European Option Princing on a GPU Cluster

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European Option Princing on a GPU Cluster

(ANR project « GCPMF »)

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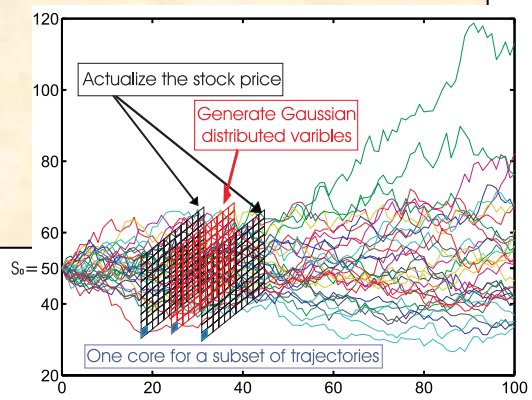
1 – Objectives

Main objectives and difficulties

Final objective:

High speed European contract pricing (for hedging)

- using Monte-Carlo computations,
- using a clusters of multi-cores (GPUs or multi-core CPUs)



Scientific and technical locks:

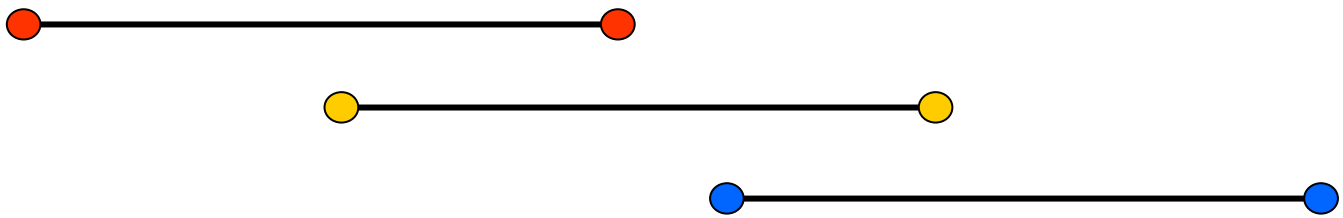
- Design a parallel algorithm for an European option pricer on GPU and cluster of GPU,
- Design and implement a right and efficient parallel RNG,
- Find out the right compromise between speedup, size up, result accuracy and energy consumption

2 – RNG parallelization and comparison

Principles

- RNG on CPU *vs* RNG on GPU
  1. Data transfer
  2. Faster if Parallel
- Parallel RNGs: two efficient alternatives for GPU
  1. Period Splitting

  2. Parametrization



Parallel RNG: PLCG

- Parallelization with parametrization

$$X_n = a.X_{n-1} \% \ m$$

- « a » is the parameter

- Chose an « a » for each sub-stream such that:

1. Good parallel independence inter-streams
   Michael Mascagni SPRNG
2. Good sequential independence intra-stream
   Donald E. Knuth Art of Computer Programming
3. Total period: 2^{41} (int 32) and 2^{65} (float 64)

Parallel RNG: CMRG

- Pierre L'Ecuyer: Combination of two MRGs (total period: 2^{185})

$$X_n = a_1 * X_{n-1} + a_2 * X_{n-2} + a_3 * X_{n-3} \ \% \ m$$
$$X'_n = a'_1 * X'_{n-1} + a'_2 * X'_{n-2} + a'_3 * X'_{n-3} \ \% \ m'$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_3 & a_2 & a_1 \end{pmatrix} \quad A' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a'_3 & a'_2 & a'_1 \end{pmatrix}$$

For example, if we want:

- one RNG/trajectory,
- 2^{18} trajectories/GPU,
- maximum of 16 = 2^4 GPUs.

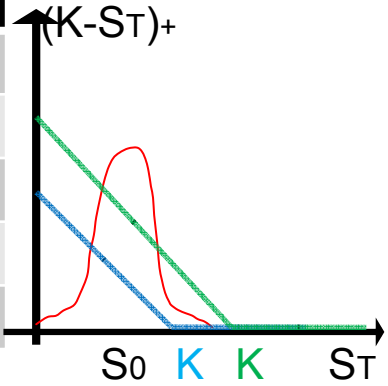
$IP = 2^{185}/2^{22} = 2^{163}$ and $X\_1 = (X_1, X_2, X_3)^T$, $X'\_1 = (X'_1, X'_2, X'_3)^T$

$X\_i = (A^{IP})^i . X \ \% \ m$ $\quad$ $X'\_i = (A'^{IP})^i . X' \ \% \ m'$

Parallel RNG: Results

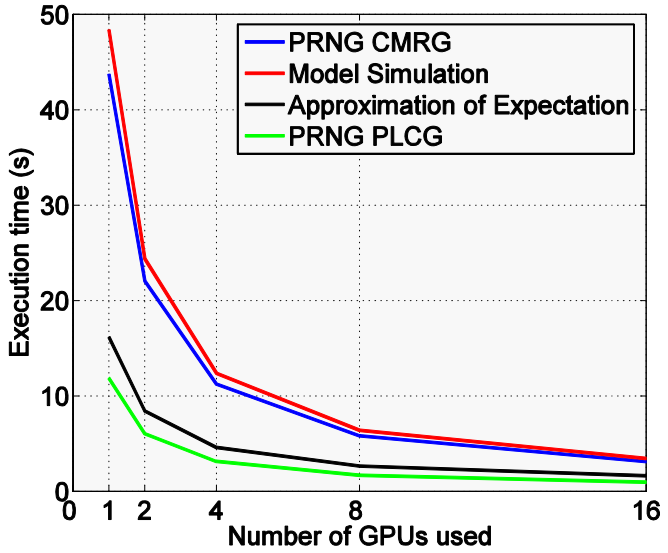
- Results Accuracy

Strike « K »	Real	PLCG MC	PLCG error	CMRG MC	CMRG error
80	0.0610	0.0621	0.0023	0.0611	0.0024
90	0.5068	0.5115	0.0078	0.5088	0.0008
100	2.1723	2.1862	0.0178	2.1740	0.0177
110	5.9208	5.9433	0.0299	5.9159	0.0296
120	11.8810	11.9023	0.0407	11.8775	0.0406



- Speedup Results for PRNG

RNG / Machine	Gaussian sample for PLCG generation/ second	Gaussian sample for CMRG generation/ second
CPU	2.24 Kg/s	2.03 Kg/s
GPU	883.58 Kg/s	239.80 Kg/s



3 – Parallel algorithm and implementations

Parallel algorithm (1)

Parallel programming paradigms:

Coarse grain: message passing (MPI on PC cluster)

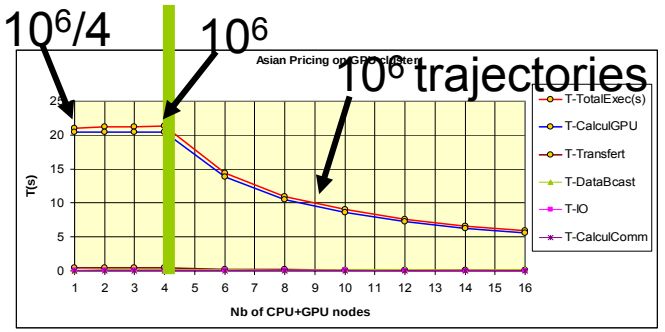
Medium grain: CPU-multithreading (OpenMP on multi-cores)

Fine grain: GPU-multithreading (CUDA on GPU)

