Outage Efficient Strategies for Network MIMO with Partial CSIT

Mari Kobayashi, Merouane Debbah, Jean-Claude Belfiore

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Mari Kobayashi,
SUPELEC
Gif-sur-Yvette, France

Mérouane Debbah,
SUPELEC
Gif-sur-Yvette, France

Jean-Claude Belfiore,
Télécom ParisTech
75013, Paris, France

***Abstract*— We consider a multi-cell MIMO downlink (network MIMO) where B base-stations (BS) with M antennas, connected to a central station (CS), transmit K messages to K single-antenna user terminals (UT) simultaneously. Although many works have shown the potential benefits of network MIMO, the conclusion critically depends on the underlying assumption of perfect channel state information at transmitters (CSIT). In this paper, we propose an outage-efficient strategy that requires only partial CSIT. Namely, with side information of all UT's messages and local CSIT, each BS applies zero-forcing (ZF) beamforming in a distributed manner, which creates K parallel MISO channels. Based on the statistical knowledge of these parallel channels, the CS performs a robust power allocation that jointly minimizes the outage probability of K UTs and achieves a diversity gain of $B(M-K+1)$ per UT. Numerical results show that even with partial CSIT network MIMO can be beneficial by providing high data rates with a sufficient reliability to individual users.**

I. Introduction

Recently, network MIMO (where neighboring BSs are connected to form an antenna array) schemes have been proposed as a mean to drastically increase the downlink capacity and solve the interference management problem of cellular systems [1]. Inspired by this result, we consider the multi-cell MIMO downlink where B BSs with M antennas connected to a CS communicate simultaneously with K UTs with a single antenna (B, K, M are finite and comparable). Fig.1 illustrates an example of a multi-cell downlink system for $B = K = 3$ and $M = 3$. Assuming a reliable (possibly wired) backbone link, we model the multi-cell downlink as the $B \times K$ MISO interference channel with partial transmit cooperation. More precisely, we let $\mathbf{h}_{ij}$ denote the frequency-flat fading channel vector between BS i and UT j. Assuming that the UTs are arbitrarily distributed in each cell, the per-component variance σ_{ij} of $\mathbf{h}_{ij}$ captures the path loss of this link. Under this setting, the number BSs by which each UT is served varies from 1 to B, while we let the number of UTs per cell $[K/B]$.

If the CS or equivalently all the BSs have perfect CSIT and share the messages of all UTs, the multi-cell MIMO downlink scheme falls down into the classical $BM \times K$ MIMO broadcast channel with per-BS power constraints. In this case, the optimal strategy to maximize the multicell throughput is joint dirty-paper coding [2]. For the case of a single-antenna BS ($M = 1$), the per-cell downlink capacity has been studied based on the circular Wyner model both in the absence and presence of flat-fading channels [3], [4], [5]. In

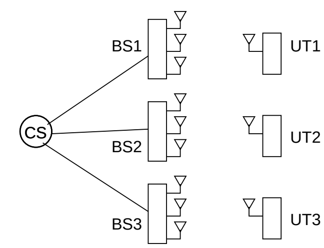

Fig. 1. A multicell downlink with $B = 3$ BSs and $K = 3$ UTs.

the regime of a large number of cells/users per cell, closed-form expressions for the per-cell capacity have been derived [3], [4]. If each BS is equipped with $M > 1$ antennas, the multi-cell downlink capacity can be naturally enhanced by exploiting the spatial degrees of freedom (see for example [6], [7]). For a small B, the MIMO multi-cell downlink channel is also referred to the MIMO interference channel or MIMO-X channel under various message sharing assumptions (see [6], [8] and reference therein). In these recent contributions, the sum degrees of freedom has been extensively studied.

Unfortunately, the global joint processing at the CS is difficult in a practical network MIMO system due to the following limitations. First, learning downlink channels at the CS requires a substantial overhead compared to a single-cell downlink. In fact, it can be easily expected that as the number of cooperative cells B or the number of users K increases, the overhead will rapidly dominate the whole downlink/uplink resource. Moreover, future network MIMO systems aim at providing high data rate to individual users, rather than providing multiuser diversity. From these perspectives, the regime of a large number of cells or users is questionable. Second, the backbone links between the BSs and the CS are typically imperfect, i.e. the capacity-limited and erroneous. The impact of the limited-capacity backbone has been recently studied (see e.g. [9] and references therein).However, to the best of our knowledge, the first limitation, which might emerge as one of the limiting factors of network MIMO, has not been explicitly addressed.

