|n+1}$. The resulting algorithm can be resumed, at time $n+1$, as follows:

$$
\begin{aligned}
A_{n+1} &= \left[\begin{array}{ccc|c} \hat{a}_{n,1} & \dots & \dots & \hat{a}_{n,L} \\ \hline 1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & 1 & 0 \end{array}\right], \\
P_{n+1|n} &= A_{n+1} P_{n|n} A_{n+1}^\top + R_\epsilon, \\
\hat{\mathbf{x}}_{n+1|n} &= A_{n+1} \hat{\mathbf{x}}_{n|n} \\
\hat{\Psi}_{n+1} &= \hat{\mathbf{x}}_{n|n} \\
\hat{y}_{n+1|n} &= c_{n+1} \hat{x}_{n+1|n} = \mathbf{c}_{n+1}^\top \hat{\mathbf{x}}_{n+1|n}
\end{aligned}
\tag{18}
$$

If x_{n+1} is available, i.e. $c_{n+1} = 1$,

$$\gamma_{n+1} = \frac{G_n \hat{\Psi}_{n+1}}{\lambda + \hat{\Psi}_{n+1} G_n \hat{\Psi}_{n+1}}, \tag{19a}$$

$$G_{n+1} = \frac{1}{\lambda}(I_d - \gamma_{n+1} \hat{\Psi}_{n+1}^\top) G_n, \tag{19b}$$

$$\hat{\mathbf{a}}_{n+1} = \hat{\mathbf{a}}_n + \gamma_n (y_{n+1} - \hat{y}_{n+1|n}), \tag{19c}$$

$$K_{n+1} = P_{n+1|n} \mathbf{c}_{n+1} (\mathbf{c}_{n+1}^\top P_{n+1|n} \mathbf{c}_{n+1})^{-1}, \tag{19d}$$

$$P_{n+1|n+1} = (I_d - K_{n+1} \mathbf{c}_{n+1}^\top) P_{n+1|n}, \tag{19e}$$

$$\hat{\mathbf{x}}_{n+1|n+1} = \hat{\mathbf{x}}_{n+1|n} + K_{n+1}(y_{n+1} - \hat{y}_{n+1|n}) \tag{19f}$$

Else if x_{n+1} is absent, $c_{n+1} = 0$, the predicted state, $\hat{\mathbf{x}}_{n+1|n}$, is not filtered by the Kalman filter, and the parameters are not updated using the RLS-like algorithm,

$$K_{n+1} = 0, \tag{20a}$$

$$P_{n+1|n+1} = P_{n+1|n}, \tag{20b}$$

$$G_{n+1} = \frac{1}{\lambda} G_n, \tag{20c}$$

$$\gamma_{n+1} = 0, \tag{20d}$$

$$\hat{\mathbf{x}}_{n+1|n+1} = \hat{\mathbf{x}}_{n+1|n}, \tag{20e}$$

$$\hat{\mathbf{a}}_{n+1} = \hat{\mathbf{a}}_n. \tag{20f}$$

This algorithm uses the Kalman filter to reconstruct the signal in the least mean square sense, the use of that predictor with the pseudo-linear RLS algorithm offers a non biased parameter estimation and a fast adaptation in the case of non stationnary AR processes with missing observations. In addition, it is simple and fast.

6. EXAMPLE

In this section, the proposed algorithm, the pseudo-linear RLS algorithm [9] and the LMS-like algorithm based on the incomplete past predictor [10] are compared. The test signal used is a non stationary AR(2) signal generated over 5.10^4 samples. The parameters of the signal are $[1.5, -0.7]$ for the first 25.10^3 samples and $[1, -0.5]$ for the last 25.10^3 samples. The Bernoulli's probability of sample loss is $q = 0.3$. For the two first algorithms, the forgetting factor used is $\lambda = 0.999$. For the LMS-like algorithm, $\mu = 14.10^{-5}$, it is an empirical value, a higher value may cause a divergence in the parameters estimation especially for a Monte Carlo of regenerations of the signal and the loss pattern. The performances of these algorithms are summarized in table 1. b_i is the bias existing in the estimation of a parameter and σ_i^2 is the variance of an estimated parameter. The MQRE is the normalized mean quadratic reconstruction error. The normalization is done with respect to the power of the signal. The CPU is the computation time (in seconds) required to simulate each one of the algorithms for the above test signal, using MATLAB on a 3 GHz processor.

Table 1. Comparison of the three algorithms performances

Algorithm	b_1	b_2
Proposed algorithm	0.0006	0.003
Pseudo-linear RLS	0.1811	0.151
LMS-like algoritm	0.0099	0.005
Algorithm	σ_1	σ_2
Proposed algorithm	0.014	0.015
Pseudo-linear RLS	0.018	0.017
LMS-like algoritm	0.035	0.035
Algorithm	MQRE	CPU
Proposed algorithm	0.066	1.5
Pseudo-linear RLS	0.073	1.3
LMS-like algoritm	0.068	18.8

The figure 1 shows the instantaneous mean value of the estimated parameter a_1 with the three algorithms for a Monte Carlo of 1000 regenerations of the signal and the loss pattern.

