Opportunistic Power Allocation for Point-to-Point Communication in Self-Organized Networks

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Abstract— **In this paper, we study the performance of point-to-point communication in self-organized networks. An information-theoretic framework is considered to determine the optimum transmission power enabling reliable communication between neighboring nodes at a certain user requested rate. Realistic channel models taking into account path-loss and fading are also considered. Optimal power allocation strategies are investigated. For this purpose, we use the *ergodic* capacity as a criterion for information-theoretic analysis. In the first case, the transmit power is the same for all pairs. In a more opportunistic case, different transmit pairs use different transmit powers according to their channel realizations. Then, for a given rate requirement C, we derive the optimal close-talker distance that meets the requirement for a given power P. Numerical results are finally presented giving insights into the design of power adaptation schemes for point-to-point communications.**

I. INTRODUCTION

Recently, self-organized networks have been drawing a lot of interest [9] and [11]-[18]. Given that these networks are self-configured, without any dependence on a central controller (as currently done in cellular systems), they offer high flexibility of deployment and maintenance. Several inherent issues pertain to self-organized networks. The notions of *cooperation* and *coalition* [9] have been addressed where users cooperate by forming coalitions to improve their individual utilities. The issue of cooperation has also been studied in [11] where intelligent nodes cooperate using distributed Multiple-Input-Multiple-Output (MIMO) techniques.

The general problem of addressing how the network throughput scales as a function of the number of source-destination (S-D) pairs has been subject to intensive research. In their seminal paper, Gupta and Kumar [12] have shown that the fundamental performance limitation comes from the fact that long-range direct communication between user pairs is infeasible due to the excessive interference coming from other nodes. As a result, most communications have to occur between neighbors, at a distance of order $\frac{1}{\sqrt{N}}$ (N is the number of users) and the throughput scales as $O(\sqrt{N/logN})$. In [19], percolation theory arguments were used to determine the capacity of the network. Finally, only recently, the trade-off between throughput and delay incurred by point-to-point communication has been investigated in [16].

The issue of point-to-point communication has been subject to intensive research, where most of the literature has focused on throughput scaling laws as a function of the number of mobile pairs for both static and mobile cases. Besides,

other papers have mainly focused on networking issues. For instance, in [1], vehicles that are close to each other use direct communication using industrial, scientific and medical (ISM) radio band modems mounted on the vehicles. When they are far away or there is an obstacle such as trucks, a cellular mode can be used instead. In [2], cellular ad-hoc united communication is proposed enabling direct terminal to terminal communication. Finally, the study of a hybrid system was used in [4]-[6].

In the first part of this contribution, a useful information-theoretic framework is provided to determine the optimum transmission power enabling reliable communication. The *ergodic* capacity is used as a relevant performance and to show that there is a bound on the achievable requested rate. In the opportunistic approach where nodes access channel state information (CSI), we apply the well-known results of waterfilling [8] to the ergodic capacity. Finally, we introduce the notion of *close-talker* where users can estimate the closest mobile within a certain distance.

The paper is organized as follows. In Section II, the system model is introduced. Different power allocation strategies are investigated in Section III. Finally, numerical results are shown in Section IV and conclusions are drawn in Section V.

II. SYSTEM MODEL

We consider a two-dimensional (2-D) network with average density of users d and radius R (where $R \to \infty$). The users are randomly distributed in the plane. The network contains therefore $N = \pi R^2 d$ mobiles. We suppose that time is slotted by a universal clock that every mobile is aware of, and that in each timeslot, $\frac{N}{2}$ communication pairs form at random between neighboring nodes, with $\frac{N}{2}$ transmitters and $\frac{N}{2}$ receivers. Each link between a communication pair experiences path-loss (depending on the distance between two mobiles) and flat fading. In addition, it is assumed that each transmitter is able to adjust its transmission power. The received signal y_j at mobile j of the communication pair (i,j) has the form:

$$y_j = \frac{h_{ji}}{r_{ji}^{\alpha/2}}\sqrt{P_i}s_i + \sum_{k \neq i}\frac{h_{jk}}{r_{jk}^{\alpha/2}}\sqrt{P_k}s_k + n_{ji} \qquad (1)$$