The above observations motivate us to consider a practical approach with limited BS cooperation that can be easily implemented in future network MIMO systems. To this end, we focus on a small B, K and $M \geq K$. We assume that each BS i knows perfectly $\{\mathbf{h}_{ij}\}_{j=1}^K$ while the CS has only statistical CSI. To concentrate on the impact of partial CSIT the underlying backbone link is assumed perfect such that B BSs can share the messages of the K users. Each BS applies ZF beamforming in a distributed manner, which creates K parallel MISO channels. Based on the statistical knowledge of these parallel channels, the CS performs a robust power allocation that jointly minimizes the outage probability of K UTs and achieves a diversity gain of $B(M-K+1)$ per UT. Numerical results show that even in a realistic scenario with partial CSIT network MIMO can be beneficial and enables to provide high data rate with a sufficient reliability to individual users.

The paper is organized as follows. The following section describes the system model. In Section III we present the power allocation policies that minimize the outage probability both under perfect and statistical CSI at the CS. Section IV provides some numerical results and concludes the paper.

II. System Model

We consider a MIMO multicell downlink where B BSs with M antennas and $K \leq M$ UTs with a single antenna communicate over a frequency-flat fading channel. The received signal at UT k is given by

$$y_k = \sum_{i=1}^{B} \mathbf{h}_{ik}^H \mathbf{x}_i + n_k \tag{1}$$

where $\mathbf{h}_{ik}^H \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma_{ik}\mathbf{I}_M)$ denotes the channel vector from BS i to UT k i.i.d. over any pair i, k with per-component variance σ_{ik} reflecting the path-loss, $\mathbf{x}_i \in \mathbb{C}^{M\times 1}$ denotes the input vector subject to the power constraint such that

$$\mathrm{tr}(\mathbb{E}[\mathbf{x}_i\mathbf{x}_i^H]) \leq P_i \tag{2}$$

where P_i denotes the short-term power constraint of BS i.

We assume that the CS broadcasts K messages intended to K UTs such that each BS encodes these messages into KT symbols by some capacity-achieving space-time coding (with a sufficiently large T channel uses). We consider a simple ZF beamforming which enables each UT to achieve a multiplexing gain of one. At each channel use, BS i transmits K symbols by ZF beamforming such that

$$\mathbf{x}_i = \sum_{j=1}^{K} \sqrt{p_{ij}}\mathbf{g}_{ij}s_{ij} \tag{3}$$

where $s_{ij} \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ denotes the symbol of UT j transmitted from BS i with power p_{ij}, $\mathbf{g}_{ij}$ is a unit-norm ZF vector orthogonal to $K-1$ vectors $\{\mathbf{h}_{ik}\}_{k\neq j}$. With this beamforming, the received signal at UT k is given by

$$y_k = \sum_{i=1}^{B} \sqrt{p_{ik}}a_{ik}s_{ik} + n_k \tag{4}$$

where we let $a_{ik} = \mathbf{h}_{ik}^H\mathbf{g}_{ik}$ denote the overall channel from BS i to UT k. Since $\mathbf{g}_{ij}$ is independent of $\mathbf{h}_{ij}$ and orthogonal to $K-1$ vectors $\mathbf{h}_{ik}$ for $k \neq j$, $|a_{ij}|^2$ is a chi-square distributed random variable with $2(M-K+1)$ degree of freedom (d.o.f.). We remark that the original $B\times K$ MISO interference channel (1) is decoupled into K parallel $B \times 1$ MISO channels and each MISO channel has $2B(K-M+1)$ d.o.f.. Therefore, each user is expected to achieve a diversity order of $B(M-K+1)$.

