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Oral presentation

Introduction

Correct modelling of damping is essential to capture the dynamic behaviour of a MEMS device. Our interest is squeeze-film damping which models the behaviour of a fluid in small gaps between a fixed surface and a structure moving perpendicular to this surface . The lateral dimensions of the surfaces are large compared to the gap and the system is considered isothermal. Squeeze film damping is then governed by the Reynolds equation [1]:

$$\nabla\left(\frac{G^3}{12\mu}P\nabla P\right)=\frac{\partial(GP)}{\partial t}, \qquad \textbf{(1)}$$

where $G(x,y,t)$ is the distance between the moving and the fixed surface, $P(x,y,t)$ is the pressure, $\boldsymbol{\varepsilon}$ the effective viscosity of the fluid [1]. For small excitation frequencies or amplitudes the squeezed film behaves as a linear damper. For larger amplitudes or frequencies, the gas has no time to flow away and the pressure builds up creating a stiffening effect coupled to a nonlinear damping. The boundary conditions for (1) are usually chosen as trivial: ñzero pressure variationò or ñzero pressure gradientò, although some authors have considered less ideal, frequency-dependent and aspect-ratio dependent boundary conditions [2-3]. A complete review on this equation and its different regimes can be found in [4].

Coupling the Reynolds equation to the equation governing the mechanical behaviour of the microstructure leads to a nonlinear system of partial differential equations (PDEs), which has no analytical solution and must be simplified thanks to some assumptions. The most commonly made assumptions are the following:

- uniform displacements, i.e. $\frac{\partial G}{\partial x}=\frac{\partial G}{\partial y}=0$ (for example, [5]).
- steady-state sinusoidal excitation, i.e. $G=G_e\sin(\omega t)$ [6-7].
- small displacements, i.e. $G=G_0+g$ and $g<<G_0$, where G_0 is the nominal gap of the structure at rest or close to a static equilibrium [8-9].
- small pressure variations, i.e. $P=P_0+p$ and $p<<P_0$, where P_0 is the ambient pressure.

These hypotheses prove to be useful in a variety of applications, if only for gaining insight of nonlinear damping phenomena. However, in many cases, it is difficult to justify their use: for example, it is clear to see that none of the first three hypotheses holds when trying to estimate the switching time of a micro-switch. Most micro-switches do not undergo uniform displacements, nor can these displacements be considered small, and the behaviour of a micro-switch is fundamentally transient.

To date, the most notable attempts to tackle the problem of reduced-order modelling (ROM) of squeeze-film damping with large, non-uniform displacements have been made by Younis *et al.* [10-12], Mehner *et al.* [6,13], Yang *et al.* [14-15], Hung and Senturia [16] and Rewienski and White [17-18]. In [10-12], the authors propose to solve the nonlinear Euler-Bernoulli beam equation to determine the static deformation of a microplate under a voltage bias. The von Karman plate equations and the compressible Reynolds equation are then linearized close to this operating point and a perturbation method is used to calculate the pressure deviation. In [6,13], the authors use a modal projection method to calculate modal frequency-dependent damping and stiffening coefficients close to a determined operating point. To extend this approach to large displacements, Mehner [6] gives an analytical expression of these coefficients as a function of mechanical modal coordinates established by fitting of simulation data for different initial deformations. These approaches are all based on several steady-state sinusoidal calculations [10-12] or simulations [6,13], which increase the time for setting up the reduced-order model. The most general approaches may well be those developed in [14-18]: the authors rely on fully-coupled, nonlinear transient simulations of the complete system (usually a micro-switch) to establish a reduced-order model of the microstructure. These approaches are very general and they can even be successfully applied to the fully nonlinear Reynolds equation (1). However, they have a high computational cost (because of the nonlinear/multiphysics/transient simulation they require) and their accuracy depends, to some degree, on the choice of the training trajectory.
We present a reduced-order model of the Reynolds equation which is valid for large, non-uniform displacements and transient excitation but with the restriction of small pressure variations. This hypothesis is valid for a microswitch as shown by the results presented in [19].

