Second-order near field localization with automatic paring operation
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2. DEFINITION OF THE MODEL AND ASSOCIATED COVARIANCE SEQUENCES

2.1. Definition of the model

We consider $M$ near-field, uncorrelated, narrowband sources. Each source is characterized by its range $r_m$, its bearing $\omega_m$ and a complex amplitude $\alpha_m(t)$. Let a linear array with $N = 2L + 1$ uniformly spaced sensors with interelement spacing $\Delta$. In near-field, using a second-order Taylor expansion, the $t$-th observation on sensor $p \in [-L : L]$ can be approximately [5] modeled as:

$$x_p(t) = \sum_{m=1}^{M} \alpha_m(t)e^{i(\omega_m t + \phi_m p^2)} + n_p(t)$$

(1)

where $n_p(t)$ is an additive gaussian noise spatially white. The pulsations $\omega_m$ and $\phi_m$ are functions of the bearing $\theta_m$ and the range $r_m$ of the $m$-th source, they can be expressed as :

$$\omega_m = -2\pi \frac{\Delta}{\lambda} \sin \theta_m$$

(2)

$$\phi_m = \pi \frac{\Delta^2}{\lambda^2} \cos^2 \theta_m$$

(3)

where $\lambda$ is the source wavelength.

2.2. Covariance sequences

If we assume that the near-field sources are uncorrelated, i.e., $E\{\alpha_m(t)\alpha_{m'}(t)^*\} = \sigma_m^2 \delta_{m-m'}$, where $\sigma_m^2$ is the variance of the $m$-th source, then

$$r_n(p) = E\{x_{p+b}(t)x_p(t)^*\}$$

(4)

$$= \sum_{m=1}^{M} \sigma_m^2 e^{i(\omega_m b + \phi_m b^2)} e^{2i\phi_{mb}}.$$ 

(5)

3. JOINT VANDERMONDE-TYPE DECOMPOSITION

3.1. Definition of the tensor

Let $\mathcal{H}$ be a $L \times L \times 2$ three-order tensor (or three-way array). We denote by $[\mathcal{H}]_{s}$ the $s$-th vertical slice of tensor $\mathcal{H}$. Then, we define each slice by

$$[\mathcal{H}]_1 = H_0^*, \quad [\mathcal{H}]_2 = H_1$$

(6)
where ∗ means conjugate and

\[ H_0 = \begin{bmatrix}
  r_{-1}\left(-\frac{L}{2}\right) & r_{-1}\left(-\frac{L}{2} + 1\right) & \cdots & r_{-1}(0) \\
  r_{-1}\left(-\frac{L}{2} + 1\right) & r_{-1}\left(-\frac{L}{2} + 2\right) & \cdots & r_{-1}(1) \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{-1}(0) & r_{-1}(1) & \cdots & r_{1}\left(\frac{L}{2}\right)
\end{bmatrix}, \]

\[ H_1 = \begin{bmatrix}
  r_{1}\left(-\frac{L}{2}\right) & r_{1}\left(-\frac{L}{2} + 1\right) & \cdots & r_{1}(0) \\
  r_{1}\left(-\frac{L}{2} + 1\right) & r_{1}\left(-\frac{L}{2} + 2\right) & \cdots & r_{1}(1) \\
  \vdots & \vdots & \ddots & \vdots \\
  r_{1}(0) & r_{1}(1) & \cdots & r_{1}\left(\frac{L}{2}\right)
\end{bmatrix}, \]

with \( r_0(p) \) defined in expression (4). Remark that tensor \( \mathcal{H} \) is partially Hankel-structured, i.e., each vertical slice has a Hankel structure.

### 3.2. "Two-side" Vandermonde-type decomposition

First remark that the two covariance sequences involved in each vertical slice is defined according to

\[ r_1(p) = \sum_m \sigma_m^2 e^{i(\omega_m + \phi_m)} e^{2i\phi_1 p}, \quad r_{-1}(p) = \sum_m \sigma_m^2 e^{-i(\omega_m - \phi_m)} e^{-2i\phi_1 p}. \]

Based on these covariance sequences, there exists a "two-side" Vandermonde-structured matrix, defined by

\[ Z = \begin{bmatrix}
  e^{-2i\phi_1} & e^{-2i\phi_2} & \cdots & e^{-2i\phi_M} \\
  \vdots & \vdots & \ddots & \vdots \\
  e^{-2i\phi_1} & e^{-2i\phi_2} & \cdots & e^{-2i\phi_M} \\
  1 & 1 & \cdots & 1 \\
  \vdots & \vdots & \ddots & \vdots \\
  e^{2i\phi_1(\frac{L}{2} - 1)} & e^{2i\phi_2(\frac{L}{2} - 1)} & \cdots & e^{2i\phi_M(\frac{L}{2} - 1)}
\end{bmatrix}_{L \times M}, \]