Assuming that each UT k perfectly knows $a_{1k}, \dots, a_{Bk}$, it decodes the space-time code and achieves the following rate

$$R_k = \log\left(1 + \sum_{i=1}^{B} |a_{ik}|^2 p_{ik}\right).$$

The deterministic capacity region of (4) for *fixed* powers $\mathbf{p} = \{p_{ij}\}$ and channels $\mathbf{a} = \{a_{ij}\}$ is given by

$$\mathcal{R}(\mathbf{a};\mathbf{p}) = \left\{\mathbf{R} \in \mathbb{R}_+^K : R_k \leq \log\left(1 + \sum_{i=1}^{B} |a_{ik}|^2 p_{ik}\right) \ \forall k\right\} \tag{5}$$

which is clearly convex (rectangular for $K = 2$). The capacity region of (4) under individual BS power constraints $\mathbf{P} = (P_1, \dots, P_B)$ for *fixed* channels $\mathbf{a}$ is given by

$$\mathcal{C}(\mathbf{a};\mathbf{P}) = \bigcup_{\sum_{k=1}^K p_{ik} \leq P_i, \forall i} \mathcal{R}(\mathbf{a};\mathbf{p}) \tag{6}$$

where the union is over all possible powers satisfying B individual power constraints. The capacity region $\mathcal{C}(\mathbf{a};\mathbf{P})$ is convex and its boundary can be explicitly characterized by solving the weighted sum rate maximization as specified in subsection III-A.

III. Power allocation minimizing outage probability

We consider that a target rate tuple $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_K)$ is fixed by the system. For a given $\boldsymbol{\gamma}$, we define the outage probability as the probability that $\boldsymbol{\gamma}$ is not inside the capacity region $\mathcal{C}(\mathbf{a}, \mathbf{P})$

$$P_{\mathrm{out}}(\boldsymbol{\gamma}) \triangleq 1 - \Pr\left(\boldsymbol{\gamma} \in \mathcal{C}(\mathbf{a}, \mathbf{P})\right). \tag{7}$$

This section provides the power allocation policies minimizing the outage probability under perfect and statistical CSIT.

A. Perfect CSI at CS

For the special case of perfect CSI at CS, we are particularly interested in the power allocation policy that provides the rate tuple proportional to the target rate tuple (rate-balancing). As seen later, this policy equalizes the outage probability of all UTs and thus provides the strict fairness among UTs, one of the most desired properties. Our objective is find the set of $\{p_{ij}\}$ satisfying

$$\frac{R_k(\mathbf{p})}{R_1(\mathbf{p})} = \frac{\gamma_k}{\gamma_1} \triangleq \alpha_k, \quad k = 2, \dots, K \tag{8}$$

where we defined $\frac{\gamma_k}{\gamma_1} = \alpha_k$ and $\alpha_1 = 1$. More precisely, the power allocation is a solution of [10]

$$\min_{\sum_k \theta_k = 1} \max_{\mathbf{R}\in\mathcal{C}(\mathbf{a},\mathbf{P})} \sum_{k=1}^{K} \theta_k \frac{R_k}{\alpha_k} \tag{9}$$

Notice that the inner problem is the weighted sum rate maximization for a fixed weight set, while the outer problem with respect to $\theta_2, \dots, \theta_K$ is a convex problem for which a $(K-1)$-dimensional subgradient method can be suitably applied [11]. Due to space limitations, we only describe a numerical method to solve the inner problem, given by

$$\max_{\sum_{k=1}^{K} p_{ik} \le P_i, \forall i} \sum_{k=1}^{K} w_k \log\left(1 + \sum_{i=1}^{B} |a_{ik}|^2 p_{ik}\right) \tag{10}$$

where $w_k = \theta_k/\alpha_k$ denotes a non-negative weight for all k. Since the above problem is a convex optimization problem, it is necessary and sufficient to solve the KKT condition, given by

$$\frac{w_k |a_{ik}|^2}{1 + \sum_{j=1}^{B} |a_{jk}|^2 p_{jk}} = \frac{1}{\mu_i}, \quad k = 1, \dots, K, i = 1, \dots, B \tag{11}$$

Unfortunately solving (11) directly seems intractable. Nevertheless, the following waterfilling approach inspired by the iterative multiuser waterfilling [12] solves the KKT conditions iteratively.

Algorithm A1 : Iterative waterfilling algorithm for B-BS weighted sum rate maximization

1) Initialize $\mathbf{p}_i^{(0)} = \mathbf{0}$ for $i = 1, \dots, B$ and $c_{ij}^{(0)} = 0$ for all i, j.
2) At each iteration n
   For $i = 1, \dots, B$
   - Compute $c_{ik}^{(n)} = \sum_{l \neq i} |a_{lk}|^2 p_{lk}^{(n)}$
   - Waterfilling step : let $\mathbf{p}_i^{(n)}$ be
   $$p_{ik}^{(n)} = \left[w_k \mu_i - \frac{1 + c_{ik}^{(n)}}{|a_{ik}|^2}\right]_+, \quad \forall k \tag{12}$$
   where μ_i is determined such that $\sum_{k=1}^{K} p_{ik}^{(n)} = P_i$.

