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# ROBUSTIFIED CONTROL OF A MULTIVARIABLE ROBOT

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**Keywords:** Model Predictive Control, Youla-Kučera parameter, unstructured uncertainties, Linear Matrix Inequality, multivariable systems, Linear Quadratic Control, robot control.

**Abstract:** This paper presents the application of several advanced control techniques to a nonlinear strongly coupled multivariable robot. The main difficulties come from the flexibility of the mechanical chain, but also from the lack of joints sensors. In a first stage, a state-feedback linear quadratic (LQG) technique and a model predictive control (MPC) are designed using optimal observers. Considering additional sensors that provide measurements of accelerations increases the robustness of the controlled system. The second stage consists into adding a supplementary robustness layer (i.e. explicitly considering the robust stability under unstructured uncertainties) on the stabilizing MPC developed at the previous stage. Comparative results are proposed highlighting the trade-off between robust stability and nominal performance for disturbances rejection.

## 1 INTRODUCTION

Robots are nonlinear, often multivariable systems, with a strong interaction between their components. Modelling procedures (Book, 1989, Spong et al., 2005, Sciavicco and Siciliano, 1996) for robots can be difficult, leading sometimes to sophisticated models, which cannot be used for control. In addition, the models have to offer an accurate image of the real robots, while preserving the simplicity. Neglected or poorly known dynamics can affect the behaviour of the controlled robots. Therefore a need for robust control techniques is identified. Different control laws have been developed: robust state-feedback controllers (Tomei, 1994), output-feedback controllers (Moreno-Valenzuela et al., 2008), robust nonlinear control for robots with parametric uncertainties (Spong, 1992), LPV (linear parameter variant) control (Halalchi et al., 2010). Predictive control has also been applied on robots (Merabet and Gu, 2009, Maalouf, 2006, Hedjar et al., 2002).

This paper proposes an application of robustified control techniques to a medical robot (Al Assad et al., 2008), which is a nonlinear multivariable strongly-coupled system. In fact, this paper is an extension of the work proposed by (Stoica et al., 2009) in which a monovariable model of the pivot

(only one axis model) of the same robot is studied. In this paper, we consider two stabilizing initial control laws (linear quadratic control and predictive control) for a two axes model of the robot. In order to explicitly guarantee robust stability under unstructured uncertainties, an offline robustification procedure of the initial stabilizing MPC (Model Predictive Control) law is proposed. This robustification method is based on the optimization of a Youla-Kučera parameter also known as the  $Q$  parameter. Addressing the robust stability under unstructured uncertainties leads to a convex optimization problem, solved with LMI (Linear Matrix Inequality) tools. The advantage of this robustification method is that it unifies the qualities of both robust control and predictive control, while keeping a simple implementation: a feedback-gain coupled with an observer gain and a Youla parameter. The main novelty of this paper is the application of the proposed robustified controllers on the multivariable two axes model of the robot.

The proposed approach is an alternative to the current implemented structure based on LQ (linear quadratic) regulators for each axis (Al Assad et al., 2007).

This paper is organized as follows. Section 2 describes the medical robot, offering a Lagrange

model for the Pivot and C-arc system. Section 3 deals with control strategies applied on the robot, while Section 4 offers some details about the technique used to robustify the MPC controller. Section 5 focuses on an analysis of the results obtained with the proposed control laws. Finally, some concluding remarks and perspectives are given in Section 6.

## 2 DESCRIPTION OF THE ROBOT

The system considered in this paper is a vascular robot (Al Assad et al., 2008) developed by General Electric Healthcare and used for medical X-ray imaging. It is a four-degree of freedom open-chain robot composed of the following links: the L-arm (revolute joint), the pivot (revolute joint), the C-arc (which can be considered as a revolute joint around a virtual axis crossing the C-arc centre) and the lift (prismatic joint). Each joint is driven using a DC motor.

The robot is a nonlinear system (especially due to the irreversibility of worm gears) and a strongly coupled multivariable system (due to the interconnection of its joints). The model takes into account the hard nonlinearities of the system such as joints friction and gear's irreversibility. The flexibility of each axis is modelled as a two-mass spring system representing one vibrating mode (Fig. 1). A detailed model of the entire robot can be found in (Al Assad et al., 2008).

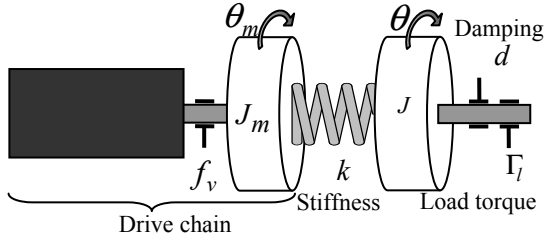


Figure 1: Two-mass spring system.