In which s_i is the useful signal, transmitted with power P_i to mobile j by mobile i, and affected by path-loss $\frac{1}{r_{ji}^{\alpha/2}}$ (where r_{ji} is the distance between mobiles i and j, and α is the path-loss exponent usually between 2 and 6). h_{ji} is the fading

component. The sum in the second term is taken over all $\frac{N}{2}-1$ transmitters for $k \neq i$. Each term in the sum represents the contribution to the interference of transmitter k. Finally, n_{ji} is the Additive White Gaussian Noise (AWGN). We make the assumption that the signals sent by users are encoded in a Gaussian codebook. All channel coefficients h_{lm} and noise n_{lm} are supposed to be independent Gaussian variables, with zero mean and variance 1 and N_0, respectively.

III. OPTIMAL POWER ALLOCATION STRATEGIES

In this section, we investigate different power allocation strategies for point-to-point communication. We derive the optimum transmit power for a high number of interferers. In the first case, the transmit power is assumed to be equal for all mobile pairs. The notion of close-talker is also presented. In the other case, different communication pairs will use different transmission powers according to their channel realizations (*opportunistic* approach).

A. Uniform Power Analysis

We would like to determine the optimal power allocation assuming that each transmitter only knows the statistics of its channel (i.e., each communication pair (i,j) knows the distribution of r_{ji}, and the variances $\zeta_{ij} = \zeta$ and N_0) so that the user's requested rate C is satisfied. By symmetry, each communication pair will use the same power P in this setting. Given the random nature of the channel, the *ergodic* capacity can be approached using an appropriate coding scheme. In our setting, it can be expressed as:

$$C(i,j) = \mathbb{E}_{h,r}\left[\log_2\left(1 + \frac{P\,|h_{ij}|^2\,r_{ij}^{-\alpha}}{P\sum_{k\neq i}|h_{jk}|^2\,r_{jk}^{-\alpha} + N_0}\right)\right] \tag{2}$$

We want to ensure that $C(i,j) > C$ for all users. For this purpose, we have to find an estimate of the interference sum:

$$\sum_{k\neq i}|h_{jk}|^2\,r_{jk}^{-\alpha} \tag{3}$$

which runs over all transmitting nodes except node i.

B. No Interference Case

In order to have an element of comparison, we first investigate the case when there is no interference. If only *one* node transmits, the expression of the *ergodic* capacity reduces to:

$$C(1,2) = \mathbb{E}_{h,r}\left[\log_2\left(1 + \frac{P\,|h|^2\,r_{12}^{-\alpha}}{N_0}\right)\right] \tag{4}$$

According to [10], the probability distribution of r in (4) can be modeled as:

$$f_1(r) = \frac{2r}{\rho^2}e^{-\frac{r^2}{\rho^2}} \text{ with } \rho = \sqrt{\frac{2d}{\pi}} \tag{5}$$

Where $d = \mathbb{E}(r)$ is the average mobile separation. Thus, we can obtain the value of the ergodic capacity for a given power
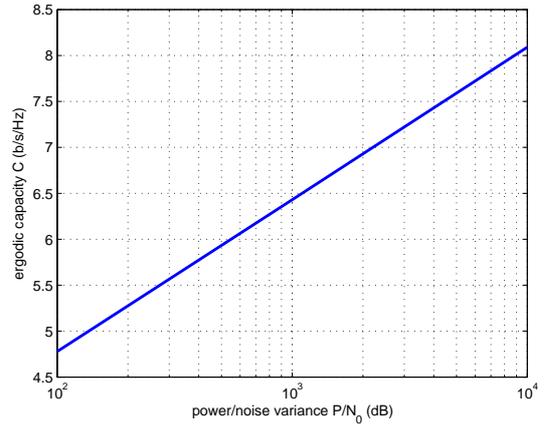

Fig. 1. No Interference Case, Same transmit power for all mobile pairs (case $N = 2$).

P. In this case, the minimum power P within which the requested rate C can be satisfied, is given by the equation:

$$\mathbb{E}_{h,r}\left[\log_2\left(1 + \frac{P\,|h|^2\,r^{-\alpha}}{N_0}\right)\right] = C \tag{6}$$

The left-hand side of (6) is an increasing function of P that does not saturate (i.e., it tends to infinity as $P \to \infty$). In this case, with a sufficient transmit power, any capacity requirement can be fulfilled.