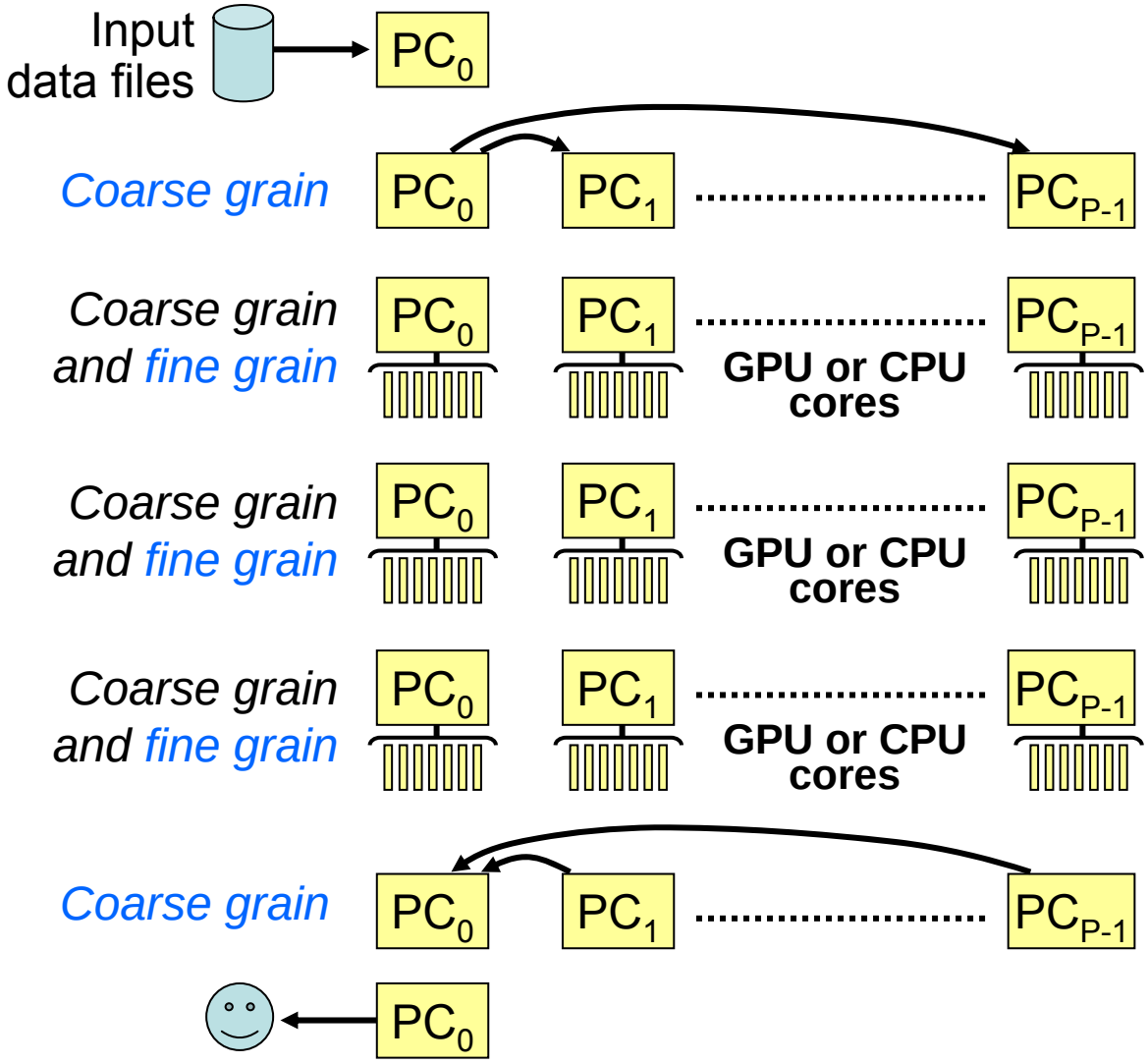
Strategy:

- Avoid concurrent input file accesses
- Minimize data transfer between CPU and GPU memories.
- Common algorithm for multi-core-CPU and GPU clusters.
- When limited by the GPU memory:

  “size up + speedup”



Parallel algorithm (2)



1 - Input data reading on P_0

2 - Input data broadcast from P_0

3 - Parallel and independent RNG initialization

4 - Parallel and independent Monte-Carlo computations

5 - Parallel and independent partial results computation

6 - Partial results reduction on P_0 and final price computation

7 – Print results and perfs

GPU/CPU code comparison (1)

OpenMP parallelization on multi-core CPU: split the external loop

```
void ActStock(double sqrtdt)
{
  int StkIdx, yIdx, xIdx;     // Loop indexes
  #pragma omp parallel private(StkIdx,yIdx,xIdx)
  {
  for (StkIdx = 0; StkIdx < NbStocks; StkIdx++) {
    Parameters_t *parPt = &par[StkIdx];
    // Process each trajectory
    #pragma omp for
    for (yIdx = 0; yIdx < Ny; yIdx++)
       for (xIdx = 0; xIdx < Nx; xIdx++) {
          float call;
          // - First pass
          call = ……;
          // - The passes that remain
          for (int stock = 1; stock <= StkIdx ; stock++)
             call = ……;
          // Copy result in the global GPU memory
          TabStockCPU[StkIdx][yIdx][xIdx] = call;
       }
  }
  }
}
```

```
float TabStockCPU[NbStocks][Ny][Nx]
```

GPU/CPU code comparison (2)

CUDA parallelization on GPU: one kernel work on one trajectory

```
__global__ void Actual_kernel(void)
{
  float call, callBis;
  // Computes the indexes and copy data into multipro sh. memory
  int xIdx = threadIdx.x + blockIdx.x*BlockSizeX;
  int yIdx = blockIdx.y;
  __shared__ float InputLine[Nx];
  __shared__ float BrownLine[Nx];
  InputLine[xIdx] = TabStockInputGPU[StkIdx][yIdx][xIdx];
  GaussLine[xIdx] = TabGaussGPU[0][yIdx][xIdx];
  // First pass
  call = ……;
  callBis = call;
  // The passes that remain
  for (int stock = 1; stock <= StkIdx; stock++) {
    GaussLine[xIdx] = TabGaussGPU[stock][yIdx][xIdx];
    call = callBis*……;
    callBis = call;
  }
  // Copy result in the global GPU memory
  TabStockOutputGPU[StkIdx][yIdx][xIdx] = call;
}
```

```
float TabStockOutputGPU[NbStocks][Ny][Nx]
```

GPU/CPU code comparison (3)

CUDA parallelization on GPU: one kernel work on one trajectory

```
void ActStock(double sqrtdt)
{
 // GPU thread management variables
 dim3 Dg, Db;
 // Set thread Grid and blocks features
 Dg.x = Nx/BlockSizeX; Dg.y = Ny; Dg.z = 1;
 Db.x = BlockSizeX; Db.y = 1; Db.z = 1;
 // Transfer a float version of the time increment on the GPU
 float sqrtdtCPU = (float) sqrtdt;
 cudaMemcpyToSymbol(sqrtdtGPU,&sqrtdtCPU,sizeof(float),0,
                    cudaMemcpyHostToDevice);
 // For each stock: transfer its index on the GPU and compute
 // its actualization (process all trajectories)
 for (int s = 0; s < NbStocks; s++) {
    cudaMemcpyToSymbol(StkIdx,&s,sizeof(int),0,
                       cudaMemcpyHostToDevice);
    Actual_kernel<<<Dg,Db>>>();  //Run the GPU computation
 }
}
```

```
float TabStockCPU[NbStocks][Ny][Nx];
float TabStockOutputGPU[NbStocks][Ny][Nx];
```