This paper considers the flexible model of only two axes: the pivot and the C-arc. The other two axes (the L-arm and the lift) were considered to be fixed. The dynamics of this model is given by the following Lagrange equations:

$$\begin{cases} \mathbf{J}_m \ddot{q}_m + \mathbf{F}_v \dot{q}_m + \mathbf{K}(q_m - q) = \Gamma_m \\ \mathbf{A}(q) \ddot{q} + \mathbf{D} \dot{q} + \mathbf{C}(q, \dot{q}) + \mathbf{Q}(q) = \mathbf{K}(q_m - q) \end{cases} \quad (1)$$

where  $q = [\theta_2 \ \theta_3]^T$  and  $q_m = [\theta_{m2} \ \theta_{m3}]^T$  are respectively the vectors of joints angular position

and motors shaft angular position of the pivot and the C-arc. More exactly, the index '2' is further used for the pivot elements, '3' denotes the C-arc elements and the index 'm' is used for each motor.

$$\mathbf{J}_m = \text{diag}(J_{m2}, J_{m3}), \mathbf{F}_v = \text{diag}(f_{v2}, f_{v3}),$$

$\mathbf{K} = \text{diag}(k_2, k_3)$  and  $\mathbf{D} = \text{diag}(d_2, d_3)$  are diagonal matrices belonging to  $\mathfrak{R}^{2 \times 2}$  which contain the parameters of each axis: inertia ( $\mathbf{J}_m$ ), viscous friction ( $\mathbf{F}_v$ ), joint stiffness ( $k$ ) and respectively joint damping ( $d$ ). The matrix  $\mathbf{A}(q) \in \mathfrak{R}^{2 \times 2}$  is the robot inertia matrix, the vector  $\mathbf{C}(q, \dot{q}) \in \mathfrak{R}^{2 \times 1}$  represents the Coriolis and centrifugal torque/forces,  $\mathbf{Q}(q) \in \mathfrak{R}^{2 \times 1}$  represents the gravitational forces vector and  $\Gamma_m = [\Gamma_{m2} \ \Gamma_{m3}]^T \in \mathfrak{R}^{2 \times 1}$  is the input torques vector (in fact the vector of control signals). The matrices from (1) can be detailed as:

$$\mathbf{A}(q) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \mathbf{C}(q, \dot{q}) = \begin{bmatrix} 0 & c_{12} & c_{13} \\ c_{21} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_2^2 \\ \dot{\theta}_2 \dot{\theta}_3 \\ \dot{\theta}_3^2 \end{bmatrix},$$

$$\mathbf{Q}(q) = \begin{bmatrix} -q_{11} - q_{12} \\ -g(MX_3 \sin \theta_3 + MY_3 \cos \theta_3) \cos \theta_2 \end{bmatrix},$$

with the following notations:

$$\begin{aligned} a_{11} &= ZZ_2 + XX_3 \cos^2 \theta_3 - 2XY_3 \cos \theta_3 \sin \theta_3 \\ a_{12} &= XZ_3 \cos \theta_3 - YZ_2 \sin \theta_3 \\ a_{21} &= XZ_3 \cos \theta_3 - YZ_3 \sin \theta_3 \\ a_{22} &= ZZ_3 \\ c_{12} &= -2XX_3 \cos \theta_3 \sin \theta_3 + 2XY_3 \cos^2(2\theta_3) \\ c_{13} &= -XZ_3 \sin \theta_3 - YZ_3 \cos \theta_3 \\ c_{21} &= XX_3 \cos \theta_3 \sin \theta_3 + XY_3 \cos^2(2\theta_3) \\ q_{11} &= g(MX_2 \sin \theta_2 + MY_2 \cos \theta_2) \\ q_{12} &= g(MX_3 \sin \theta_3 + MY_3 \cos \theta_3) \sin \theta_2 \end{aligned}$$

The notations  $YZ_2, ZZ_2, XX_3, XY_3, XZ_3, YZ_3, ZZ_3, MX_2, MY_2, MX_3, MY_3$  refer to the inertia moments of the pivot or the C-arc, expressed in the corresponding coordinate.

Equation (1) can be rewritten as a nonlinear state-space representation:

$$\dot{x} = \begin{bmatrix} \mathbf{A}(q)^{-1}(\mathbf{K}(q_m - q) - \mathbf{D} \dot{q} - \mathbf{C}(q, \dot{q}) - \mathbf{Q}(q)) \\ \mathbf{J}_m^{-1}(\Gamma_m - \mathbf{F}_v \dot{q}_m - \mathbf{K}(q_m - q)) \\ \dot{q} \\ \dot{q}_m \end{bmatrix} \quad (2)$$

where the state of the model is defined as  $x = [\dot{q}^T \ \dot{q}_m^T \ q^T \ q_m^T]^T$ .

### 3 CONTROL STRATEGIES

This section presents the theoretical background of the LQG and MPC control techniques that will be applied on the nonlinear system (1).

