Synthesis of Complex Subband Hybrid Filter Banks A/D Converters using Adaptive Filters

Zhiguo Song, Caroline Lelandais-Perrault, Philippe Benabes

Synthesis of Complex Subband Hybrid Filter Banks A/D Converters using Adaptive Filters

Zhiguo Song, Caroline Lelandais-Perrault (*member, IEEE*) and Philippe Bénabes (*member, IEEE*)
Department of Signal Processing and Electronic Systems
SUPELEC
91192 Gif sur Yvette, France
Email: `Firstname.Lastname@supelec.fr`

***Abstract*—Hybrid Filter Banks (HFB) A/D converters (ADC) may be an attractive solution for future cognitive radio application. In this paper, using complex adaptive finite impulse response (FIR) filter whose coefficients are initialized with zero, their optimal values are achieved by iterative adjustments. This synthesis method is validated for a complex subband HFB by simulation in the time-domain.**

I. INTRODUCTION

As one of the key technologies for the cognitive radio, A/D converters (ADCs) are expected to convert alternatively a wide band with a low resolution (in sensing mode) or a narrow bandwidth with a higher resolution (in communication mode). Hybrid Filter Banks (HFB) ADCs may be an appropriate solution for these requirements [1] [2]. Being parallel architecture, HFB ADCs are composed of an analog frequency decomposition (analysis bank), a conversion-stage (ADC bank) and a digital reconstruction (synthesis bank). With a given ADC sampling rate, classical HFBs enlarge the conversion bandwidth. The resolution of conversion is related to the digital reconstruction complexity and the resolution of each ADC [3],[4],[5]. Furthermore, when a narrow band conversion is needed, the digital part may be reconfigured to reconstruct this subband only, which defines subband HFB ADCs proposed in [6].

The synthesis methods of the HFBs proposed in [4] and [5] synthesize jointly the analog part and digital part. These methods require high-precision analog components to obtain a given transfer function. Therefore, their hardware implementation becomes more difficult and more expensive with the scaling of integrated circuit technologies [1]. Relaxing the constraints upon the realization of the analog components, and assuming that the knowledges of the analysis frequency response are obtained, the synthesis filters can be achieved by calculating the inverse FFT of the ideal synthesis filter frequency response [7]. With the same hypothesis, [1] proposed a least mean square global approximation (LMSGA) synthesis method that minimizes a criterion describing the near prefect reconstruction of the classical HFB. This method is applied also for synthesizing the subband HFB [6]. We proposed in [8] an adaptive method to compensate analog errors digitally for a subband HFB. Initializing with the pre-calculated coefficients by the LMSGA method, although the analog errors degrade dramatically the performance of the subband HFB, the iterative adjustments of the synthesis filters can recover a good resolution finally.

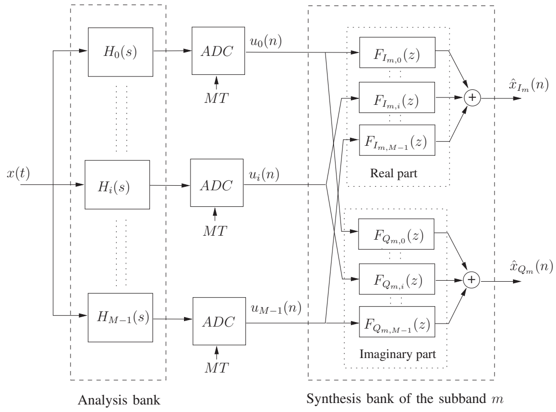

Fig. 1. Continuous-time M-channel complex subband HFB ADC, T denotes the Nyquist sampling period of input wideband signal. The subscripts denote the subband number and the channel number respectively, $m, i \in [0, M-1]$.

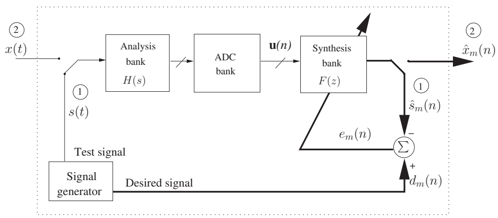

Fig. 2. Principle of synthesis for complex subband HFB ADC: adaptive channel equalization (phase 1). Once the synthesis process is achieved, the subband HFB is ready to convert the extern wideband signal $x(t)$ (phase 2). The think line designates the signal in complex form.