→ Identical data structures for CPU and GPU versions when using one GPU-thread per trajectory

Compilation

OpenMPI + Cuda:

```
nvcc --host-compilation C++
        -O3 -I/opt/openmpi/include
        -DOMPI_SKIP_MPICXX -c X.cu

nvcc -O3 -L/opt/openmpi/lib
        -o pricer  X.o Y.o .... -lmpi -lm
```

OpenMPI + OpenMP

```
g++ -O3 -fopenmp -I/opt/openmpi/include
       -c X.cc

g++ -O3 -fopenmp -L/opt/openmpi/lib
       -o pricer  X.o Y.o ...  -lmpi -lm
```

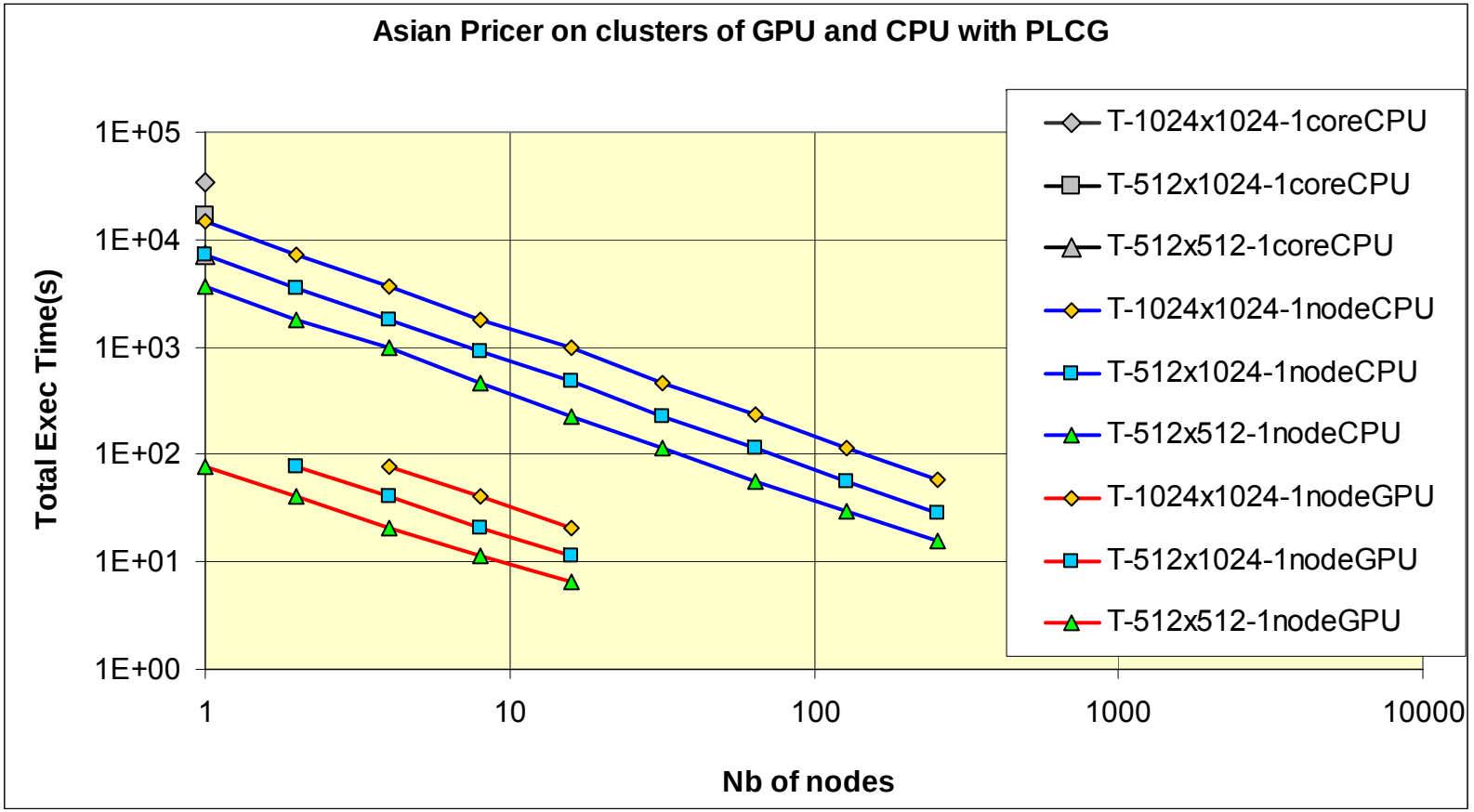
This is a basic C++ code.

Compilation with CUDA code is easy.

4 – Experiments and performance analysis

Computing perf (1)