Construction of the reduced-order model

We work on the variable $p = P - P_0$, supposing $p << P_0$. (1) has then the following form:

$$\nabla\left(\frac{G^3}{12\mu}\nabla p\right) = \frac{\partial}{\partial t}\left(G\left(1+\frac{p}{P_0}\right)\right) \qquad \textbf{(2)}$$

The first reduction step is based on modal projection of (2) which is first transformed via a change of variable on *p*. The aim of this change of variable is to obtain a spatial operator for which the Laplacian eigenmodes are more relevant than for the operator in (2), conserving its self-adjoint property thus guarantying convergence of the solution. The reduced-order model may be written as:

$$\frac{d}{dt}(\mathbf{A}(\mathbf{x})\mathbf{s} - \mathbf{f}(\mathbf{x})) = \mathbf{H}(\mathbf{x})\mathbf{s}\,, \qquad \textbf{(3)}$$

where **x** and **s** are respectively the vectors of the mechanical modal coordinates of the moving structure, and of the modes corresponding to the Reynolds equation. For a structure under electrostatic actuation, one may write the full coupled model as:

$$\frac{d}{dt}(\mathbf{g}(\mathbf{z}(t)) = \mathbf{f}(\mathbf{z}(t)) + \mathbf{B}(\mathbf{z}(t))u(t) \qquad \textbf{(4)}$$

where **z** is the state vector including all modal coordinates. The cost of evaluating the terms **g**(**z**(t)), **f**(**z**(t)), and **B**(**z**(t)) is reduced using a piecewise linear approach described in [17]. The fact that the coefficients in (3) only depend on the mechanical modal coordinates reduces the cost of construction of the piecewise linear model, which has the following structure:

$$\frac{d}{dt}\left[\sum_{i=0}^{s-1} w_i(\mathbf{z})(\mathbf{g}(\mathbf{z_i}) + \mathbf{JG_i}(\mathbf{z} - \mathbf{z_i}))\right] = \sum_{i=0}^{s-1} w_i(\mathbf{z})(\mathbf{f}(\mathbf{z}_i) + \mathbf{JF}_i(\mathbf{z} - \mathbf{z}_i)) + \mathbf{F}_{elec}(\mathbf{z}) \qquad \mathbf{(5)}$$

where $\mathbf{JG}_i$ and $\mathbf{JF}_i$ are respectively the jacobians of the functions **g(z)** and **f(z)** at the linearization points $\mathbf{z_i}$ and $\mathbf{F_{elec}(z)}$ corresponds to the electrostatic force. Two problems arise from the piecewise linearization: the choice of the linearization points and the weighting procedure. We choose the linearization points from a simulation of the first reduced model. A new linearization point is chosen when a point is far enough from the already chosen points. We normalize the state variables to calculate the distances so as to take into account all state variables with the most relevance. The weighting procedure is the one described in [17].

Validation of the reduced-order model

We work on the example of a microswitch also treated in [16-18]. We use the piecewise linear reduced order model to determine the switching time of the device for a step voltage between 9 and 10.5 V at atmospheric ambient pressure. The 21 linearization points are chosen along a 9.5V input training trajectory. Fig.1 shows the response to a 10V input using a linear model, the modal projection model, and the piecewise linearized model. Fig.2 shows the experimental and simulated data presented in [16] and results of our piecewise linear reduced model of order 6 for the switching time.