In this paper, we propose the adaptive equalization [9] [10] as a synthesis method for the subband HFB. The digital filters are all initialized to zero. Their coefficients are adjusted iteratively to reach an optimal reconstruction. Furthermore, as complex signals are widely used in digital processing for radiocommunication, we propose here an architecture (Figure 1) that reconstruct complex digital signals which are more convenient for the downstream treatments.

Figure 2 presents the principle of the proposed synthesis method in the M-chennal complex subband HFB context. Two linked signals are provided by the generator: the test signal $s(t)$ and the desired signal $d_m(n)$. $s(t)$ and $d_m(n)$ are respectively analog wideband signal and complex digital signal of the subband $m \in [0,\ M-1]$. The error $e_m(n)$ is used to adjust the synthesis filter coefficients in the subband m. The synthesis process will be achieved until the mean-square error (MSE) of $e_m(n)$ is small enough to meet the reconstruction accuracy requirements. Thus, the subband HFB is ready to convert the wideband signals $x(t)$. The quantization errors are neglected here.

In section II, the synthesis method using adaptive channel equalization is theoretically justified for a complex subband HFB ADC. In section III, initializing the synthesis filters coefficients to zero, an eight-channel complex adaptive subband HFB ADC is simulated in the time-domain. A wideband signal different from the test signal $s(t)$ is converted after the synthesis processing. Then, the performances of the complex subband HFB are discussed.

II. Adaptive equalization for synthesizing a complex subband HFB ADC

For each subband, the synthesis process is similar, so we can just focus on the reconstruction of one subband m.

A. Test signal $s(t)$

The test signal is a critical point to realize the proposed synthesis method. As a wideband signal, $s(t)$ should cover spectrally all of the band to convert, i.e. its spectrum should be "rich" enough. Furthermore, it is necessary to avoid the additional interferences generated by the frequencies apart the passband. Filtered by bandpass filters with high selectivity, zero-mean white noise signal may satisfy the needs of the test signal for synthesizing the complex subband HFB.

B. Desired signal $d_m(n)$

The desired signals provide the references for reconstructing digitally the subband signals of the test signal. Denoting $S(j\Omega)$ the Fourier transform of $s(t)$, Ω is the pulsation and its normalization is given by $w = \Omega T$, where T is the Nyquist period of sampling. Using complex modulated passband filter $G_m(e^{j\omega})$, $S(j\Omega)$ is split into M subbands which bandwidth is $\frac{2\pi}{M}$ (Figure 3). Let define $G_m(e^{j\omega})$ in two parts: $G_{m'}(e^{j\omega})$ and $G_{m''}(e^{j\omega})$, where $m' \in [0,\ \frac{M}{2}-1]$ and $m'' \in [\frac{M}{2},\ M-1]$. So, $G_m(e^{j\omega})$ can be presented as follows

$$\begin{cases} G_{m'}(e^{j\omega}) = 1; \quad R' = (n+2m')\frac{\pi}{M} \leq \omega \leq (n+2m'+2)\frac{\pi}{M} \\ G_{m'}(e^{j\omega}) = 0; \quad \omega \notin R' \end{cases} \tag{1}$$

$$\begin{cases} G_{m''}(e^{j\omega}) = 1; \quad R'' = -(n+2(m''-\frac{M}{2}))\frac{\pi}{M} \geq \omega \geq -(n+2(m''-\frac{M}{2})+2)\frac{\pi}{M} \\ G_{m''}(e^{j\omega}) = 0; \quad \omega \notin R'' \end{cases} \tag{2}$$

where $n \in \mathbb{Z}$. After being sampled at the rate $\frac{1}{MT}$, the selected subband signals can be founded in the bass frequency. According to (1) (2) and the extraction of the subband signal illustrated in Figure 3, the desired signal of the subband m' can be given by (3) for n is odd

$$D_{m'}(e^{j\omega}) = \begin{cases} \frac{1}{M}S(e^{j(\frac{\omega}{M}+2m'\frac{\pi}{M}+(n+1)\frac{\pi}{M})}) \\ \frac{1}{M}S(e^{j(\frac{\omega}{M}+(2m'+1)\frac{\pi}{M}+(n+1)\frac{\pi}{M})}) \end{cases} \tag{3}$$