which jointly decomposes each vertical slice of tensor \( \mathcal{H} \) according to

\[ H_0^* = Z\Theta_0 Z^T, \quad H_1 = Z\Theta_1 Z^T. \]

where

\[ \Theta_0 = \begin{bmatrix}
  \sigma_1^2 e^{i(\omega_1 - \phi_1)} & 0 \\
  0 & \ddots & \sigma_M^2 e^{i(\omega_M - \phi_M)}
\end{bmatrix}, \]

\[ \Theta_1 = \begin{bmatrix}
  \sigma_1^2 e^{i(\omega_1 + \phi_1)} & 0 \\
  0 & \ddots & \sigma_M^2 e^{i(\omega_M + \phi_M)}
\end{bmatrix}. \]

### 4. ESTIMATION OF THE BEARING AND THE RANGE BY ALS-PARAFAC ANALYSIS

#### 4.1. Decomposition of the modes of tensor \( \mathcal{H} \)

The particular structure described in the previous section implies a suitable structure into the modes (or also called matrix unfoldings) of tensor \( \mathcal{H} \). Let \( H^{(1)} \), \( H^{(2)} \) and \( H^{(3)} \) be these modes. The explicit definition of these matrices can be found in reference [6] and by tacking into account expressions (10) and (11), it comes

\[ H^{(1)} = H^{(2)} = \left[ H_0^* H_1 \right] = Z \left[ \Theta_0 Z^T \Theta_1 Z^T \right] = Z (\Omega \otimes Z)^T \]

\[ H^{(3)} = \left[ \text{vec}(H_0) H^T \right] = \text{vec}(\Theta_1)^T (Z \otimes Z)^T \]

where

\[ \Omega = \begin{bmatrix}
  \sigma_1^2 e^{i(\omega_1 - \phi_1)} & \sigma_2^2 e^{i(\omega_2 - \phi_2)} & \cdots & \sigma_M^2 e^{i(\omega_M - \phi_M)} \\
  \sigma_2^2 e^{i(\omega_1 + \phi_1)} & \sigma_2^2 e^{i(\omega_2 + \phi_2)} & \cdots & \sigma_M^2 e^{i(\omega_M + \phi_M)} \\
  \vdots & \vdots & \ddots & \vdots \\
  \sigma_M^2 e^{i(\omega_1 + \phi_1)} & \sigma_M^2 e^{i(\omega_2 + \phi_2)} & \cdots & \sigma_M^2 e^{i(\omega_M + \phi_M)}
\end{bmatrix}_{2 \times M} \]

and vec(\( X \)) is the vector formed by concatenating all the columns of \( X \), \( \otimes \) (resp. \( \otimes \)) stands for the Khatri-Rao product (resp. Kronecker product). Note that the first and the second modes are equal since the first and the second sizes of the tensors are also equal (\( = L \)).

#### 4.2. Extraction of the model parameters

We can extract the model parameters by using the ALS-PARAFAC algorithm [7]. According to expressions (10) and (11), we can see that the \( m \)-th bearing and the \( m \)-th range are obtained simultaneously, i.e., if matrix \( Z \) is column-permuted then the diagonal matrices \( \Theta_0 \) or \( \Theta_1 \) are accordingly column-permuted. This structural property explains why the pairing operation is automatic. Now, suppose that the resulting matrices of the ALS-PARAFAC analysis are denoted by \( Z_0, Z_1 \) and \( \Omega \). These matrices are known up to a scaling of each of their columns. This corresponds to the indeterminacies in expressions (10) and (11) since there exists two invertible diagonal matrices \( D_0 \) and \( D_1 \) according to

\[ H_0^* = Z_0 (D_0^{-1} D_0^{T}) \Theta_0 (D_0 D_0^{-1}) Z_0^T \]

\[ H_1 = Z_1 (D_1^{-1} D_1^{T}) \Theta_1 (D_1 D_1^{-1}) Z_1^T. \]

But, we know that the \( (L + 1) \)-th term of matrices \( Z_0, Z_1 \) must be one (cf. matrix \( Z \)) so the scaling operation is characterized by

\[ Z = Z_0 D_0^{-1}, \quad Z = Z_1 D_1^{-1}. \]

where

\[ D_0 = \begin{bmatrix}
  [Z_0] \frac{L}{2} + 1, 1 & 0 \\
  \vdots & \ddots & \vdots \\
  [Z_0] \frac{L}{2} + 1, M \\
  [Z_1] \frac{L}{2} + 1, 1 & 0 \\
  \vdots & \ddots & \vdots \\
  [Z_1] \frac{L}{2} + 1, M
\end{bmatrix}, \]

\[ D_1 = \begin{bmatrix}
  [Z_1] \frac{L}{2} + 1, 1 & 0 \\
  \vdots & \ddots & \vdots \\
  [Z_1] \frac{L}{2} + 1, M
\end{bmatrix}, \]

in which \([Q]_{ij}\) means the \((i, j)\)-th entry of matrix \( Q \). From expressions (14), (15), (17) and (18), we deduce

\[ \Omega = \Omega D_0 D_1. \]