Figure 1 shows the ergodic capacity as a function of the transmission power, in the case of no interference. Capacity is logarithmically increasing without bound along with the power. Hence, without interference, any capacity requirements can be met if the power is sufficiently increased

C. General Case

Let us now investigate the case $N > 2$. In this case, the communication pairs impose interference on each other given by the sum (3). In our setting, the number of interferers tends to infinity, therefore we can use *asymptotic* results. The sum given by (11) will be approximated by its expectation, which is simply $\sum_{k\neq i}\mathbb{E}[|h_{jk}^2| * r_{jk}^{-\alpha}] = \sum_{k\neq i}\mathbb{E}[|h_{jk}^2|] * \mathbb{E}[r_{jk}^{-\alpha}] = \sum_{k\neq i}\mathbb{E}_r[r_{jk}^{-\alpha}]$ since the random variables r and h are independent and $\mathbb{E}_{h_{ij}}\left[|h_{jk}|^2\right] = 1$ for all k.

According to [10], the probability distribution of the distance between mobile j and its kth closest mobile can be modeled by:

$$f_k(r_{jk}) = \frac{2r_{jk}^{2k-1}}{(k-1)!\rho^{2k}}e^{-\frac{r_{jk}^2}{\rho^2}} \text{ with } \rho = \sqrt{\frac{2d}{\pi}} \tag{7}$$

A mobile has probability $\frac{1}{2}$ to be transmitting. Thus, using (7), the expectation of (3) becomes:

$$\sum_{k\neq i}\mathbb{E}_r\left[r_{jk}^{-\alpha}\right] = \frac{1}{2}\sum_{k=2}^{+\infty}\int_0^{+\infty}\frac{2u^{2k-1-\alpha}}{(k-1)!\rho^{2k}}e^{-\frac{u^2}{\rho^2}}\,du. \tag{8}$$
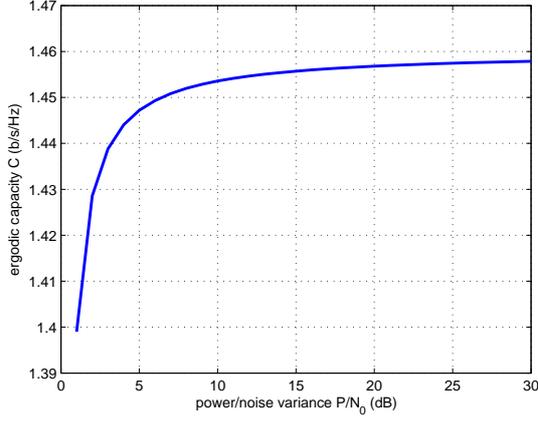
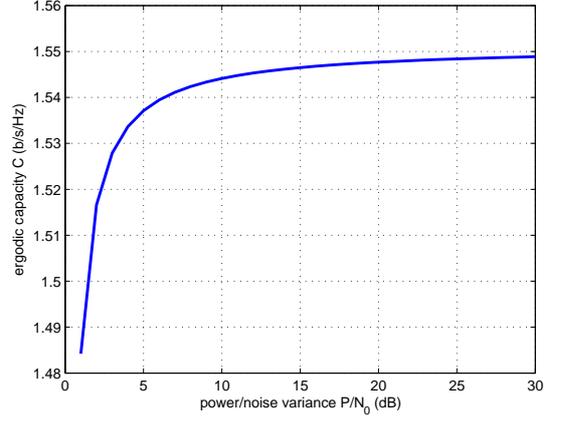Fig. 2. Interference case, same Power, $\alpha = 3$.

Fig. 3. Same Power, Maximum Distance $a = 1$, $\alpha = 3$.