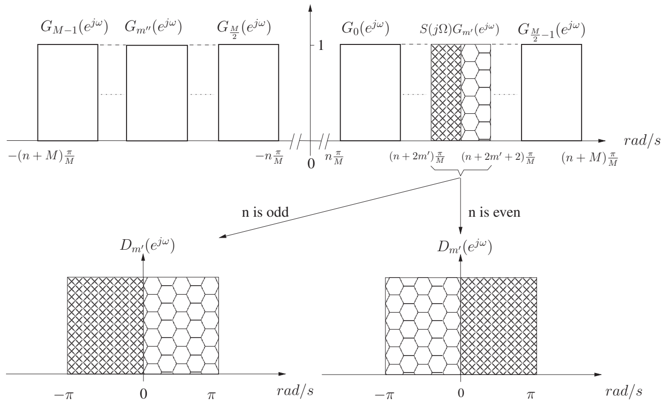

Fig. 3. Above: the passband filters for selecting the subband signals, where where $n \in \mathbb{Z}$, $m' \in [0,\ \frac{M}{2}-1]$, $m'' \in [\frac{M}{2},\ M-1]$. Below: extraction of each subband desired signal $D_{m'}(e^{j\omega})$ from $S(j\Omega)$.

When n is even, $D_{m'}(e^{j\omega})$ is given by

$$D_{m'}(e^{j\omega}) = \begin{cases} \frac{1}{M}S(e^{j(\frac{\omega}{M}+(2m'+1)\frac{\pi}{M}+(n+2)\frac{\pi}{M})}) \\ \frac{1}{M}S(e^{j(\frac{\omega}{M}+2m'\frac{\pi}{M}+n\frac{\pi}{M})}) \end{cases} \tag{4}$$

$D_{m''}(e^{j\omega})$ can be achieved by using the same method for extracting $D_{m'}(e^{j\omega})$. Hence, the digital complex desired signals are ready for synthesizing the subband HFB adaptively.

C. Synthesis of the complex subband HFB

The wideband signal of test $s(t)$ is decomposed by the analog filters into M channels, then undersampled at the rate of $\frac{1}{MT}$ to be $\mathbf{u}(n)$ denoting the inputs of the synthesis filters, $\mathbf{u}(n)$ is given by

$$\mathbf{u}(n) = [\mathbf{u}_0(n), \mathbf{u}_1(n), \cdots, \mathbf{u}_{M-1}(n)]^T, \tag{5}$$

where $\mathbf{v}^T$ means the transpose of the vector $\mathbf{v}$, and $\mathbf{u}_i(n)$ is denoted by

$$\mathbf{u}_i(n) = [u_i(n), u_i(n-1), \cdots, u_i(n-L+1)] \tag{6}$$

where L is the length of the synthesis filter. Let the vectors $\tilde{\mathbf{f}}_{m,i} = [\tilde{f}_{m,i}(0), \tilde{f}_{m,i}(1), \cdots, \tilde{f}_{m,i}(L-1)]$ denote the complex synthesis filters coefficients in subband m channel i, which will be adjusted adaptively to minimize the reconstruction error. Let $\tilde{\mathbf{f}}_m$ denote all the synthesis filter coefficients in the subband m

$$\tilde{\mathbf{f}}_m = [\tilde{\mathbf{f}}_{m,0}, \tilde{\mathbf{f}}_{m,1}, \cdots, \tilde{\mathbf{f}}_{m,M-1}] \tag{7}$$

Let $e_m(n)$ denote the difference between the desired signal and the estimated subband signal $\hat{s}_m(n)$. $\hat{s}_m$ and $e_m(n)$ are defined respectively as follows

$$\hat{s}_m(n) = \tilde{\mathbf{f}}_m^* \mathbf{u}(n) \tag{8}$$

$$e_m(n) = d_m(n) - \hat{s}_m(n) \tag{9}$$

Let define $\mathbf{J}_m$ the chosen cost function MSE, its dependence on the complex synthesis coefficients is shown as follows

$$\begin{aligned}\mathbf{J}_m &= \mathbf{E}(|e_m(n)|^2) \\ &= \mathbf{E}(|d_m(n)|^2) - \tilde{\mathbf{f}}_m^* \mathbf{E}(\mathbf{u}(n) d_m^*(n)) - \\ &\quad \tilde{\mathbf{f}}_m \mathbf{E}(\mathbf{u}(n) d_m(n)) + \tilde{\mathbf{f}}_m^* \mathbf{E}\left(\mathbf{u}(n)^T \mathbf{u}(n)\right) \tilde{\mathbf{f}}_m^T \quad (10)\end{aligned}$$