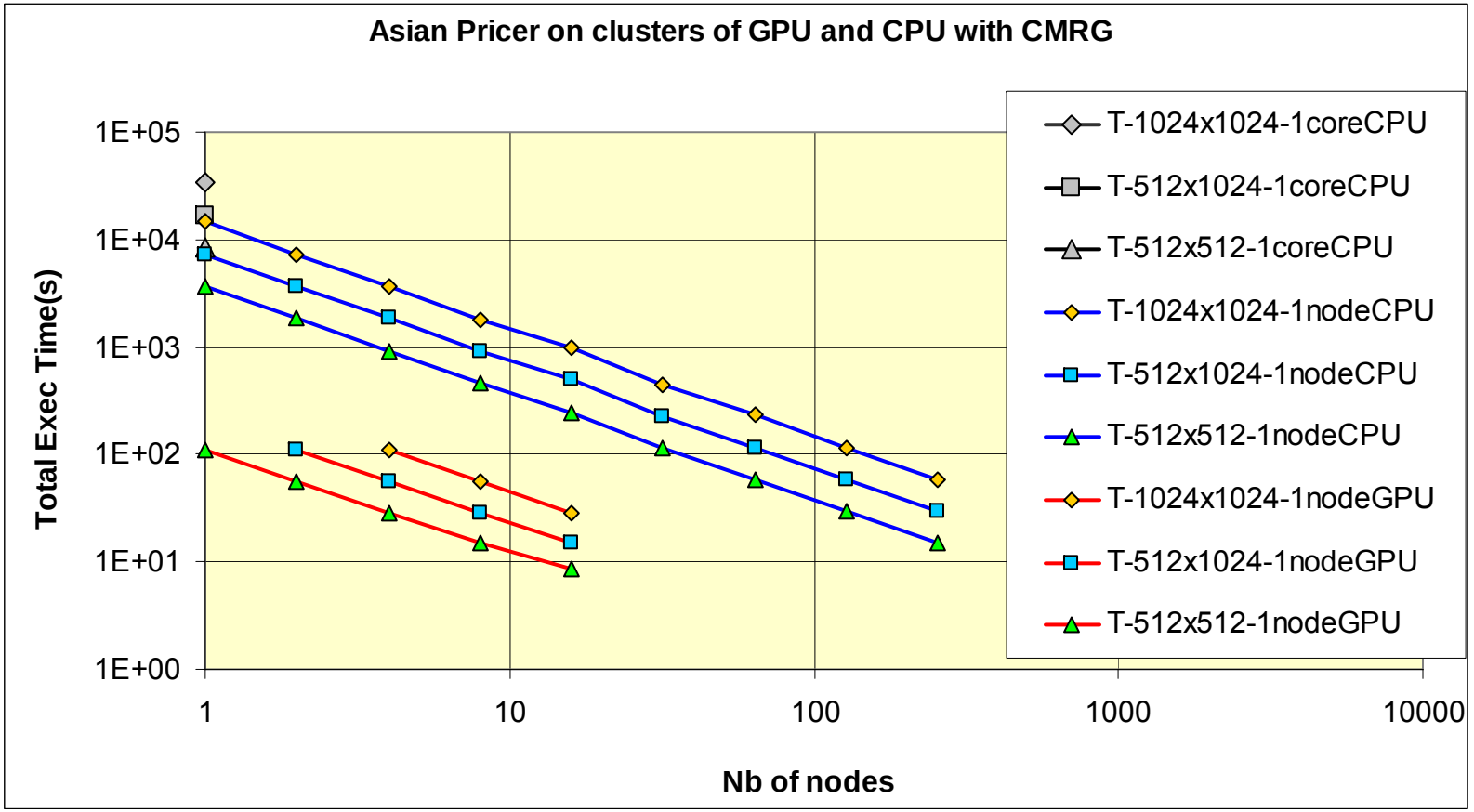
Pricing execution time with PLCG



Good scaling on both systems.

Computing perf (2)

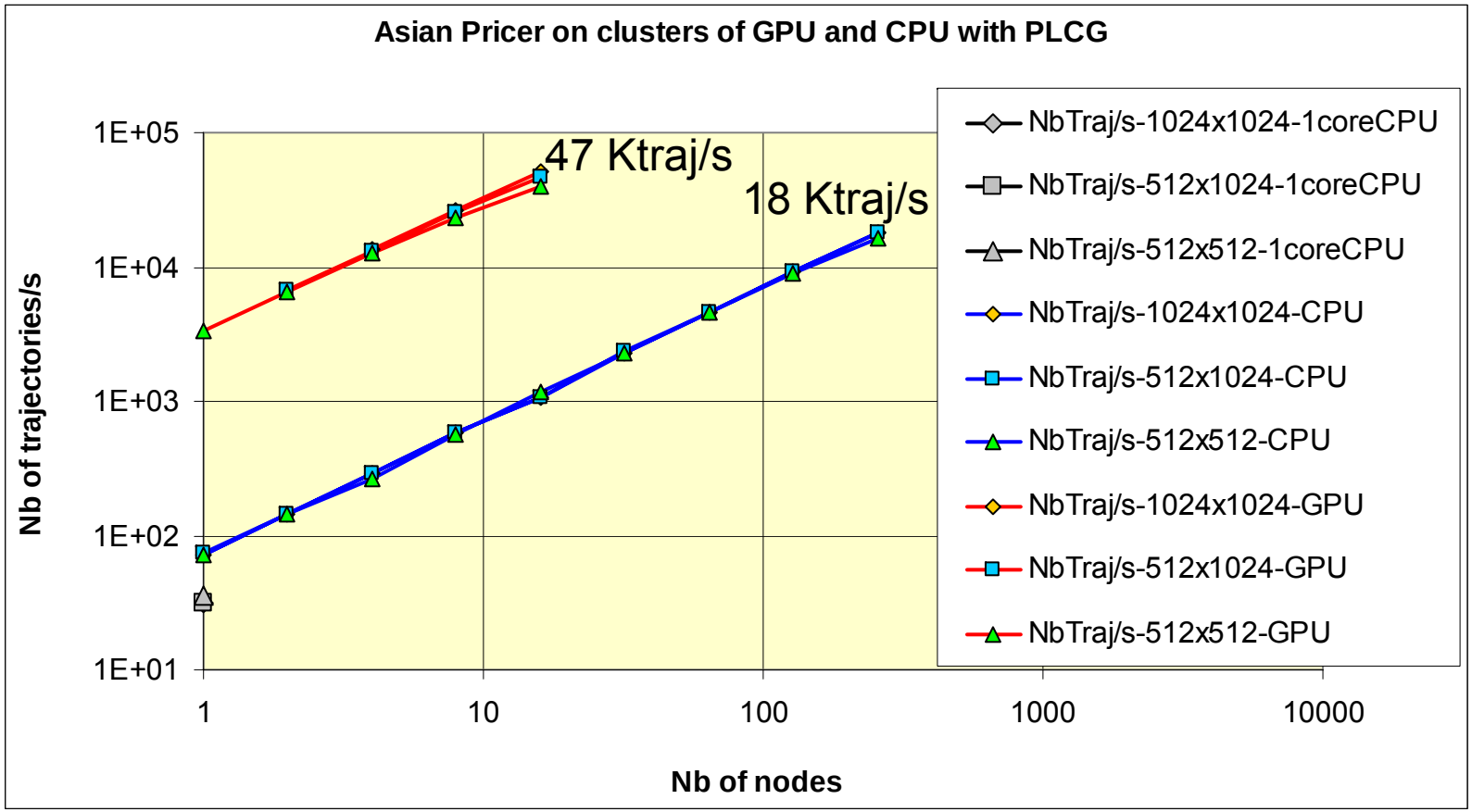
Pricing execution time with CMRG



Good scaling on both systems.
GPU time is impacted by the RNG.

Computing perf (3)