For control design purposes the nonlinear model (2) is linearized around an operating point  $x_0$  leading to the following continuous time LTI (linear time invariant) state-space representation:

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = C_y x(t) \\ z(t) = C_z x(t) \end{cases} \quad (3)$$

where  $A_c \in \mathfrak{R}^{n \times n}$ ,  $B_c \in \mathfrak{R}^{n \times m}$ ,  $C_y \in \mathfrak{R}^{p_y \times n}$ ,

$C_z \in \mathfrak{R}^{p_z \times n}$ .  $u(t) = [\Gamma_{m2} \ \Gamma_{m3}]^T$  represents the vector of the control signals,  $y(t) = q_m$  represents the vector of the controlled signals and  $z(t)$  is the vector of the measured signals. Usually, available sensors in robot arms can provide only the velocity and the position of the motor shaft. In order to increase the robustness of the control law, additional sensors will be considered to measure the joints accelerations. This leads to  $z(t) = [q_m^T \ \ddot{q}^T]^T$  and

$C_z = \begin{bmatrix} C_y \\ C_{\ddot{q}} \end{bmatrix}$ , where  $C_{\ddot{q}}$  contains only the first two

lines of the  $A_c$  matrix.

#### 3.1 State Feedback LQG Control

A state feedback control scheme is considered as a first approach. Consider the LTI system (3). If the pair  $(A_c, B_c)$  is stabilizable and  $(A_c, C_z)$  is detectable then the control law that optimizes the cost function (4) is given by  $u(t) = -L_1 x(t)$ .

$$J_1 = \int_0^{\infty} (x^T(t) Q_{J_1} x(t) + u^T(t) R_{J_1} u(t)) dt \quad (4)$$

In order to cancel the static errors an integral action on the motor shaft position error  $\int (q_m - q_{sp}) dt = \int \varepsilon dt$  is added (with  $q_{sp}$  the set-

point of the motors shaft angular position), leading to the following augmented state vector

$x_I = \left[ \dot{q}^T \ \dot{q}_m^T \ q^T \ q_m^T \ \left( \int \varepsilon dt \right)^T \right]^T$ . The new matrices

of the augmented state representation are:

$$A_{cl} = \begin{bmatrix} A_c & 0_{n, p_y} \\ C_y & 0_{p_y, p_y} \end{bmatrix}, B_{cl} = \begin{bmatrix} B_c \\ 0_{p_y, m} \end{bmatrix} \quad (5)$$

$$C_{cl} = \begin{bmatrix} C_z & 0_{p_z, p_y} \end{bmatrix}$$

Finally, since the sensors do not provide all the states, an observer (6) is incorporated into the control procedure:

$$\dot{\hat{x}}(t) = A_c \hat{x}(t) + B_c u(t) + K_1 (z(t) - C_z \hat{x}(t)) \quad (6)$$

In order to minimise the influence of the observer on the control law robustness (Doyle and Stein, 1979), an asymptotic Kalman filter is proposed as observer:

$$K_1 = \Sigma_1 C_z^T W_1^{-1} \quad (7)$$

where  $\Sigma_1$  is the unique positive definite solution of the Riccati equation:

$$A_c \Sigma_1 + \Sigma_1 A_c^T + V_1 - \Sigma_1 C_z^T W_1^{-1} C_z \Sigma_1 = 0 \quad (8)$$

$Q_{J_1}, R_{J_1}, V_1$  and  $W_1$  are symmetric positive definite weighting matrices that will be used as tuning parameters.

#### 3.2 Model Predictive Control (MPC)

The second control technique applied on the two axis model (1) is an unconstrained MPC. The motivation of this choice consists in the MPC's capacity of handling multivariable systems (Camacho and Bordons, 2004, Maciejowski, 2001). The model used by MPC is obtained from (3) after discretizing with a sample time  $T_s$ :

$$\begin{cases} x(k+1) = A_d x(k) + B_d u(k) \\ y(k) = C_y x(k) \\ z(k) = C_z x(k) \end{cases} \quad (9)$$

with  $A_d \in \mathfrak{R}^{n \times n}$  and  $B_d \in \mathfrak{R}^{n \times m}$ . For the steady state errors cancellation, an integral action on the control signal  $u(k) = u(k-1) + \Delta u(k)$  is added leading to the extended state-space representation described by:

$$A_{dl} = \begin{bmatrix} A_d & B_d \\ 0_{m,n_y} & I_m \end{bmatrix}, B_{dl} = \begin{bmatrix} B_d \\ I_m \end{bmatrix}, \quad (10)$$

$$C_{dl} = \begin{bmatrix} C_z & 0_{p_z,m} \end{bmatrix}$$

with  $x_I(k) = [x^T(k) \ u^T(k-1)]^T$ .