The expectation $\mathbf{E}(\mathbf{u}(n)d_m(n))$ is equal to the cross-correlation between $\mathbf{u}(n)$ and $d_m(n)$, noted $\mathbf{p}_m$. The expectation $\mathbf{E}(\mathbf{u}(n)^T\mathbf{u}(n))$ is equal to the auto-correlation of the vector $\mathbf{u}(n)$, noted $\mathbf{R}_{uu}$. So, (10) can be written by

$$\mathbf{J}_m = \sigma_{d_m}^2 - \tilde{\mathbf{f}}_m^* \mathbf{p}_m^* - \tilde{\mathbf{f}}_m^* \mathbf{p}_m + \tilde{\mathbf{f}}_m^* \mathbf{R}_{uu} \tilde{\mathbf{f}}_m^T \tag{11}$$

Since $\mathbf{R}_{uu}$ is a matrix nonsingular, its inverse matrix $\mathbf{R}_{uu}^{-1}$ exists. Minimizing $\mathbf{J}_m$ in terms of $\tilde{\mathbf{f}}_m$, the optimal synthesis coefficients can be achieved by solving the Wiener-Hopf equations [10] and given by

$$\mathbf{f}_m^{opt} = \mathbf{R}_{uu}^{-1} \mathbf{p}_m \tag{12}$$

The calculation of $\mathbf{f}_m^{opt}$ with (12) is obviously too costly for inversing the large matrices. Thus, the complex LMS algorithm is chosen here due to its simplicity [9] [10], which updates iteratively the synthesis filter coefficients $\tilde{\mathbf{f}}_m$ for approximating $\mathbf{f}^{opt_m}$ the (Figure 4) with the formulas as follows

$$\begin{aligned}\tilde{\mathbf{f}}_{I_m}(n+1) &= \tilde{\mathbf{f}}_{I_m}(n) + \mu e_{I_m}(n)\mathbf{u}(n) \quad (13) \\ \tilde{\mathbf{f}}_{Q_m}(n+1) &= \tilde{\mathbf{f}}_{Q_m}(n) + \mu e_{Q_m}(n)\mathbf{u}(n) \quad (14)\end{aligned}$$

where μ is named step-size. The condition of stability of long complex LMS FIR filters is shown below [10]

$$0 < \mu < \frac{1}{ML\mathbf{E}[u_i^2(n)]}, \tag{15}$$

where $\mathbf{E}\left[u_i^2(n)\right]$ denotes the expectation of the power spectral density of the synthesis bank inputs.

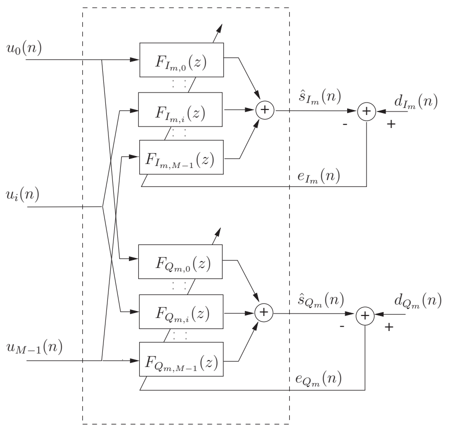

Fig. 4. Synthesis of the complex subband HFB ADC by using complex LMS algorithm.

D. Performance of the complex subband HFB ADC

Figure 1 shows the architecture of the complex subband HFB, which reconstruction function [8] can be presented in frequency-domain by

$$T_{m,p}(e^{j\omega}) = \sum_{i=0}^{M-1} H_i(j(\Omega - \frac{2\pi p}{MT}))F_{m,i}(e^{j\omega}) \tag{16}$$

where the index $p \in [-(M-1),\ M-1]$. Theoretically, the conditions for perfect reconstruction of the subband are shown as follows

$$\begin{cases} T_{m,0}(e^{j\omega}) = e^{-j\omega\rho}, & p = 0, \rho \in \mathbb{R}_*^+ \\ \displaystyle\sum_{-(M-1)}^{M-1} T_{m,p}(e^{j\omega}) = 0, & p \neq 0 \end{cases} \tag{17}$$

where $T_{m,0}(e^{j\omega})$ is defined as the distortion function, which is expected to a pure delay of ρ; the sum of $T_{m,p\neq 0}(e^{j\omega})$ is defined as the aliasing function, which is expected to be 0. Hence, the performance characteristic of the subband HFB for reconstruction of the subband m can be defined here by the Distortion-to-Aliasing Ratio (DAR)

$$DAR_{dB}(\omega) = 20log \frac{\left|\frac{T_{m,m}(e^{j\omega})}{ce^{-j\omega\rho}}\right|}{\sqrt{\sum_{i\neq m} \left|T_{m,i}(e^{j\omega})\right|^2}} \tag{18}$$