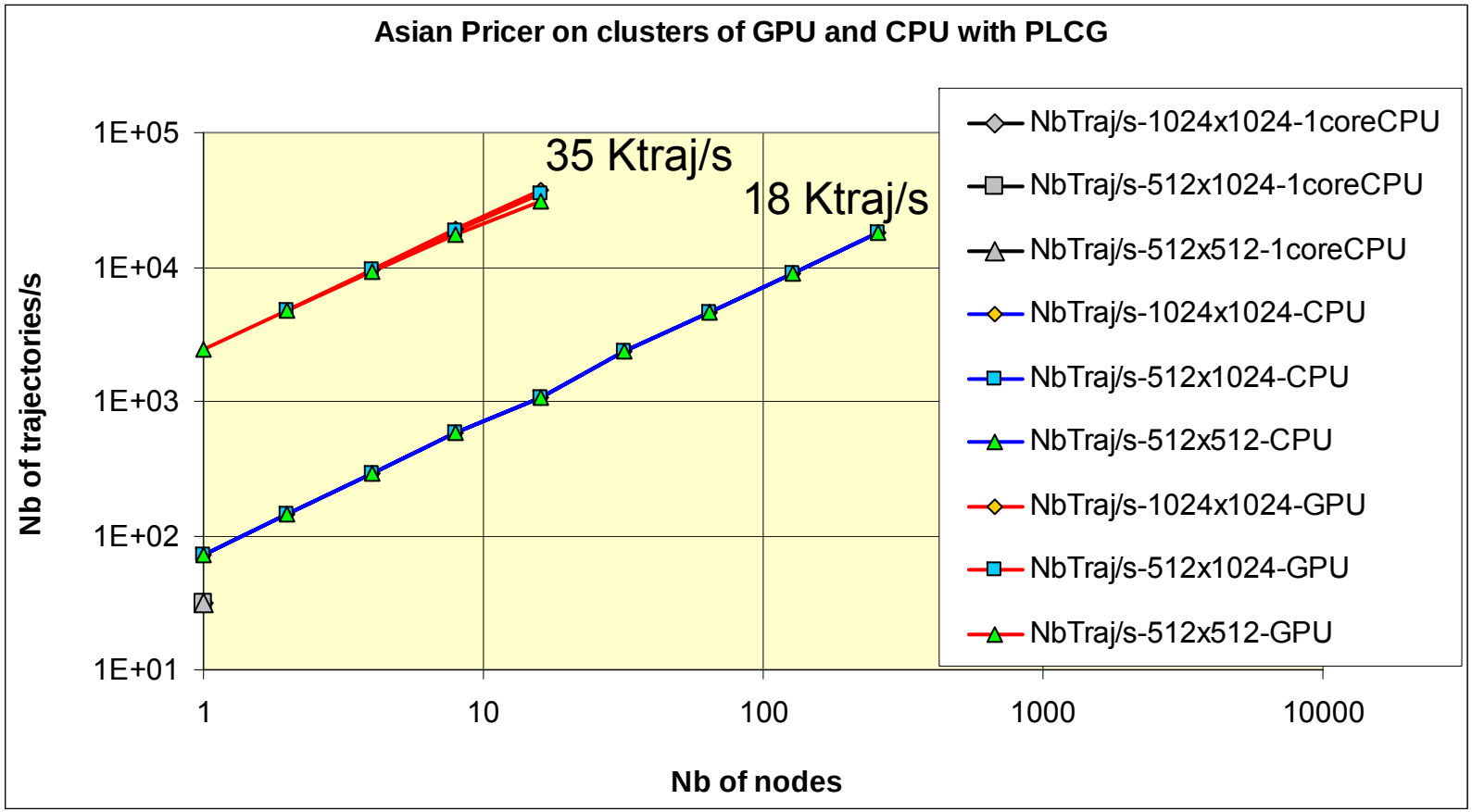
Pricing speed with PLCG



Computation speed is independent of the problem size.

Computing perf (4)

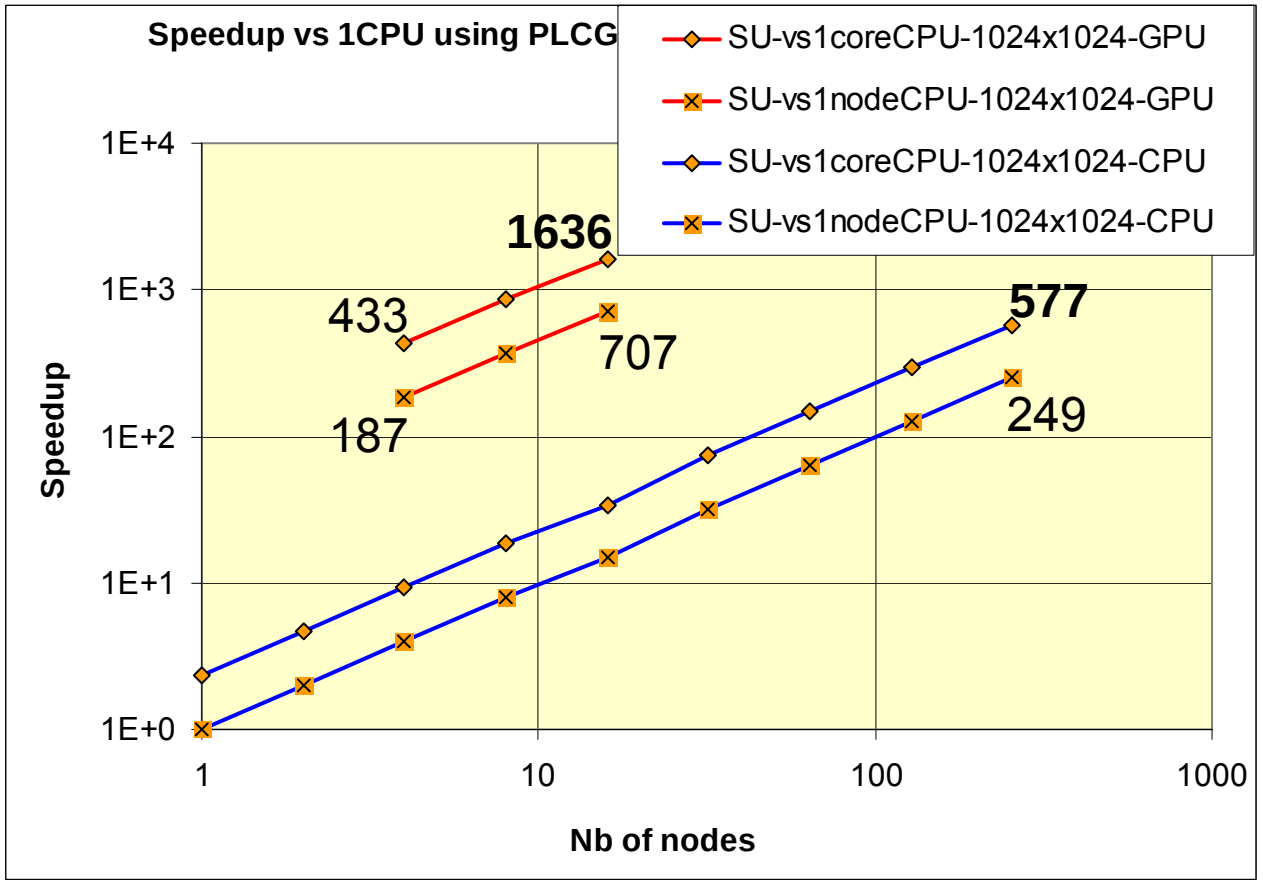
Pricing speed with CMRG



Computation speed is independent of the problem size.
GPU computation speed is impacted by the RNG choice.

Computing perf (5)