In order to design the MPC gain the following criterion is minimised:

$$J_2 = \sum_{i=N_1}^{N_2} \|\hat{x}(k+i) - x_{sp}(k+i)\|_{Q_{J_2}}^2 + \sum_{i=1}^{N_u-1} \|\Delta u(k+i)\|_{R_{J_2}}^2 \quad (11)$$

using as tuning parameters: the prediction horizons  $N_1, N_2$ , the control horizon  $N_u$  and the weightings  $Q_{J_2}, R_{J_2}$ . Here  $x_{sp}$  represents the set-point of the state vector. It is considered that  $\Delta u(k+i) = 0, \forall i \geq N_u$ . The states are calculated using the prediction model (9) as follows:

$$\hat{x}_I(k+i) = A_{dl}^i \hat{x}_I(k) + \sum_{j=0}^{i-1} A_{dl}^{i-j-1} B_{dl} u(k+j) \quad (12)$$

with the state estimation  $\hat{x}_I(k)$  obtained from the optimal Kalman filter:

$$\hat{x}_I(k+1) = A_{dl} \hat{x}_I(k) + B_{dl} \Delta u(k) + K_2(z(k) - C_{dl} \hat{x}_I(k)) \quad (13)$$

where  $K_2 = A_{dl} \Sigma_2 C_{dl}^T (C_{dl} \Sigma_2 C_{dl}^T + W_2)^{-1}$  and  $\Sigma_2$  is the unique positive definite solution of the algebraic Riccati equation:

$$A_{dl} \Sigma_2 A_{dl}^T + V_2 - A_{dl} \Sigma_2 C_{dl}^T (C_{dl} \Sigma_2 C_{dl}^T + W_2)^{-1} C_{dl} \Sigma_2 A_{dl}^T = \Sigma_2 \quad (14)$$

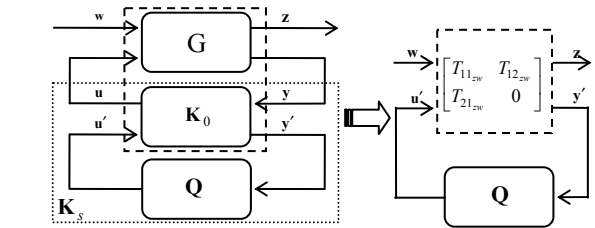
$V_2$  and  $W_2$  are symmetric positive definite weighting matrices used as tuning parameters for the observer. Next, applying the receding horizon principle, which is specific to predictive control, the following control law is obtained:

$$\Delta u(k) = F_r x_{sp}(k) - L_2 \hat{x}_I(k) \quad (15)$$

with the set-point pre-filter  $F_r$  and the MPC gain  $L_2$  (Fig. 3 and Fig. 4).

## 4 ROBUSTIFIED MPC

This subsection focuses on the procedure used to enhance robustness to the Model Predictive Control law developed in the Section 3.2. The basic idea is to add a stable Youla-Kučera parameter (Kučera, 1974, Youla et al., 1976) to parameterize the class of all stabilizing controllers starting from an initial stabilizing state-feedback controller coupled with an observer. This approach is known in the literature as the modified controller paradigm (Boyd and Barratt, 1991, Maciejowski, 1989) and consists into modifying the initial stabilizing controller by adding an auxiliary input  $u'$  and an output  $y'$  with a zero transfer in between (Fig. 2). This procedure enables to find a controller belonging to the class of all stabilizing controllers that will improve the robustness of the initial control law, without changing the initial Input/Output behaviour (i.e. the transfer from  $w$  to  $z$ ) of the initial closed-loop in the absence of disturbances.



(13) Figure 2: Class of all stabilizing controllers.

The transfer  $T_{zw}$  can be represented using the LLFT (Lower Linear Fractional Transformation) form of the initial controlled system coupled with the Youla-Kučera parameter, denoted Q parameter, with  $T_{22_{zw}} = 0$ :

$$T_{zw} = T_{11_{zw}} + T_{12_{zw}} Q T_{21_{zw}} \quad (16)$$

Note that this structure is affine in the Q parameter, allowing convex specifications in closed-loop.

The next step is to apply this strategy to the MPC law proposed in Section 3.2. Different scenarios can be considered depending on the choice

of the transfer  $T_{zw}$ . For instance, if the aim is to improve stability robustness under additive unstructured uncertainties, then the following choice has to be done  $T_{zw} = T_{z_u, b}$  (Fig. 3). For robust stability under multiplicative unstructured uncertainties, the following transfer has to be considered  $T_{zw} = T_{z_y, b}$  (Fig. 4), which is equivalent to the complementary sensitivity function.

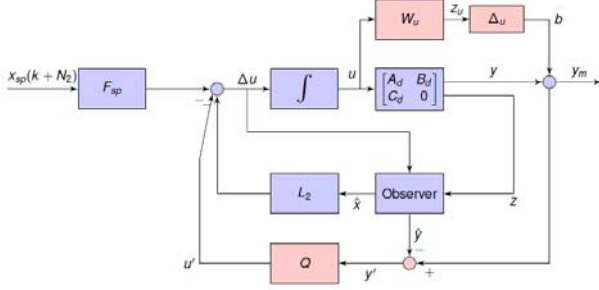


Figure 3: Robustification under additive unstructured uncertainties.