III. Simulation results

A. Conditions of simulation

Considering the wideband signal to convert is in the frequency band [0.85 1.25]GHz. The analog signal is split into an analog bandpass filter bank which consists in eight pure Gm-LC resonators [1], whose central frequencies and quality factors are respectively $2\pi\Omega_i$ and Q_i. The transfer function of the analysis filter of each channel is given by

$$\mathbf{H}_i(s) = \frac{\frac{\Omega_i}{Q_i}s}{s^2 + \frac{\Omega_i}{Q_i}s + \Omega_i^2} \tag{19}$$

According [2], the dynamic range of the radiocommunication application like GSM is defined to be 82 dB i.e a resolution of 14 bit. This can be considered as the goal of reconstruction accuracy. Each synthesis subband filters of the subband m are complex FIR filters whose coefficients are initialized by zero. The process of synthesis of the complex subband HFB is simulated in the time-domain with Matlab using C MEX-functions.

A pseudo random sequence is filtered by the complex subband anti-aliasing filters, their sum forms the test signal $s(t)$. The signals of each subband m is sampled at the frequency $\frac{1}{MT}$. Thus, complex desired signal $d_m(n)$ is obtained. Here, the purpose of the simulations is to synthesize the subband 0. In order to reduce the aliasing terms, it's efficiency to allocate a small part of frequency borders (low and high frequencies) at each subband spectrum as guard band (GB) [6], which equals

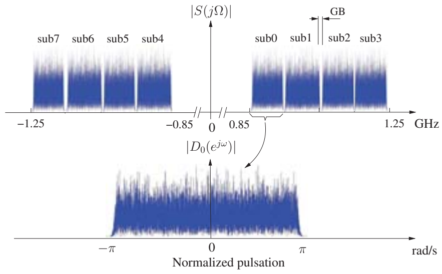

Fig. 5. Specturum of the test signal $s(t)$ and the desired signal $d_0(n)$.

7% of the subband bandwidth in this paper. Figure 5 shows the spectrums of $s(t)$ and $d_0(n)$.

B. Simulation results of synthesis

The synthesis process is stopped as soon as the MSE of $e_0(n)$ is small enough, i.e. the performance of the subband HFB satisfies the application requirements. Here, the stop condition is defined by dynamic range of 82 dB, which corresponds to a resolution of 14 bit. In the end of the synthesis process, the complex synthesis coefficients may be considered as the optimal coefficients noted by $\mathbf{f}_0^{opt}$. In Figure 6, the above part shows the convergence of the coefficients VS the iteration number. The performance of the subband HFB is calculated in the time-domain by desired signal to error ratio, which noted SNR conventionally. The convergence of the SNR is shown in the below part, which progresses for 14.6 bit.

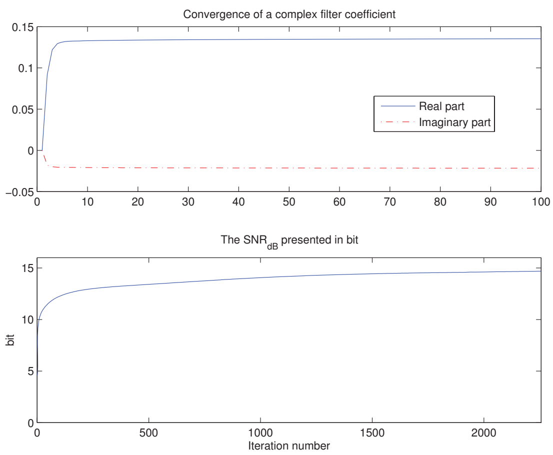

Fig. 6. Above: the convergence of the coefficient indexed by 64 VS the iteration number. Below: the evolution of the SNR presented in bit for the reconstruction of the subband 0.