The integrals in (8) are well-defined for all k if $\alpha < 4$[1]. In this case[2], since for all $N \geq 2$,

$$\left| \sum_{k=2}^{N} \frac{2u^{2k-1-\alpha}}{(k-1)! \rho^{2k}} e^{-\frac{u^2}{\rho^2}} \right| \leq \frac{2u^{1-\alpha}}{\rho^2} \left(1 - e^{-\frac{u^2}{\rho^2}} \right),$$

Lebesgue's dominated convergence theorem [20] allows us to intervert sum and integral, and finally obtain:

$$I = \sum_{k \neq i} \mathbb{E}_r \left[r_{jk}^{-\alpha} \right] = \frac{1}{2} \int_0^{+\infty} \frac{2u^{1-\alpha}}{\rho^2} \left(1 - e^{-\frac{u^2}{\rho^2}} \right) du. \quad (9)$$

Using this result, we can deduce the transmission power corresponding to the requested rate C thanks to the equation:

$$\mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{P |h|^2 r^{-\alpha}}{PI + N_0} \right) \right] = C. \quad (10)$$

In this case, the ergodic capacity is an increasing function of P that has an horizontal asymptote for some capacity C_0, given by:

$$C_0 = \mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{|h|^2 r^{-\alpha}}{I} \right) \right]. \quad (11)$$

All capacity requirement strictly below C_0 can be met; but capacity requirements above C_0 will lead to users increasing their transmission power without bound and will not be attained. The value C_0 for $\alpha = 3$ is around 1.457 (b/s/Hz).

Fig. 2 shows the ergodic capacity as a function of the transmission range. In this case, the capacity obtained with a given power is much lower, and there exists a maximum capacity C_0, given by (15). If mobiles try to meet capacity requirements above C_0, they will increase their power without bound, without reaching those requirements. Hence, since not all the values can be met, we need to define a communication range where any rate C can be satisfied. This is developed in the next section and is referred to the close-talker case.

[1] $1 + \alpha - 2k < 0$

[2] The probability that n mobiles are within r is: $1 - e^{-\frac{r^2}{\rho^2}} \sum_{k=0}^{n-1} \frac{1}{k!} (\frac{r^2}{\rho^2})^k$

D. Close-Talker Case

Let us suppose that mobiles can estimate whether the closest mobile to them is within a distance a. Then, a communication pair forms between two mobiles only if their distance is smaller than a. This pertains to the notion of clusters. According to [11] and [12], the optimal strategy is to confine to nearest neighbor communication and maximize the number of simultaneous transmissions, through spatial reuse.

In this case, the probability distribution of the distance to the closest mobile changes to:

$$f_1^a(r) = \begin{cases} 0 & \text{if } r > a, \\ \frac{f_1(r)}{1 - e^{-\frac{a^2}{\rho^2}}} & \text{if } r \leq a. \end{cases} \quad (12)$$

The interference sum (3) has to be multiplied by $\left(1 - e^{-a^2/\rho^2} \right)$, that is the probability that the closest mobile is indeed at distance at most a of the receiving mobile in the communication pair. Under this setting:

$$I = \left(1 - e^{-\frac{a^2}{\rho^2}} \right) \int_0^{+\infty} \frac{u^{1-\alpha}}{\rho^2} \left(1 - e^{-\frac{u^2}{\rho^2}} \right) du. \quad (13)$$

We can deduce the transmission power corresponding to the requested rate C thanks to the equation:

$$\mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{P |h|^2 r^{-\alpha}}{PI + N_0} \right) \right] = C. \quad (14)$$

This case is similar to the one in subsection III-C. Hence, the attainable capacity is bounded by:

$$C_0^a = \mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{|h|^2 r^{-\alpha}}{I} \right) \right]. \quad (15)$$

For a given C, the optimum communication range a is given by:

$$\int_0^a \mathbb{E}_h \left[\log_2 \left(1 + \frac{P |h|^2 r^{-\alpha}}{PI + N_0} \right) \right] = C. \quad (16)$$