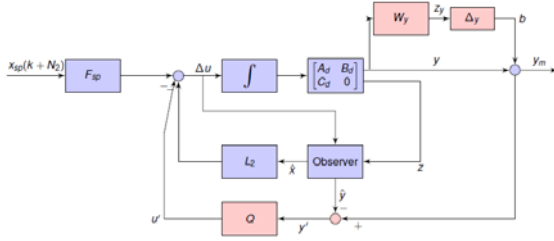


Figure 4: Robustification under multiplicative unstructured uncertainties.

In order to improve the robustness of the initial control the following optimization problem has to be solved:

- a. Find  $Q \in \mathfrak{RH}_\infty$  that improves the robust stability under additive unstructured uncertainties solving:

$$\min_{Q \in \mathfrak{RH}_\infty} \|T_{z_b}\|_\infty = \min_{Q \in \mathfrak{RH}_\infty} \|W_u T_{ub}\|_\infty \quad (17)$$

- b. Find  $Q \in \mathfrak{RH}_\infty$  that improves the robust stability to multiplicative unstructured uncertainties solving:

$$\min_{Q \in \mathfrak{RH}_\infty} \|T_{z_y, b}\|_\infty = \min_{Q \in \mathfrak{RH}_\infty} \|W_y T_{yb}\|_\infty \quad (18)$$

Here  $W_u$  and  $W_y$  denote appropriate weighting terms chosen in order to accomplish the desired

robustness specifications in a specified frequency range.

As the robustification procedure is identical for both additive and multiplicative uncertainties, the notation  $T_{zw}$  will be further used for the general case.

Since the Q parameter initially varies in the infinite-dimensional space of stable transfers ( $\mathfrak{RH}_\infty$ ), it is suitable to restrict the search. A sub-optimal solution (Scherer 2000) is to consider a FIR (Finite Impulse Response) filter. The state-space form  $(A_Q, B_Q, C_Q, D_Q)$  of this multivariable Q parameter will be further used, with a known (a priori fixed) pair  $(A_Q, B_Q) \in \mathfrak{R}^{p_y n_Q \times p_y n_Q} \times \mathfrak{R}^{p_y n_Q \times p_y}$

and an unknown pair  $(C_Q, D_Q) \in \mathfrak{R}^{m \times p_y n_Q} \times \mathfrak{R}^{m \times p_y}$  that will result from the optimization procedure (see (Stoica et al., 2007) for more details). Here  $n_Q$  denotes the degree of the Q polynomial. The optimization problems (17) and (18) can be reformulated as a matrix inequality using the following theorem.

**Bounded real lemma** (Boyd et al., 1994, Scherer, 2000, Clement and Duc, 2000). A linear discrete time invariant system with the state-space representation  $(A_{cl}, B_{cl}, C_{cl}, D_{cl})$  is stable and has a  $H_\infty$  norm lower than  $\gamma$  if and only if:

$$\exists X_1 = X_1^T > 0 / \begin{bmatrix} -X_1^{-1} & A_{cl} & B_{cl} & 0 \\ A_{cl}^T & -X_1 & 0 & C_{cl}^T \\ B_{cl}^T & 0 & -\gamma I & D_{cl}^T \\ 0 & C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (19)$$

with “ $> 0$ ” (“ $< 0$ ”) denoting a strictly positive (negative) definite matrix.

Using a change of variables and two congruence transformations (Scherer, 2000, Clement and Duc, 2000), the expression (19) can be further transformed into a LMI (Linear Matrix Inequality) with the decision variables:  $X_1$ ,  $\gamma$  and the Q parameter hidden in the closed-loop matrices. An exact form of this LMI and also the entire procedure (which is outside the aim of this paper) leading to this LMI can be found in (Stoica et al., 2007).

Hence, depending on the considered transfer minimisation, the resulting optimization problem is the following:

- a. only robust stability under additive unstructured uncertainties: minimize  $\gamma$  subject to the Linear Matrix Inequality  $LMI_0$  using the state-

space representation  $(A_{cl}, B_{cl}, C_{cl}, D_{cl})$  of the transfer  $T_{zw} = T_{z_u, b}$ :

$$\min_{LMI_0} \gamma \quad (20)$$

b. only robust stability under multiplicative unstructured uncertainties: minimize  $\gamma$  subject to the Linear Matrix Inequality  $LMI_{CS}$  using the state-space representation  $(A_{cl}, B_{cl}, C_{cl}, D_{cl})$  of the transfer  $T_{zw} = T_{z_y, b}$ :

$$\min_{LMI_{CS}} \gamma \quad (21)$$

c. robust stability under both additive and multiplicative unstructured uncertainties: minimize a given cost function subject to the two LMIs defined before:

$$\min_{LMI_0, LMI_{CS}} (c_0 \gamma_0 + c_{CS} \gamma_{CS}) \quad (22)$$

where  $c_0, c_{CS} \in \Re$  are weighting terms and  $\gamma = \gamma_0$  in  $LMI_0$  and  $\gamma = \gamma_{CS}$  in  $LMI_{CS}$ .