With $\mathbf{f}_0^{opt}$, the performances are calculated in the frequency-domain. The frequency responses of the synthesis filters in the subband 0 are presented in the above part of Figure 7. According to (18), the performance DAR is calculated to be 90 dB, which is presented in the below part.

C. Simulation results of validation

After being synthesized with good performance, the complex subband HFB is ready to convert the others signals

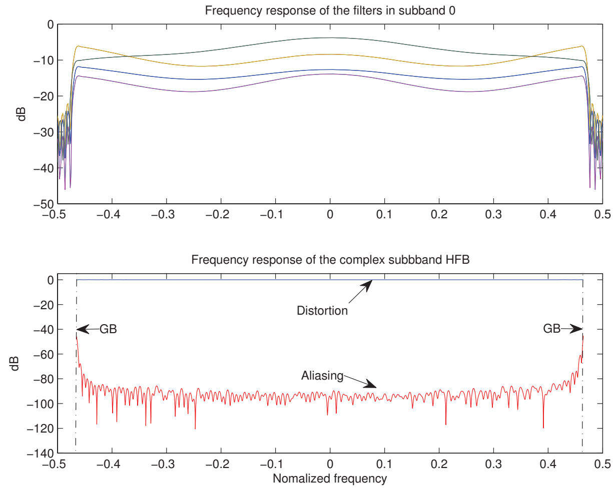

Fig. 7. Above: the frequency responses of the complex filters in the subband 0. Below: frequency response of complex subband HFB ADC.

differed from $s(t)$ (Figure 1). In order to validate this aspect, fixing $\mathbf{f}_0^{opt}$, the comb sinus signals are applied as $x(t)$. The simulation result shows that the resolution of conversion of the subband 0 reaches 14.5 bit.

IV. Conclusion

In this paper, negating the quantization errors, a synthesis method using adaptive filters for a complex subband HFB ADC is proposed. The optimal synthesis coefficients can be approached iteratively by using the complex LMS algorithm. This method is easy for implementing in a digital circuit. The resolution of conversion for one subband can reach 14 bit, this meets severals needs of cognitive radio.

References

[1] C. Lelandais-Perrault, T. Petrescu, D. Poulton, P. Duhamel, J. Oksman, “Wide-band, band pass and versatile Hybrid Filter Bank A/D, conversion for software radio”, in *IEEE TCAS-I (Transactions Circuits and Systems I)*, Accepted to publication, 2009.

[2] C. Lelandais-Perrault, Z. Song, D. Poulton, P. Benabes, “Hybrid Filter Bank A/D conversion systems applied to future telecommunication scenarios”, *Joint IEEE-NEWCAS and TAISA Conference*, June 2009.

[3] A. Petraglia and S. Mitra, “High speed A/D conversion incorporating a QMF bank”, *IEEE Transactions on Instruments and Mesurement*, vol. 41, pp.427-431, June 1992.

[4] S. R. Velazquez, T. Q. Nguyen, S. R. Broadstone, “Design of hybrid filter bank for analog/digital conversion”, *IEEE Transaction on Signal Processing*, Vol. 46, no. 4, pp.956-967, April 1998.

[5] P. Löwenborg, H. Johansson, and L. Wanhammar, “A class of two-channel hybrid analog/digital filter banks”, in *Processing of IEEE Midest Symposium on Circuit and Systems*, vol. 1, pp 14-17, August 1999.

[6] D. Asemani, J. Oksman and P. Duhamel “Subband architecture for Hybrid Filter Bank A/D converters”, in *IEEE Journal on selected Topics in Signal Processing*, Vol.2, No. 2, pp.191-201, April 2008.

[7] S. R. Velazquez, “Hybrid filter banks for analog/digital conversion”, Ph.D. dissertation, Massachusetts Institute of Technology, June 1997.

[8] Z. Song, C. Lelandais-Perrault, D. Poulton, P. Benabes, “Adaptive Equalization for Calibration of Subband Hybrid Filter Banks A/D Converters”, in *IEEE 19th European Conference on Circuit Theory and Design*, August 2009.

[9] B. Widrow, J. McCool, and M. Ball, “The complex LMS algorithm”, in *Proceedings of the IEEE*, vol.63, pp.719-720, April 1975.

[10] S. Haykin(Editor), “Adaptive Filter Theory” Prentice Hall, 2002.