   End
3) Continue until convergence

Some remarks are in order; 1) Algorithm A1 is a generalization of the classical (non-iterative) waterfilling algorithm for the K parallel channels under the total power constraint to the case with B transmitters with individual power constraints. Indeed, for a single-BS case ($B = 1$), the objective function reduces to the weighted sum rate of the K parallel channels. 2) The set of powers in (12) correspond to the solution to the new objective function

$$\mathbf{p}_i^{(n)} = \arg \max_{\mathbf{q} \ge \mathbf{0}, \sum_k q_k \le P_i} \sum_{k=1}^{K} w_k \log\left(1 + \frac{|a_{ik}|^2 q_k}{1 + c_{ik}^{(n)}}\right) \tag{13}$$

It can be easily seen that this new problem and the original problem (10) yield the same KKT conditions (11) for $c_{ik}^{(n)} = \sum_{l \neq i} |a_{lk}|^2 p_{lk}^{(n)}$. In other words, the solution (12) of BS i corresponds to treating $\{c_{ik}\}_{k=1}^{K}$, function of the powers $\{\mathbf{p}_l\}_{l \neq i}$ of the other BSs, as additional noise (constant), as if they did not depend on $\mathbf{p}_i$. Under the individual power constraints at each BS, the sequential iteration over different BSs shall converge. The convergence proof follows the same steps as the proofs of [12, Theorem 1, Theorem 2] and shall not be repeated here [13].

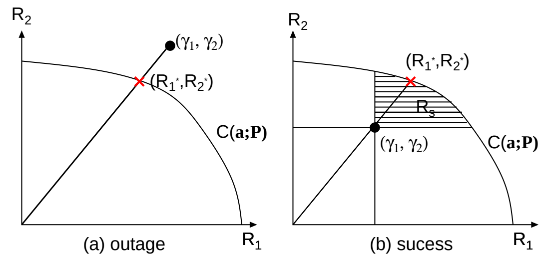

Fig. 2. Rate region $\mathcal{C}(\mathbf{a}; \mathbf{P})$ and subregion $\mathcal{R}_s$

The overall algorithm, the inner problem solved by algorithm A1 and the outer problem solved by the subgradient method, implements the rate-balancing by allocating the rates proportional to the target rate tuple. We have the following result.

Theorem 1: The rate-balancing power allocation minimizes the outage probability.

Proof: First we remark that for a given channel realization $\mathbf{a}$ the following two cases occur;

(a) The target rate tuple is outside the region $\boldsymbol{\gamma} \notin \mathcal{C}(\mathbf{a}, \mathbf{P})$
(b) The target rate tuple is inside the region $\boldsymbol{\gamma} \in \mathcal{C}(\mathbf{a}, \mathbf{P})$

Two cases are illustrated in Fig.2 (a), (b) respectively for $K = 2$. For the case (a), we are in an outage event regardless of the power allocation. For the case (b), we define the subregion $\mathcal{R}_s$

$$\mathcal{R}_s \triangleq \{\mathbf{R} | \mathbf{R} \in \mathcal{C}(\mathbf{a}, \mathbf{P}), R_k \ge \gamma_k, \ k = 1, \dots, K\} \tag{14}$$

depicted in a shadow area in Fig. 2 (b). We remark that any power allocation policy $\mathcal{P}_s$ that maps $\mathbf{a}$ into the rate tuple $\mathbf{R}$ inside $\mathcal{R}_s$ results in a successful transmission. Hence, we can conclude that such a policy that allocates the rates inside $\mathcal{R}_s$ whenever $\boldsymbol{\gamma} \in \mathcal{C}(\mathbf{a}, \mathbf{P})$ minimizes the outage probability. Since the proposed rate balancing scheme allocates the rate-tuple $(R_1^\star, \dots, R_K^\star)$ proportional to $\boldsymbol{\gamma}$ on the boundary of $\mathcal{C}(\mathbf{a}, \mathbf{P})$ whenever $\boldsymbol{\gamma} \in \mathcal{C}(\mathbf{a}, \mathbf{P})$, it belongs to the class $\mathcal{P}_s$. This establishes the proof. ∎