Note that this robustification procedure can be applied to every state-feedback controller coupled with an observer. The particular case of MPC is used here due to its good performance and simplicity of implementation when dealing with multivariable systems.

## 5 SIMULATION RESULTS

The control strategies proposed in this paper (LQG control, MPC and robustified MPC) are now applied to the nonlinear model (1). The LQG and MPC control laws are designed to achieve the same performances and to respect the admissible motors torques.

The LQG controller is designed in continuous-time in accordance to an existing LQ controller (Stoica et al., 2009) which is already implemented on the real robot.

The linearized continuous-time model (3) used to design the LQG control law was obtained via the Matlab<sup>®</sup> function 'linmod'. A sample time  $T_s = 0.005$  s and a zero-order hold on the inputs discretization method were used to determine the prediction model (9) used by MPC.

### 5.1 Tuning parameters

First of all, based only on the information of the angular position sensors from the Pivot and C-arc a LQG controller is designed (denoted LQG<sub>p</sub>), using the weighting matrices  $R_{J_1} = \text{diag}(10^{-7}, 10^{-9})$  and  $Q_{J_1} = \text{diag}(1, 1, 1, 1, 20, 80, 50, 80, 5000, 4000)$ . The observer weightings are chosen as  $W_1 = \text{diag}(10^{-6}, 10^{-6}, 0.1, 0.1)$  and  $V_1 = \alpha B_c B_c^T$ , with  $\alpha = 10^5$ .

Another LQG controller (denoted LQG<sub>a</sub>) which uses additional information from the accelerometers is next developed. This increases the initial robustness of the controller. The poles of the closed-loop obtained with LQG<sub>a</sub> are presented in Fig. 5.

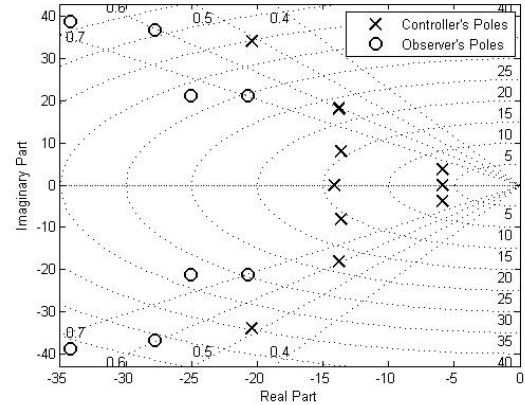


Figure 5: Closed-loop LQG<sub>a</sub> poles

Secondly, the initial MPC (denoted MPC0) is designed with the following tuning parameters:  $N_1 = 8, N_2 = 58, N_u = 14, R_{J_2} = \text{diag}(R_{J_0}, \dots, R_{J_0}), R_{J_0} = \text{diag}(10^{-7}, 10^{-8}), Q_{J_2} = \text{diag}(Q_{J_0}, \dots, Q_{J_0}), Q_{J_0} = \text{diag}(1, 1, 1, 1, 80, 30, 30, 30)$ . The MPC controller uses the information of both angular positions and accelerations of the Pivot and C-arc.

Thirdly, two robustified controllers are further developed:

- RMPC0 that considers only robust stability under additive unstructured uncertainties, obtained from MPC0 with the Q parameter which is the solution of (20);
- RMPC1 that takes into account robust stability under both additive and multiplicative unstructured uncertainties. This robustified controller is obtained from MPC0 coupled with the Q parameter from (22). The coefficients

$c_0 = 1$  and  $c_{CS} = 50$  are used in the optimization problem (22).

In both cases the degree of the Q polynomial is chosen equal to  $n_Q = 10$ . The weighting  $W_u$  (Fig. 3) on the control increment is chosen as a high-pass filter  $W_u = (1 - 0.95z^{-1})/0.05$  and the weighting  $W_y$  (Fig. 4) is chosen as  $W_y = (1 - 0.9z^{-1})/0.1$ , in order to favor the high frequency range. The total number of scalar decision variables associated with the LMI problem (20) is 948 and with the LMI optimisation problem (22) is 1387.

## 5.2 Frequency analysis

In the case of a multivariable system, the classical criteria for the analysis of stability margins such as the Nyquist criterion are no longer valid. This is the main reason why an analysis of the maximal singular values, which can give a meaningful assessment of the robustness of the controlled system, is further proposed.

In a first stage, the maximal singular values of the transfer from  $b$  to  $u$  obtained with the LQG controller that uses only the motor shaft positions (LQG<sub>p</sub>) and for the LQG with additional measurements of the joints accelerations (LQG<sub>a</sub>) are illustrated in Fig. 6. A significant improvement of the controlled system behavior with the LQG<sub>a</sub> controller can be noticed at the high frequency range. Thus the LQG<sub>a</sub> controller is kept for further comparisons with MPC0, RMPC0 and RMPC1. For simplicity reason the LQG<sub>a</sub> controller is further denoted LQG.