Pricing speedup with PLCG

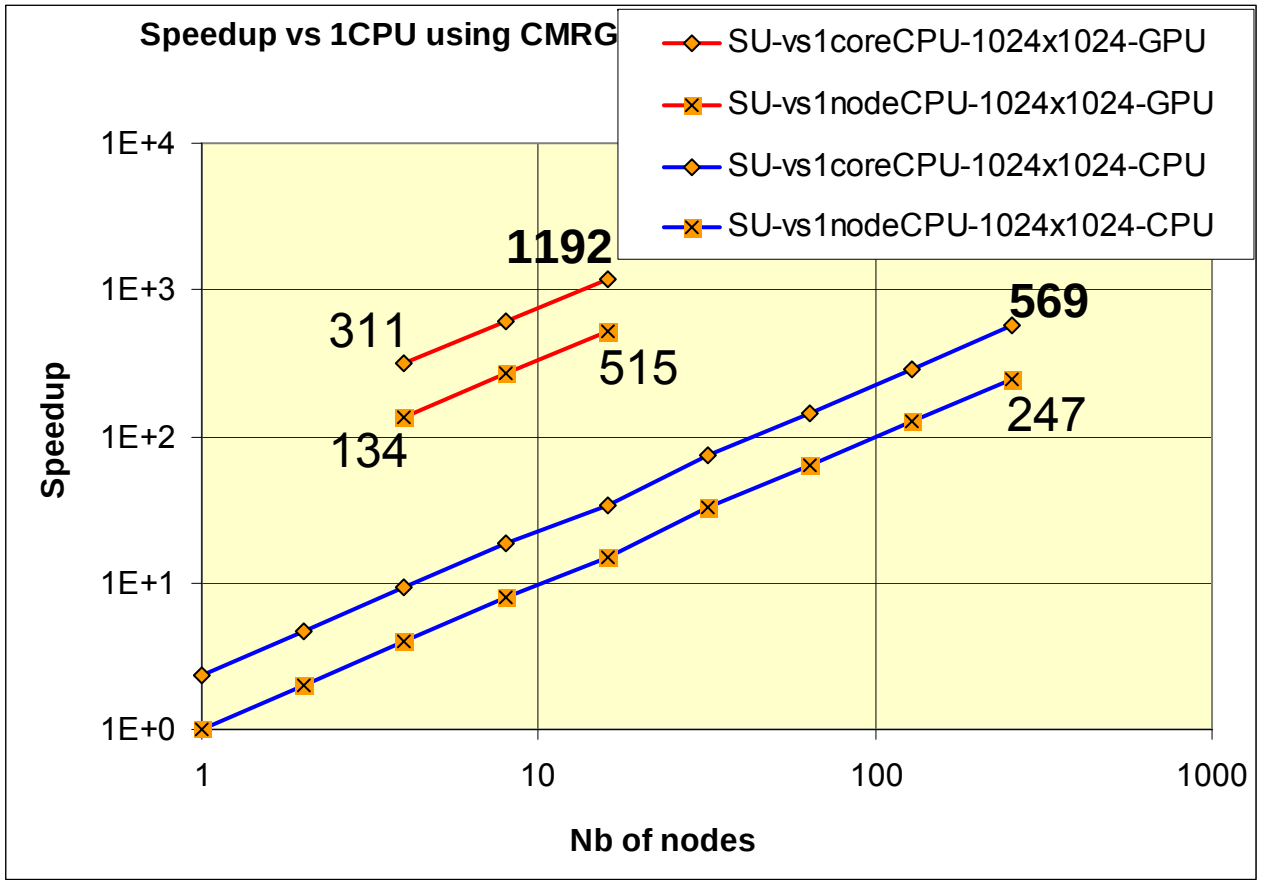


x 2.83

With 16 GPU nodes: speedup *vs* 1-core-CPU reaches 1636,
speedup *vs* 1-node-CPU reaches 707,
speedup *vs* 256-nodes-CPU cluster reaches 2.83.

Computing perf (6)

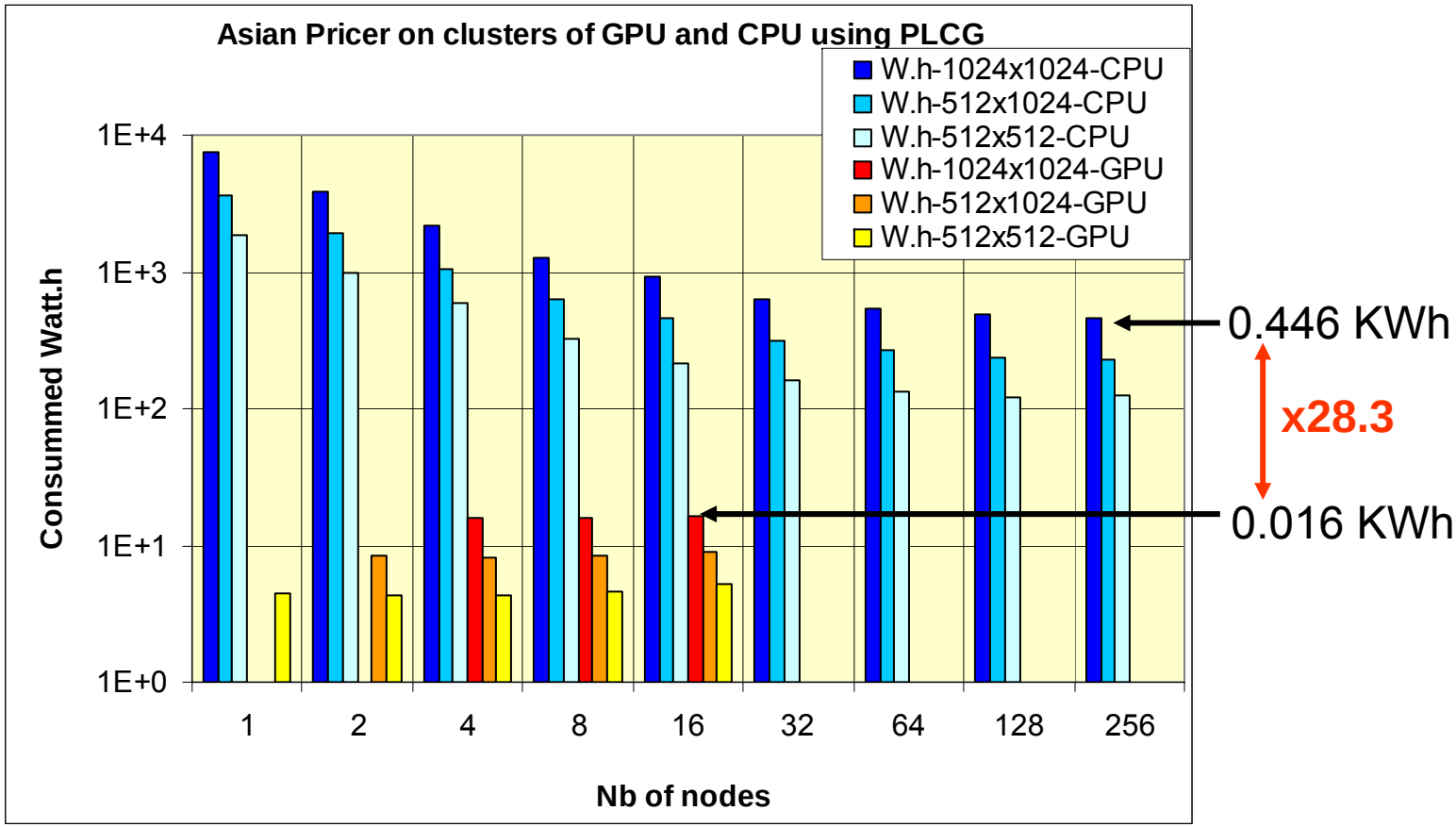
Pricing speedup with CMRG



With 16 GPU nodes: speedup *vs* 1-core-CPU reaches 1192,
speedup *vs* 1-node-CPU reaches 515,
speedup *vs* 256-nodes-CPU cluster reaches 2.09.

Energetic perf (1)