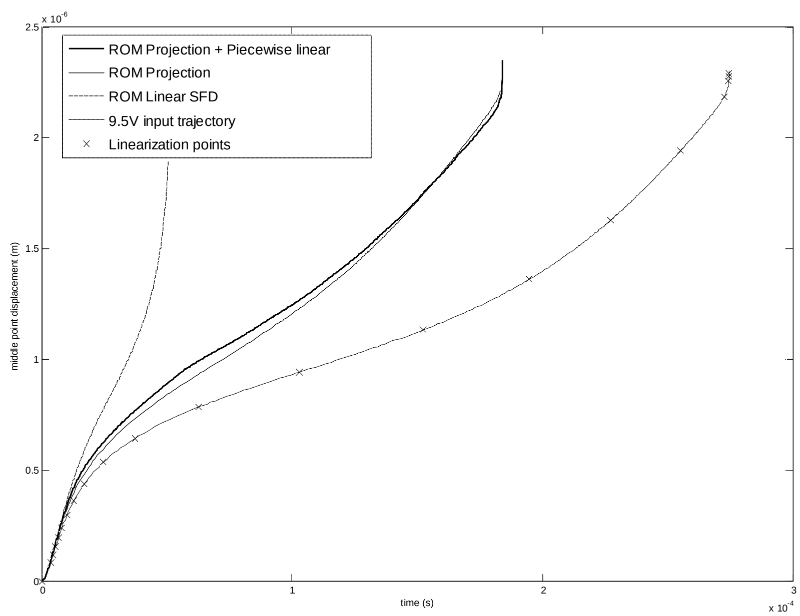

Figure 1- Middle point displacement of a microswitch for a linear model, the projected model and the projected and piecewise linearized model (1 mechanical mode, 4 ñsqueezeò modes) for a 10V step voltage. The 21 linearization points are chosen along a 9.5V input trajectory.

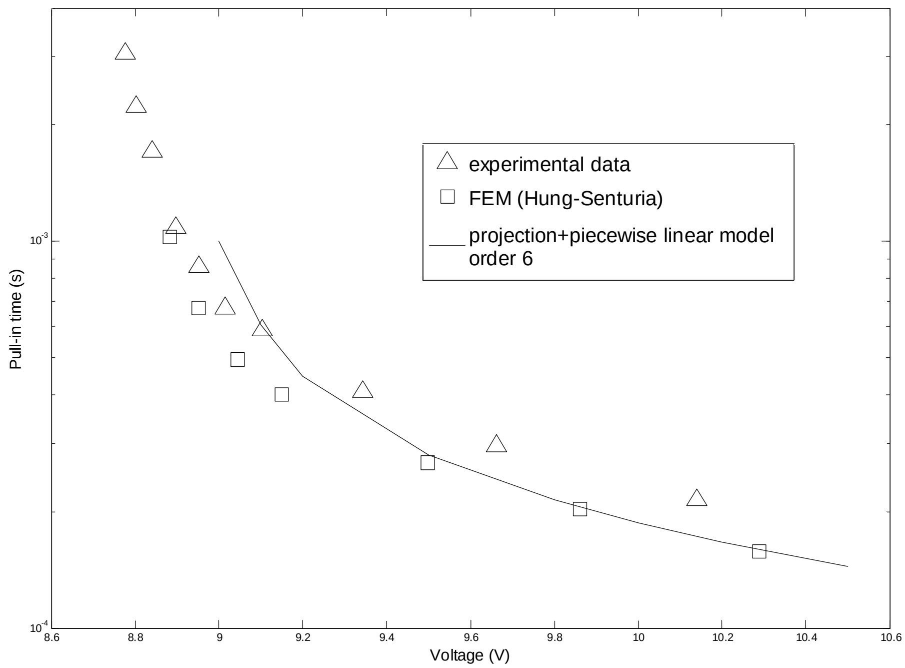

Figure 2 - Pull-in time (s) versus applied voltage (V) for P_0 =1.013×10^5 Pa. Comparison of the experimental and simulated results presented in [16] to the simulated results obtained with our reduced-order model. The chosen squeeze modes correspond to k_1 =0, 2 and k_2 =1, 3 in (38). For the mechanical part, the first eigenmode is used.

Further work

We can notice that the nonlinear terms depend only on the mechanical modal coordinate and that the model is linear in the modes concerning the Reynolds equation. It can be written as:

$$\frac{d}{dt}(\mathbf{G}(\mathbf{x})\mathbf{z}-\mathbf{FP}(\mathbf{x}))=\mathbf{B}(\mathbf{x})\mathbf{z} \qquad \textbf{(6)}$$

The piecewise linearization described above doesnôt take advantage of this structure. Another piecewise linearized model can be based on the linearization of the terms **G**, **FP** and **B.** As they only depend on the on mechanical coordinate, there is no need for a training trajectory. It is sufficient to discretize in an appropriate way the one dimensional space corresponding to the mechanical coordinate. This is a great advantage as the resulting model does not depend on the relevance of a training trajectory, one the main drawbacks of the model presented above. On the other hand the resulting equation is nonlinear which increases the resolution cost. Full results of this model will be presented.

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Alexia Missoffe received her Electrical Engineering Degree from Supelec, France, and has an Msc in Microsystems from Heriot Watt University, Scotland UK. She is currently working towards her PhD in reduced-order modelling of MEMS at Supelec.