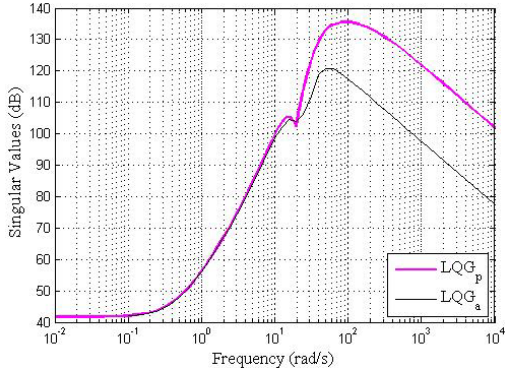


Figure 6: Maximal singular values for the transfer from  $b$  to  $u$ .

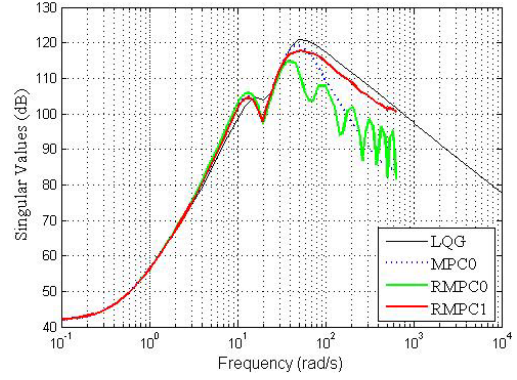


Figure 7: Maximal singular values for the transfer from  $b$  to  $u$ .

In a second stage, a comparative analysis of the maximal singular values from  $b$  to  $u$  is offered in Fig. 7. The  $H_\infty$  norm of transfer  $T_{ub}$  (which is the maximum of the maximal singular values) is progressively decreased from LQG to MPC0 and then to RMPC0. The MPC and the robustified MPC controllers have better frequency responses in the high frequency range than the LQG controller. The robustified controller RMPC1 offers a good trade-off in terms of  $H_\infty$  norm between MPC0 and RMPC0.

From the analysis of the maximal singular values of the complementary sensitivity function depicted in Fig. 8, the LQG controller has the largest bandwidth leading to a better behavior in the time domain. The  $H_\infty$  norm of the transfer  $T_{yb}$  is very similar for all the considered controllers.

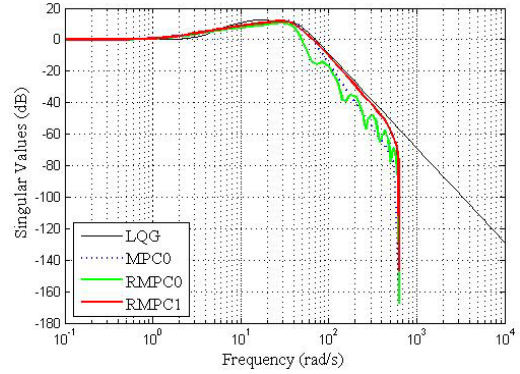


Figure 8: Maximal singular values of the complementary sensitivity function.

## 5.3 Time domain comparison

The time domain responses are obtained using a step set-point of magnitude  $\theta_{2,sp} = 0.1$  rad for the pivot



and  $\theta_{3_{sp}} = -0.15$  rad for the C-arc. A step disturbance of magnitude 200Nm at time 2s on both axes was also considered.

Figure 9 presents the outputs  $q_m$  of the nonlinear model of the 2 axes of the robot. All the controllers offer good tracking, a time response without overshoot (which is imposed by the demand specifications) and an admissible disturbance rejection. The time responses are very similar for all the controllers ( $t_{rPivot} = 0.72s$  and  $t_{rCarc} = 0.62s$ ). The LQG controller offers the fastest disturbances rejection. The disturbances rejection is slower after the frequency domain behavior. In fact the Youla-Kučera parameter will improve the robust stability under additive uncertainties and will slow down the disturbances rejection. The controller RMPC1 offers a good trade-off between the considered controllers (see the corresponding zoom of Fig. 9).

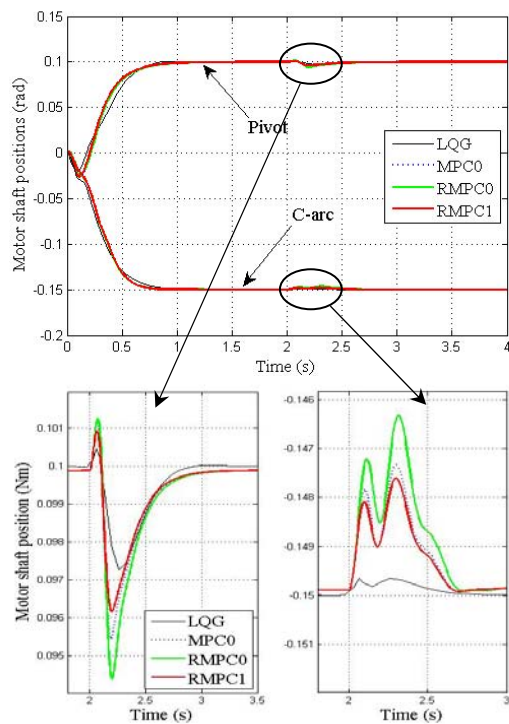


Figure 9: Output  $q_m$  – Motor shaft positions comparison.