In Fig. 3, the ergodic capacity is plotted for $a = 1$. The shape of the curve is the same as in the general case of
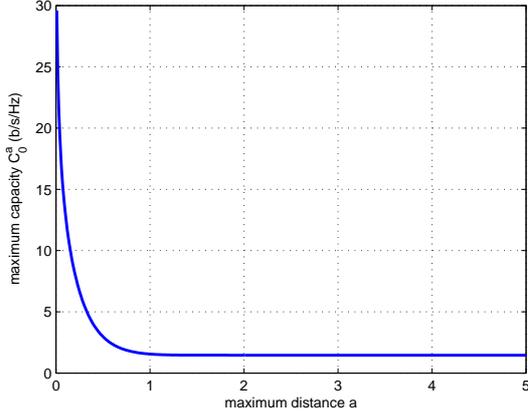
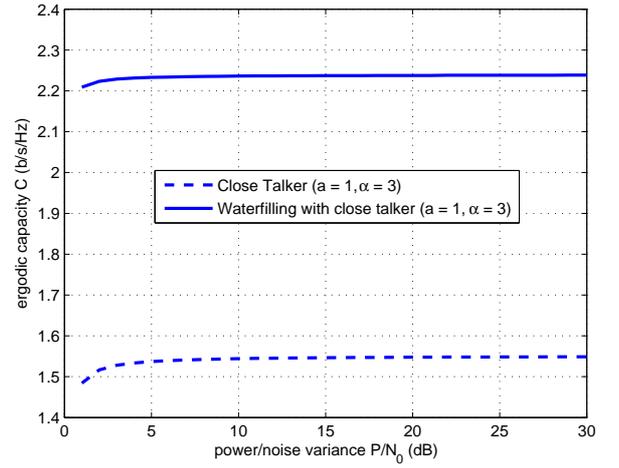Fig. 4. Same Power, maximum distance a, $\alpha = 3$.

Fig. 5. Comparison of the ergodic capacity for the opportunistic case (with close-talker) versus the close-talker approach, for $a = 1$ and $\alpha = 3$.

Subsection III-C, but the capacity obtained with a given power is higher. This is due to the fact that the capacity is higher within the communication range a. There is also a maximum capacity C_0^a, that depends on the value of a as shown in Fig. 4. As a decreases, the maximum attainable capacity C_0^a given by (15) increases.

Even though any rate can be satisfied, one has to notice that a delay is incurred by this scheme depending on the mobility pattern [16].

E. Different Mobile Pairs Transmit Power (Opportunistic Approach)

In this section, we suppose that the communication pairs have perfect channel state information, i.e. communication pair (i, j) knows the exact value of h_{ji}. In this case, different communication pairs will use different transmission powers, according to the realization of their channel. Moreover, we assume the close-talker case as discussed before.

We can readily apply the power adaptation described in [8] and allow the power $P(t)$ to vary with $t = |h^2|$ following a Rayleigh distribution. Thus, we have a time waterfilling where the average power will be:

$$\bar{P} = \int_0^a \int_0^{+\infty} P(t, r) e^{-t} f_1^a(r) dt dr. \tag{17}$$

Given an average power constraint $\bar{P}$, the capacity of the pair (i, j) is given by:

$$C(i, j) = \max_{\bar{P}} \mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{P_{ij} |h_{ij}|^2 r_{ij}^{-\alpha}}{\bar{P}I + N_0} \right) \right] \tag{18}$$

where the interfering term is given by:

$$I = \left(1 - e^{-\frac{a^2}{\rho^2}} \right) \int_0^{+\infty} \frac{u^{1-\alpha}}{\rho^2} \left(1 - e^{-\frac{u^2}{\rho^2}} \right) du.$$

Note that the average power defined in (17) uses waterfilling on a channel that is independent of the channel h_{jk} where

$k \neq i$. The power adaptation which maximizes (18) is:

$$\frac{P(t, r)}{\bar{P}I + N_0} = \begin{cases} \frac{1}{t_0} - \frac{1}{tr^{-\alpha}} & \text{if } t \geq t_0 r^\alpha \\ 0 & \text{if } t < t_0 r^\alpha \end{cases}$$

for some "cutoff" value t_0 that satisfies

$$\int_0^a \int_{t_0 r^\alpha}^{+\infty} \left(\frac{1}{t_0} - \frac{1}{t \cdot r^{-\alpha}} \right) e^{-t} f_1^a(r) dt dr = \frac{\bar{P}}{\bar{P}I + N_0}. \tag{19}$$

Is is worth mentioning that in contrast to [8], this waterfilling approach takes the path-loss α and interference into account. These parameters are necessary in order to compute the cutoff value t_0. Finally, the capacity is given by:

$$C(i, j) = \int_0^a \int_{t_0 \cdot r^\alpha}^{+\infty} \log_2 \left(1 + \left(\frac{t}{t_0} - r^\alpha \right) r^{-\alpha} \right) e^{-t} f_1^a(r) dt dr. \tag{20}$$

Figure 5 shows a comparison between the ergodic capacity for both the opportunistic and non-opportunistic approach. In particular, we focus on the close-talker approach where nodes communicate within a distance a. Clearly, the capacity is higher (46% increase) in the opportunistic approach where mobile terminals have knowledge of their channels.