Moreover, whenever the outage event occurs $\boldsymbol{\gamma} \notin \mathcal{C}(\mathbf{a}, \mathbf{P})$, the proposed rate balancing scheme equalizes the individual outage probabilities of all UTs. It is immediate to see that with the rate balancing scheme, we have for any $k = 2, \dots, K$

$$\Pr(\gamma_k < R_k^\star(\mathbf{p})) = \Pr(\alpha_k \gamma_1 < \alpha_k R_1^\star(\mathbf{p})) = \Pr(\gamma_1 < R_1^\star(\mathbf{p}))$$

where the first equality follows from (8). Therefore, the proposed rate-balancing policy offers a strict fairness among UTs. The outage probability with the rate balancing scheme can be always written as

$$\begin{aligned} P_{\text{out}}(\boldsymbol{\gamma}, \mathbf{p}) &= 1 - \Pr(\gamma_1 < R_1^\star(\mathbf{p}), \dots, \gamma_K < R_K^\star(\mathbf{p})) \\ &= 1 - \Pr(\gamma_1 < R_1^\star(\mathbf{p})). \end{aligned}$$

B. Statistical CSI at CS

Under statistical CSI of $\mathbf{a}$ at the CS, the individual outage events that UTs $1,\dots,K$ cannot support the target rate $\gamma_1,\dots,\gamma_K$ are independent. For a fixed power allocation $\mathbf{p}$, the outage probability is given by

$$P_{\rm out}(\boldsymbol{\gamma},\mathbf{p}) = 1-\prod_{k=1}^{K}\Pr\left(\Delta_k(\mathbf{p}^k)>2^{\gamma_k}-1\right) \quad (15)$$

where we let $\Delta_k(\mathbf{p}^k)=\sum_{i=1}^{B}|a_{ik}|^2p_{ik}$. First, we remark that for a fixed power $\mathbf{p}^k=(p_{1j},\dots,p_{Bk})$ of UT k , $\Delta_k(\mathbf{p}^k)$ is a Hermitian quadratic form of complex Gaussian random variables given by

$$\begin{aligned}\Delta_k(\mathbf{p}^k) &= (a_{1k}^*,\dots,a_{Bk}^*)\begin{pmatrix} p_{1k} & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & p_{Bk}\end{pmatrix}\begin{pmatrix} a_{1k}\\ \vdots \\ a_{Bk}\end{pmatrix}\\ &= (\mathbf{w}_{1k}^H\dots\mathbf{w}_{Bk}^H)\begin{pmatrix} p_{1k}\mathbf{I} & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & p_{Bk}\mathbf{I}\end{pmatrix}\begin{pmatrix} \mathbf{w}_{1k}\\ \vdots \\ \mathbf{w}_{Bk}\end{pmatrix}\end{aligned}$$

where the second equality follows by replacing a chi-square random variable $|a_{ik}|^2$ with $2(M-K+1)$ d.o.f. with $||\mathbf{w}_{ik}||^2$ where $\mathbf{w}_{ik}\sim\mathcal{N}_{\mathbb{C}}(\mathbf{0},\frac{\sigma_{ik}}{M-K+1}\mathbf{I}_{M-K+1})$. The outage probability that UT k cannot support γ_k for a fixed $\mathbf{p}^k$ is

$$\Pr(\Delta_k(\mathbf{p}^k)\le c_k)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{e^{sc_k}\Phi_{\Delta_k(\mathbf{p}^k)}(s)}{s}ds \quad (16)$$

where we let $c_k=2^{\gamma_k}-1$ denote the target SNR of UT k, and the Laplace transform of $\Delta_k(\mathbf{p}^k)$ is given by

$$\Phi_{\Delta_k(\mathbf{p}^k)}(s)=\prod_{i=1}^{B}\frac{1}{(1+s\frac{p_{ik}\sigma_{ik}}{M-K+1})^{M-K+1}} \quad (17)$$

The widely used upper bound is the Chernoff bound for fixed powers $\mathbf{p}^k$ given by

$$\Pr(\Delta_k(\mathbf{p}^k)\le c_k)\le\min_{\lambda\ge 0}e^{\lambda c_k}\Phi_{\Delta_k(\mathbf{p}^k)}(\lambda)\stackrel{\Delta}{=}\overline{F}(c_k,\mathbf{p}^k). \quad (18)$$