Figure 1 and Table 1 show that the proposed algorithm and the LMS-like algorithm based on the incomplete past predictor converge, in the mean sense, toward unbiased parameters which is not the case for the pseudo-linear RLS algorithm. Moreover, referring to figure 1, the proposed algorithm and the pseudo-linear RLS algorithm show the same

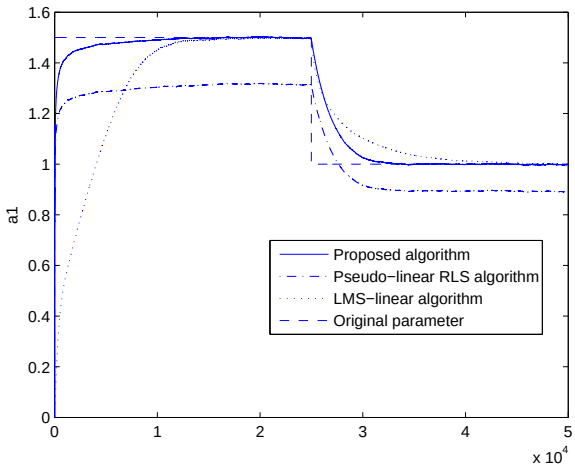

Fig. 1. Estimation of a_1 with the three algorithms.

fast convergence in comparison to the LMS-like algorithm. The rate of convergence of the LMS-like algorithm depends on the choice of μ, indeed, for a higher μ, the convergence is faster, however, the variance of the estimated parameters becomes higher, and the algorithm may diverge.

Table 1 shows that the proposed algorithm and the LMS-like algorithm offers the smallest MQRE in comparison to the pseudo-linear RLS algorithm. However, the LMS-like algorithm is computationally expensive in comparison to the other algorithms and simulations show that the CPU time increases with q. Indeed, for the prediction using the incomplete past predictor, the inversion of a matrix is required at each time, moreover the size of the matrix depends highly on the loss scheme and may become arbitrarily high for large q. The pseudo-linear algorithm shows the best computational time, it is slightly faster than the proposed algorithm. Moreover, the proposed algorithm and the pseudo-linear RLS algorithm show approximatively the same small variance of the estimated parameters in comparison to the LMS-like algorithm. This variance increases with the forgetting factor λ in the case of RLS identification algorithms, and with μ in the case of LMS identification algorithms.

7. CONCLUSION

A new adaptive algorithm for simultaneous optimal reconstruction and identification of an AR process with missing observations is described. It is based on the pseudo-linear RLS identification algorithm where the predictor used is a Kalman filter. This algorithm allows, at the same time, an unbiased estimation of the parameters, and an optimal reconstruction in the least mean square sense. In addition, due to simplifications, this algorithm is fast. Its complexity is $O(L)$ for a missing sample and $O(L^3)$ when a sample is available and is thus less computationnaly intensive than the recursive EM algorithm [7] and the LMS-like algorithm based on the incomplete past predictor [10]. Moreover, thanks to RLS identification, this algorithm shows fast convergence toward the true parameters, which is necessary for the processing of non stationary signals.

8. REFERENCES

[1] A. Ferrari and G. Alengrin, "AR spectral analysis with randomly missing observations," *IEEE Proceedings on Statistical Signal and Array Processing*, pp. 320–323, September 1998.

[2] A. Isaksson, "Identification of ARX models subject to missing data," *IEEE Transactions on Automatic Control*, vol. 38, no. 5, 1993.

[3] R. H. Jones, "Fitting a continous time autoregression to discrete data," *Academic, Applied Time Series Analysis II*, 1981.

[4] R. Wallin and A. Isaksson, "Multiple optima in identification of ARX models subject to missing data," *EURASIP Journal on Applied Signal Processing*, vol. 1, pp. 30–37, 2002.

[5] A. Janssen, R. Veldhuis, and L. Vries, "Adaptive interpolation of discrete-time signals that can be modeled as autoregressive processes," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 34, no. 2, April 1986.

[6] P. Albertos, R. Sanchis, and A. Sala, "Output prediction under scarce data operation: control applications," *Automatica, 35*, pp. 1671–1681, 1999.

[7] A. Isaksson, "A recursive EM algorithm for identification subject to missing data," *SYSID, Copenhagen, Denmark*, pp. 679–684, 1994.

[8] S. Mirsaidi, G. Fleury, and J. Oksman, "LMS like AR modeling in the case of missing observations," *IEEE Transactions on Signal Processing*, vol. 45, no. 6, 1997.

[9] R. Sanchis and P. Albertos, "Recursive identification under scarce measurements-convergence analysis," *Automatica, 38*, pp. 535–544, 2002.

[10] R. Zgheib, G. Fleury, and E. Lahalle, "New fast recursive algorithms for simultaneous reconstruction and identification of ar processes with missing observations," *Proc. Fourteenth European Signal Processing Conference, EUSIPCO, Florence, Italy*, 2006.

[11] P. Bondon, "Prediction with incomplete past of a stationary process," *Elsevier, Stochastic processes and their applications*, pp. 67–76, 2002.

[12] P. Brockwell and R. Davis, *Time Series: Theory and methods*, Springer-Verlag, 1991.

[13] A. Fletcher, Sundeep Rangan, and Vivek Goyal, "Estimation from lossy sensor data: Jump linear modeling and kalman filtering," *The 3rd International Symposium on Information Processing in Sensor Networks, California, USA*, 2004.