F. Frequency Reuse for Point-to-Point Communication

Here, we seek the impact of frequency reuse on the ergodic capacity of point-to-point communication. We suppose one frequency band $B = f_{ru} \times W$ of width W and f_{ru} is the frequency reuse. R is the radius of the considered area. We further assume concentric circles centered in the origin as illustrated in Figure 6. Equation (9) rewrites as:

$$I = \sum_{k \neq i} \mathbb{E}_r \left[r_{jk}^{-\alpha} \right] \tag{21}$$

$$= \sum_{k=1}^R \int_{a f_{ru}(k-1)}^{a(f_{ru}k - f_{ru}+1)} \frac{u^{1-\alpha}}{\rho^2} \left(1 - e^{-\frac{u^2}{\rho^2}} \right) du$$
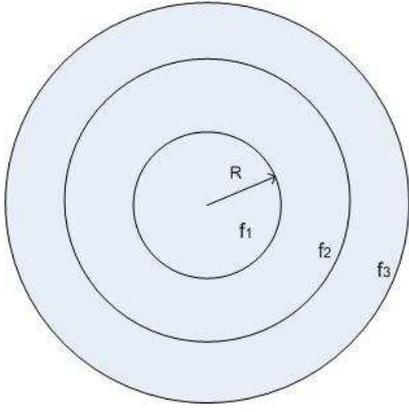
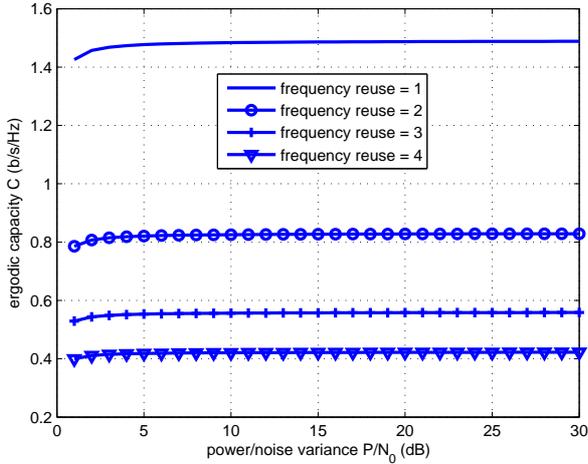Fig. 6. Illustration of the frequency reuse scheme.

Fig. 7. The impact of the frequency reuse on the ergodic capacity, for frequency reuse of 1, 2, 3 and 4.

The capacity is defined as:

$$C = \frac{B}{f_{ru}} \times \mathbb{E}_{h,r} \left[\log_2 \left(1 + \frac{P \left| h \right|^2 r^{-\alpha}}{PI + N_0} \right) \right]. \quad (22)$$

Finally, Figure 7 depicts the impact of frequency reuse on the ergodic capacity. Frequency reuse *one* yields a higher capacity (75% increase) with respect to frequency reuse 2. Moreover, the capacity decreases with increasing frequency reuse.

IV. CONCLUSION

In this paper, we investigated different power allocation strategies for point-to-point communication. We showed that neighboring nodes can communicate reliably at a requested rate by adjusting their power, if the requested rate is below a certain bound. The bound on the requested rate can be increased only if neighboring nodes that are close to each other are allowed to communicate. Additionally, for a given rate requirement C, we derived the optimal close-talker distance that meets the requirement for a given power P.

In an opportunistic approach, we derived the expression of the *ergodic* capacity under the assumption that nodes know the realization of their channel by applying the waterfilling principle. Finally, the impact of frequency reuse was also studied where frequency reuse *one* yields higher capacity.

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