It can be easily seen that (also from (4)) that each UT achieves a diversity gain of $B(M-K+1)$. Using the last expression of the individual Chernoff upper bound for each k, the outage probability for a fixed $\mathbf{p}$ is upper bounded by

$$P_{\rm out}(\boldsymbol{\gamma},\mathbf{p}) \le 1-\prod_{k=1}^{K}(1-\overline{F}(c_k,\mathbf{p}^k)) \quad (19)$$

Since the power optimization based on the exact outage probability is not amenable, we search the power allocation that minimizes the Chernoff upperbound, equivalently solves

$$\begin{aligned} \text{maximize}\quad & f(\{\lambda_k\},\{p_{ij}\})\stackrel{\Delta}{=}\prod_{j=1}^{K}(1-h_k(\lambda_k,\mathbf{p}^k)) \quad (20)\\ \text{subject to}\quad & \sum_{k=1}^{K}p_{ik}\le P_i,\quad i=1,\dots,B\\ & \lambda_k\ge 0,\quad k=1,\dots,K\\ & p_{ik}\ge 0,\quad i=1,\dots,B,k=1,\dots,K\end{aligned}$$

where we define

$$h_k(\lambda_k,\mathbf{p}^k) = \frac{e^{\lambda_k c_k}}{\prod_{i=1}^{B}(1+\alpha_{ik}\lambda_k p_{ik})^{M-K+1}} \quad (21)$$

where $\alpha_{ik}=\frac{\sigma_{ij}}{M-K+1}$. If $h_k\ge 1$ for some k, the objective becomes null regardless of the power allocation. In this case a reasonable choice is to let $p_{ik}=0,\forall i$ for such k and equally allocate the power to $\{p_{ij}\}$ for $j\ne k$. In the following, by focusing on the case $h_k<1$ for all k, we provide an efficient numerical method to solve the problem (20). We remark first that the maximization of f with respect to $\{\lambda_k\}$ can be decoupled into the minimization of h_k over λ_k for each k, where h_k is convex in λ_k. Moreover, since f is concave in $\{p_{ik}\}$, the overall problem is convex.

Minimization of h_k over λ_k It can be easily verified that h_k is monotonically decreasing in λ_k. The optimal λ_k for a fixed set of powers is the solution of

$$\frac{c_k}{M-K+1}=\sum_{i=1}^{B}\frac{\alpha_{ik}p_{ik}}{1+\lambda_k\alpha_{ik}p_{ik}} \quad (22)$$

which is a polynomial of degree B. For $B=2$, the solution is given in a closed form.

Maximization of f over p_{ij} Since f is concave in $\{p_{ij}\}$, we form the Lagrangian function by introducing B Lagrangian multipliers $\{\mu_i\}$ each of which is associated to the power constraint of BS i. By arranging the term common for all k, we obtain the KKT conditions for $k=1,\dots,K$

$$\frac{h_k(\mathbf{p}^k)}{1-h_k(\mathbf{p}^k)}\frac{\alpha_{ik}\lambda_k}{1+p_{ik}\alpha_{ik}\lambda_k}=\mu_i \quad (23)$$

When treating $p_{1k},\dots,p_{i-1,k},p_{i+1,k},\dots,p_{B,k}$ fixed, the LHS of (23), denoted by ϕ_{ik}, is a strictly positive and monotonically decreasing function of p_{ik} (since we exclude the case $\lambda_k=0$). It remains to determine μ_i such that the power constraint of BS i is satisfied , i.e. $p_{i1}+\dots+p_{iK}=P_i$. When treating the powers $\{\mathbf{p}_j\}_{j\ne i}$ of the other BSs $j\ne i$ freezed, the powers $\mathbf{p}_i$ of BS i can be found by a simple line search of μ_i. Under individual BS power constraints, a sequential update of the powers $\mathbf{p}_1,\mathbf{p}_2,\dots,\mathbf{p}_B,\mathbf{p}_1$... shall converge. The detailed of the convergence proof is referred to [13].

The following summarizes our proposed iterative algorithm to minimize the Chernoff upperbound, equivalently solve (20).