Energy consumption with PLCG

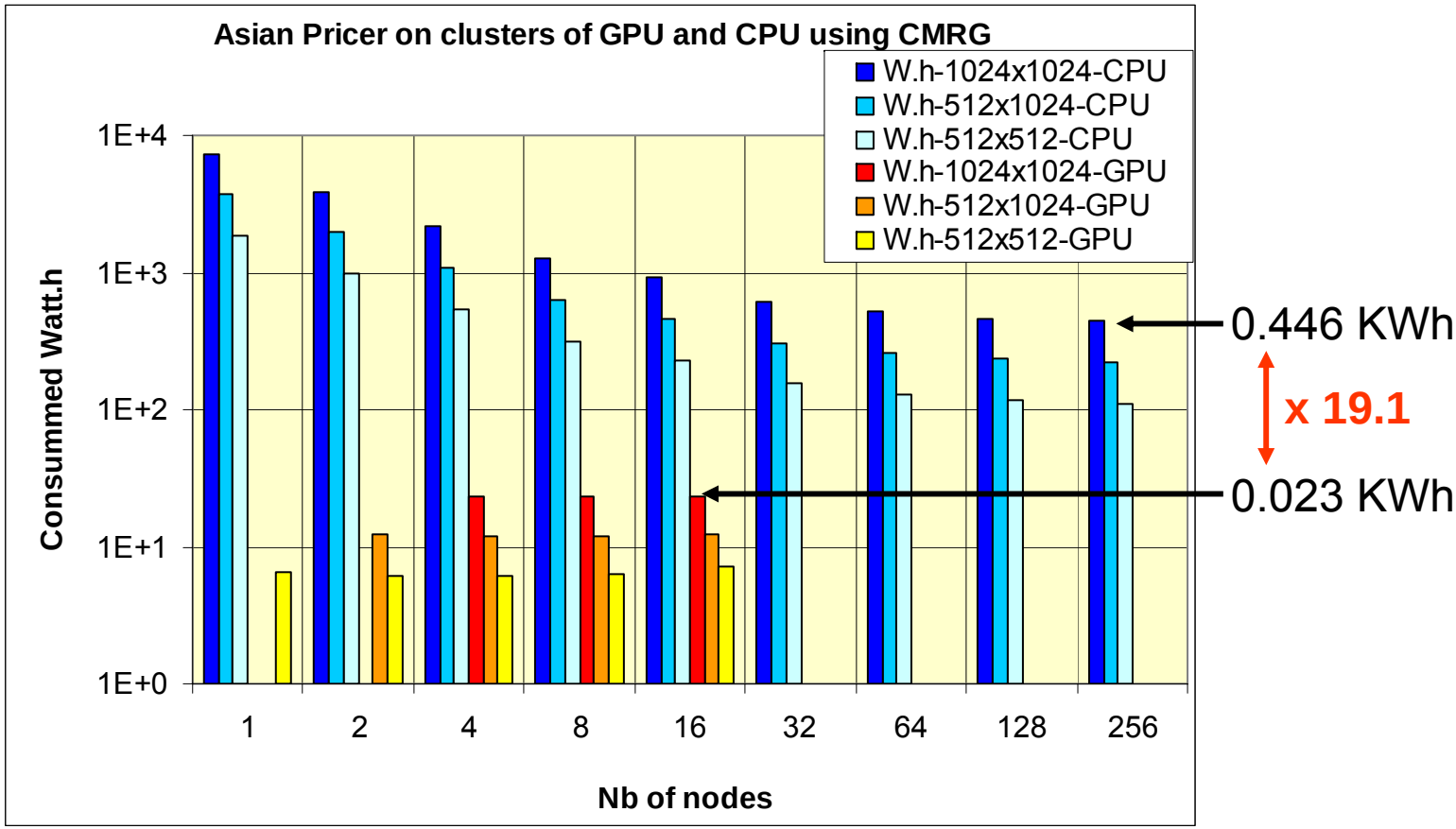


Air conditioned has not been considered.

16-nodes GPU cluster consumes 28.3 times less than a 256-nodes CPU cluster.

Energetic perf (2)

Energy consumption with CMRG

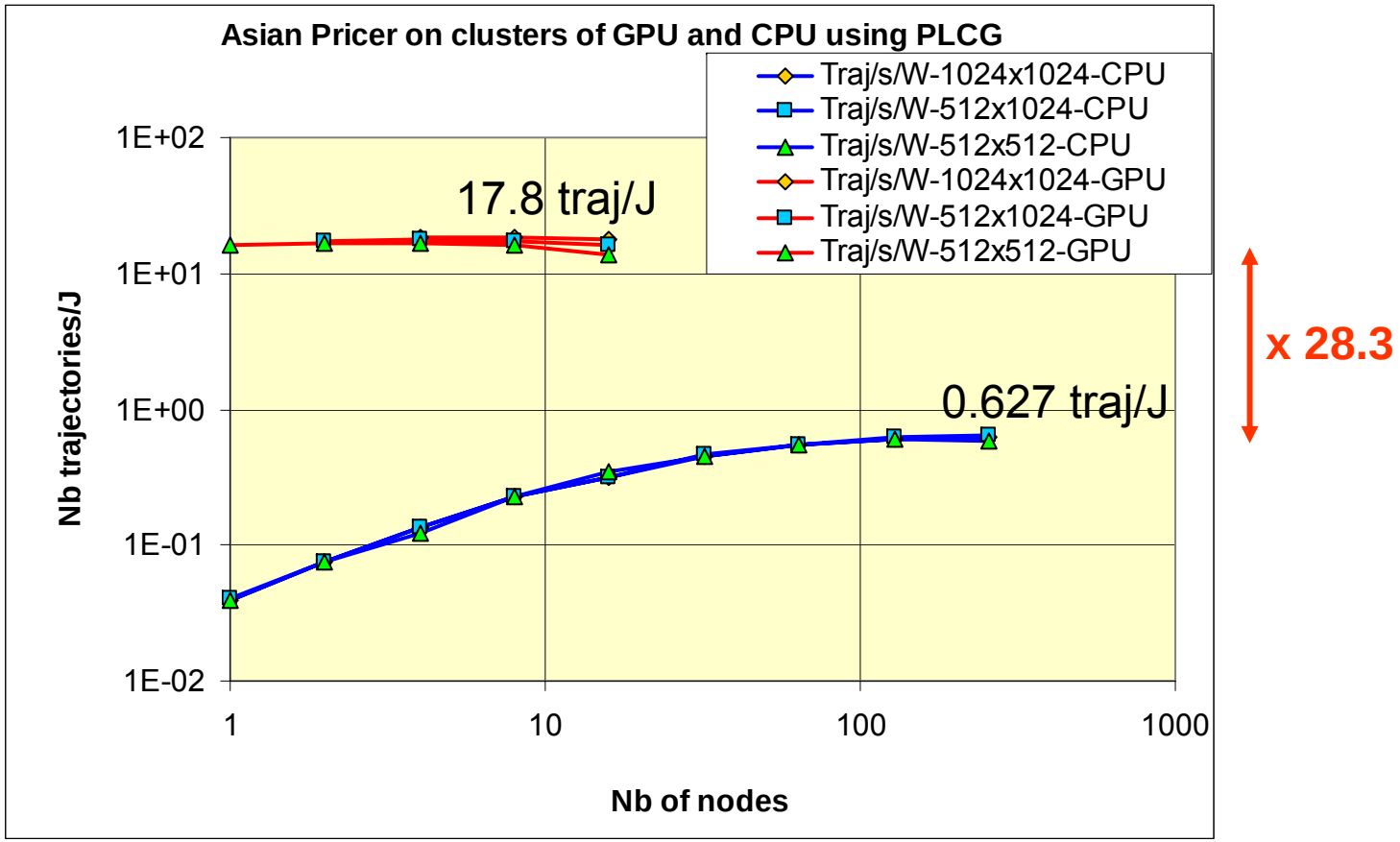


Air conditioned has not been considered.

16-nodes GPU cluster consumes 19.1 times less than a 256-nodes CPU cluster.