Finally, based on matrices defined in (19) and (22), the model parameter \( \{\omega_m, \phi_m\}_{m=1, \ldots, M} \) are estimated according to

\[ \hat{\omega}_m, \hat{\phi}_m = \left( \frac{\hat{Z}_{1, m}}{|\hat{Z}_{1, m}|^2} \right)^{\frac{1}{2}}, \quad \hat{\omega}_{n+1, m}, \hat{\phi}_{n+1, m} = \frac{1}{L} \sum_{k=1}^{L} \left( \frac{\hat{Z}_{n+1, m}}{|\hat{Z}_{n+1, m}|^2} \right)^{\frac{1}{2}} \]

2570
where \( \angle(\cdot) \) is the angle argument. Using (2) and (3), the bearing a
range parameters can be easily estimated from \( \omega_m \) and \( \phi_m \).

Note that as the first and the second modes are identical, we can
use a symmetric ALS-PARAFAC model to decrease the complexity
burden. In this case, the ALS-PARAFAC algorithm needs only to
two pseudo-inverses per iteration instead of three.

5. SIMULATION RESULTS

In this simulations we consider an array of \( N = 17 \) sensors with
a spacing of \( \Delta = \lambda/4 \). In order to compare the results with the
HOS-ESPRIT method [2], the source signal is not gaussian and is
given by \( e^{j\xi} \) where \( \xi \) is uniformly distributed in \([0, 2\pi]\).

The covariance sequences \( r_{-1}(p) \) and \( r_1(p) \) defined in expres-
sion (4) are estimated using the sample covariance estimate :

\[
\tilde{r}_{-1}(p) = \frac{1}{T} \sum_{t=1}^{T} x_{p-1}(t)x_p(t)^* \tag{24}
\]

\[
\tilde{r}_1(p) = \frac{1}{T} \sum_{t=1}^{T} x_{p+1}(t)x_p(t)^* \tag{25}
\]

where \( T \) is the number of snapshot.

5.1. One source

In this example, we consider a single source with unit power,
bearing is fixed to \( \theta_1 = 5^\circ \) and range is \( r_1 = 0.1\lambda \). The number of
snapshots used to estimate the covariance sequence is 10. The SNR
is varying from \(-10 \) dB to \( 30 \) dB.

On Fig. 1, the Root Mean Square Error (RMSE) of the bearing and
range estimate is plotted versus the SNR. Performances of the
proposed method (markers ‘*) are compared with those of the HOS
ESPRIT (markers ‘o’). In addition, the Cramér-Rao Bound (CRB) is
also plotted in dashed line. Expression of CRB can be easily derived
for the near-field joint bearing and range estimator (see for instance
[8]).

Performances plotted in Fig. 1 show that the RMSE of the proposed
method is lower than which of the HOS-ESPRIT method, in particu-
lar for the range estimate. Notice also that in this case, the RMSE of
the tensor-based method approaches the CRB.
5.2. Two sources

In this example, we consider two sources both with unit power, bearing is fixed to $\theta_1 = 5^\circ$ and $\theta_2 = 40^\circ$, range is $r_1 = 0.4\lambda$ and $r_2 = 0.1\lambda$. The number of snapshots is 10 and the SNR is varying from $-10$ dB to $30$ dB.

The RMSE plotted in figure 2 show that even if the CRB is not reached, the proposed method achieves an accuracy which is enough for practical applications. Notice that in this case the HOS-ESPRIT method is unable to estimate the bearing and range parameters.

5.3. Performance with respect to the number of snapshot

In this example, we consider the same two sources of the previous example, but the SNR is fixed to $10$ dB and the number of snapshots is varying from 5 to 100.

The RMSE is plotted on figure 3, these simulations show that the interest of the proposed method is mainly for a small or moderate number of snapshots. In this case the HOS methods are known to fail because of the difficulties to estimate cumulant with a small number of snapshots.

All these simulations have shown the advantages of the proposed approach with respect to the HOS-ESPRIT, in particular for small number of snapshots and for low SNR. Notice that these conditions correspond to the situation of interest in practical use of source localization algorithm.

6. CONCLUSION

In the context of localization of near-field sources. We propose a new method which exploits a tensor-based scheme of the covariance sequence in order to provide directly an estimation of the pair of bearing and range for each source without an additional pairing operation. Since it is based on the SOS properties of the model, the method is more accurate than the HOS-ESPRIT for small/moderate number of snapshots and in noisy environment. Numerical simulations and comparison with HOS-ESPRIT have demonstrated that the proposed tensor-based algorithm is well suited for a practical application to source localization in near-fields.

7. REFERENCES