Algorithm A2 : iterative algorithm for the Chernoff upperbound minimization

1) Initialize $\mathbf{p}^{(0)}$
2) At iteration n
   For $i=1,\dots,B$
   - Update $\boldsymbol{\lambda}^{(n)}$ by solving the polynomial (22)
   - Find the new power vector $\mathbf{p}_i^{(n)}$ of BS i by line search

   End
3) Continue until converge f

IV. Numerical Examples

This section provides some numerical examples to verify the behavior of our proposed power allocation strategies in a simple network MIMO configuration with $B = K = 2$. For the sake of comparison, we also consider the case without network MIMO (no message sharing) where each BS sends a message to its UT in a distributed fashion. In order to make the comparison fair in terms of complexity, we let each BS i send the symbol s_i by ZF beamforming, i.e. $\mathbf{x}_i = \sqrt{P_i}\mathbf{g}_i s_i$ where $\mathbf{g}_i$ is a unit-norm vector orthogonal to $\mathbf{h}_{ij}$ for $j \neq i$. It can be easily shown that such a system offers a diversity order of $M - K + 1$ for each UT.

Fig. 3 shows the outage probability performance as a function of per-BS SNR $P_1 = P_2$. The target rate is fixed to $\gamma_1 = 1, \gamma_2 = 3$ bit/channel use, respectively, and we let $\sigma_{ij} = 1$ for all i, j. We compare the different power allocation strategies, algorithm A1 with perfect CSIT, algorithm A2 with statistical CSIT, and equal power allocation ($p_{i1} = p_{i2} = P_i/2$ for $i = 1, 2$). The diversity gain of $2(M - K + 1)$ is achieved for any power allocation strategy with network MIMO, which doubles the diversity gain without network MIMO. Moreover, the proposed algorithms provide a significant power gain compared to equal power allocation.

In order to evaluate the impact of asymmetric path loss, we consider a simple 1-D configuration such that UT2 is located at $x = 3$ and UT moves from $x = 0$ to $x = 2$. Assuming that BS1, 2 is at $x = 1, 3$, we vary $d_{11} = \sqrt{1 + (1 - x)^2}, d_{12} = \sqrt{1 + (3 - x)^2}$ while we fix the position of UT2 by letting $d_{12} = \sqrt{5}, d_{22} = 1$. By taking into account the path loss $\sigma_{ij} = d_{ij}^{-3}$, Fig. 4 shows the outage probability as a function of the position x of UT 1 for $M = 4$ and $\gamma_1 = \gamma_2 = 1$ with SNR $P_1 = P_2 = 10$ dB. We observe that even in a moderate SNR regime the proposed power allocation provides a significant gain compared to the case without network MIMO.

These numerical examples show that even with partial CSIT network MIMO enables to provide high data rates with a sufficient reliability to individual users. Such a merit of network MIMO has been overlooked in most of existing works assuming perfect CSIT.

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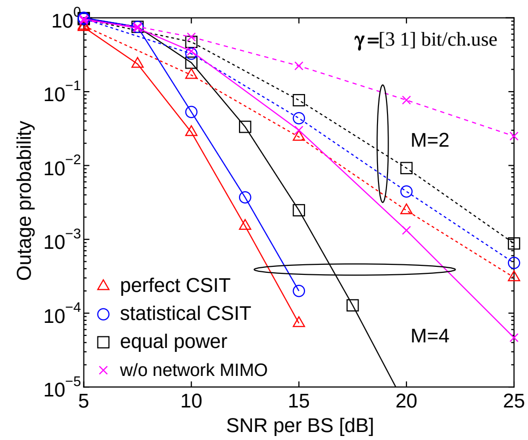

Fig. 3. Outage probability vs. SNR

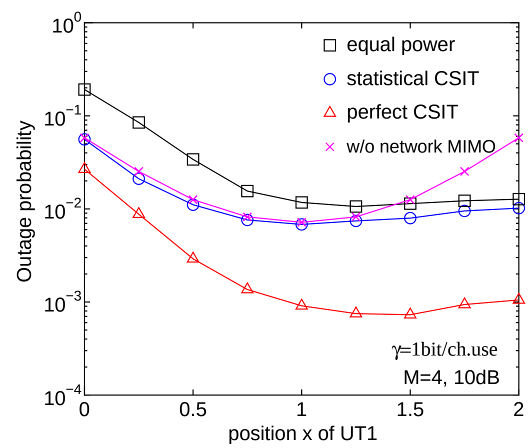

Fig. 4. Outage probability vs. location of UT1

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