Energetic perf (3)

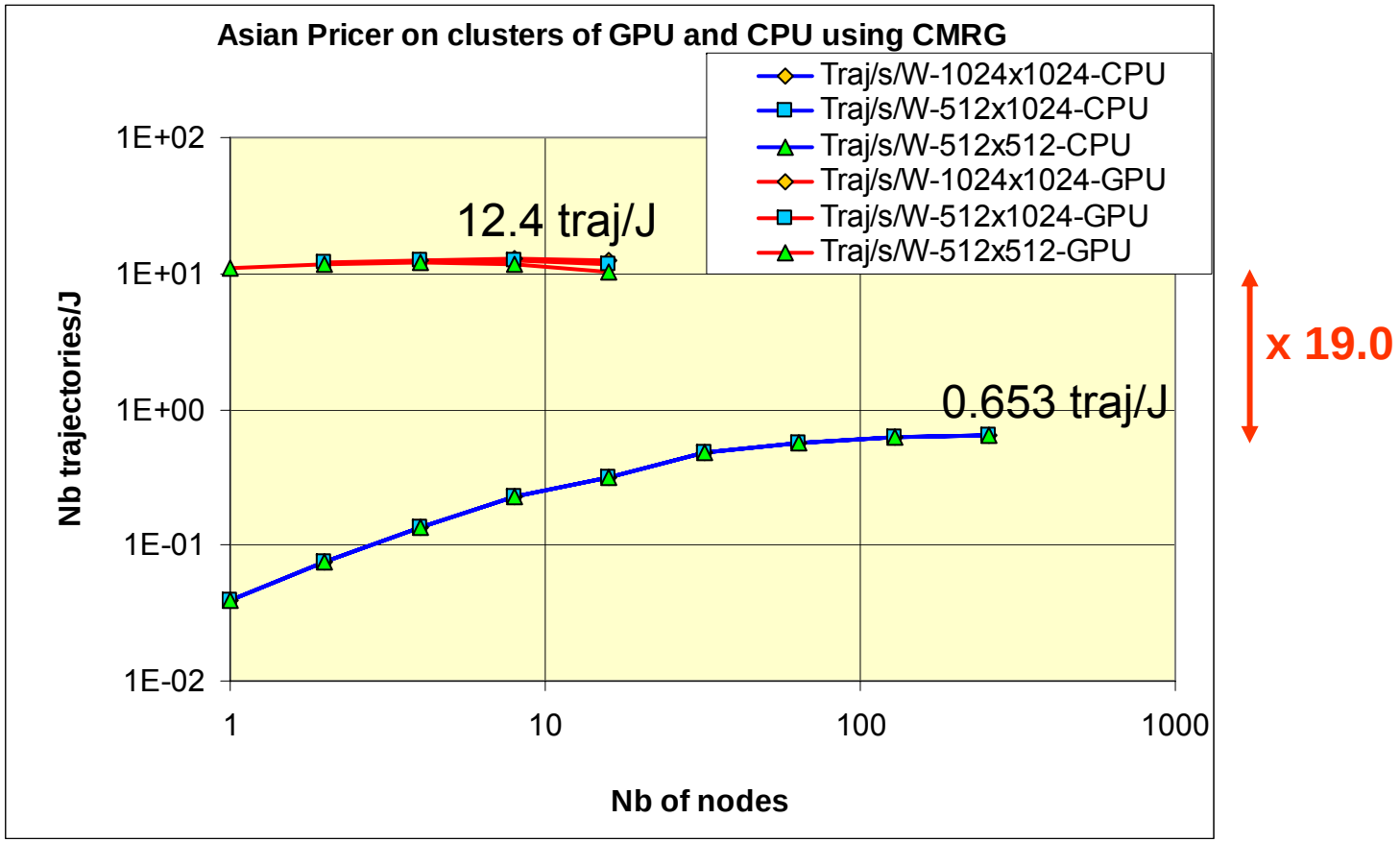
Effectiveness of computing energy with PLCG



Currently, air conditioning has not been considered.
Effectiveness of computing energy is 28.3 times higher on the 16-nodes GPU cluster.

Energetic perf (4)

Effectiveness of computing energy with CMRG



Currently, air conditioning has not been considered.
Effectiveness of computing energy is 19.0 times higher on the 16-nodes GPU cluster.

Bi-core CPU cluster *vs* GPU cluster

Global comparison of GPU and CPU clusters

256 bi-core Xeon 3075, RAM 4Go, cache 4Mo 1 CISCO 256-ports switch, gigabit-eth	16 bi-core Intel E8200, RAM 4Go, cache 6Mo Asus GeForce 8800 GT 1 DELL 24-ports switch, gigabit-eth

Benchmark : 1024x1024 trajectories
40 stocks European contract pricing
RNG: PLCG

58.70s	20.72s	Speedup = **2.83**
464.3Wh	16.4Wh	Saving = **28.3**

GPU cluster is 2.83x28.3 = **80.1** times more efficient

5 – Conclusion and perspectives

Conclusion & Perspectives

Developments:

- OpenMPI + CUDA + “C+” were compatible.
- Learning CUDA + all dev ≈ 3 weeks (2 people).
- Debug on GPU was hard.

Performances:

- Good scaling
- GPU cluster computes faster
- GPU cluster consumes less energy
- Too high quality RNG is harmful
- PLCG + 16-node GPU cluster
  - → 1636 times faster than 1-core CPU
  - → 2.8 times faster than 256-node CPU cluster
  - → consumes 28.3 times less than 256-node CPU cluster

Next steps:

- Experiment others Monte-Carlo simulations
- Design algorithm with better perf & mixed perf (speedup x energy saving)

European Option Pricing on a GPU Cluster

(ANR project « GCPMF »)

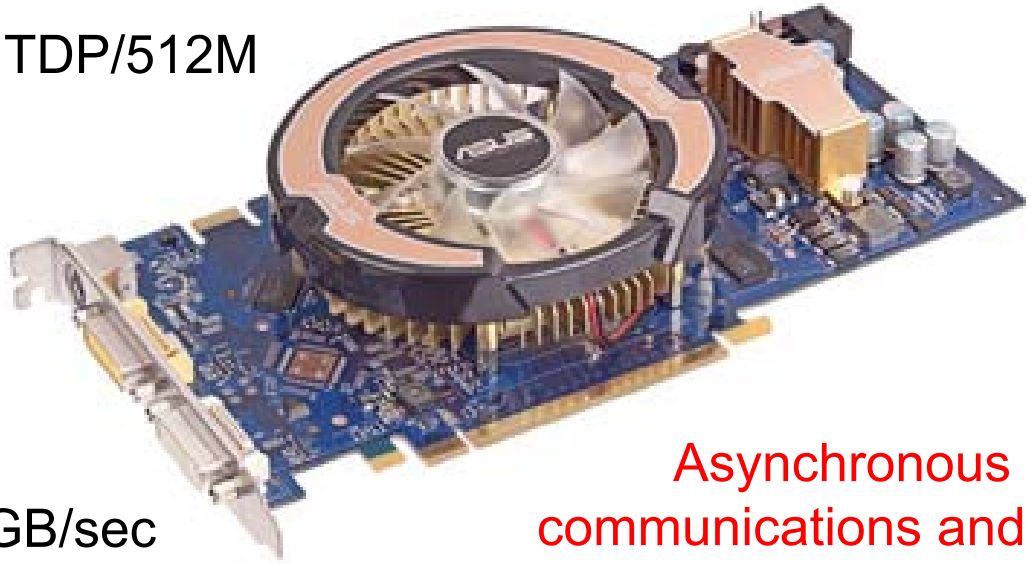
Questions ?

Hardware choice

GPU on each node: ASUS GeForce 8800 GT

Product model: EN8800GT/G/HTDP/512M

- Multiprocessors: 14
- Stream processors: 112
- Core clock: 600 MHz
- Memory clock: 900 MHz
- Memory amount: 512 MB
- Memory interface: 256-bit
- Memory bandwidth: 57.6 GB/sec
- Texture fill rate: 33.6 billion/sec



Asynchronous communications and Cuda 1.1 supported

CPU on each node: 1 processor dual-cores Intel E8200, 2.66 GHz
front side bus:1333MHz
RAM : 4Go DDR3, cache : 6Mo