Figure 10 shows the control signals applied on the nonlinear model. All the controllers offer admissible control signals that can be implemented on the real robot. A small oscillation (which could come from numerical problems) can be seen on the LQG controller.

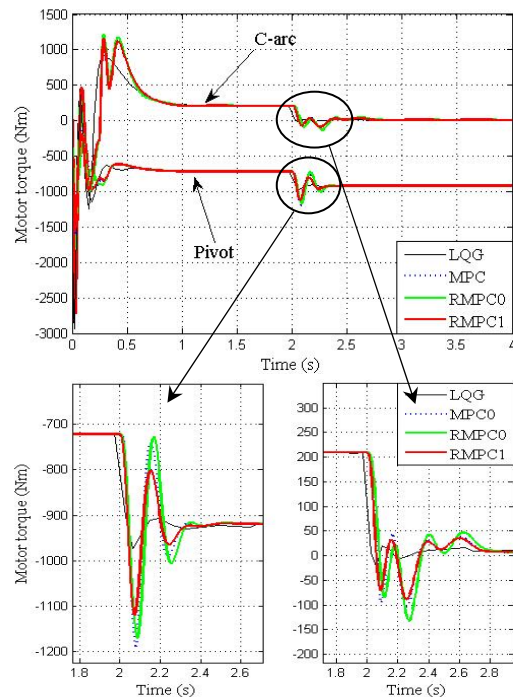


Figure 10: Control signals – Motor torques comparison.

The robust synthesis algorithms, usually offer large controllers. In this case the Youla-Kučera parameter increases the dimension of the RMPC1 controller with 20 states. In order to reduce the controller states a balanced reduction of the Youla-Kučera parameter based on the Hankel singular values is considered. The final controller (denoted RMPC1r) uses a reduced Youla-Kučera parameter that has only 4 states. Figure 11 illustrates the singular values of the Youla-Kučera parameter before and after the order reduction.

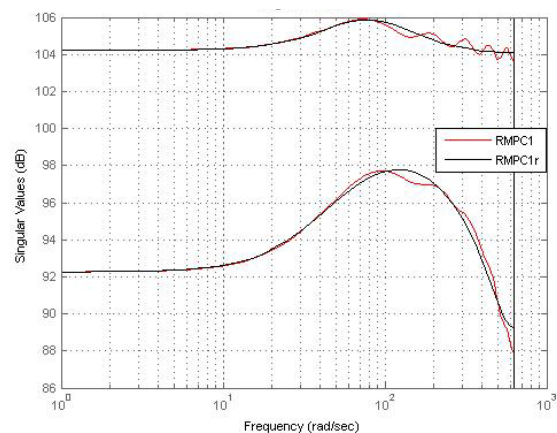


Figure 11 Singular values of Youla parameter before and after order reduction.

The influence of an unstable transmission zero (determining the behavior at the beginning of the simulation) over the pivot axis can be easily noticed in Fig. 9 and Fig. 10. The existence of this unstable zero explains the choice of the prediction horizons  $N_1 = 8 > 1, N_u = 14 > 1$ .

A robust analysis of the results is also proposed. The aim is to verify the stability of the controllers with the nonlinear model, considering structured uncertainties on the joint stiffness  $\mathbf{K}$  and motor inertia  $\mathbf{J}_m$ . Only RMPC1 and RMPC1r remain stable for all the considered parameters variations as synthesised in Table 1, where the following notations have been used:

- Case 1:  $\mathbf{K} - 20\%\mathbf{K}$  ;
- Case 2:  $\mathbf{K} + 20\%\mathbf{K}$  ;
- Case 3:  $\mathbf{J}_m - 20\%\mathbf{J}_m$  ;
- Case 4:  $\mathbf{J}_m + 20\%\mathbf{J}_m$  .

Table 1: Structured uncertainties robustness.

	LQG	MPC	RMPC0	RMPC1	RMPC1r
Case 1	✗	✓	✗	✓	✓
Case 2	✓	✓	✓	✓	✓
Case 3	✓	✗	✗	✓	✓
Case 4	✓	✓	✓	✓	✓

Figure 12 illustrates the case where an uncertainty of  $-20\%$  is considered on the motors inertia:  $\mathbf{J}_m - 20\%\mathbf{J}_m$ . Despite this uncertainty and the nonlinearities of the system, the robustified controller RMPC1 still stabilises the system. Moreover, it can be observed that this property is conserved even after the order reduction.

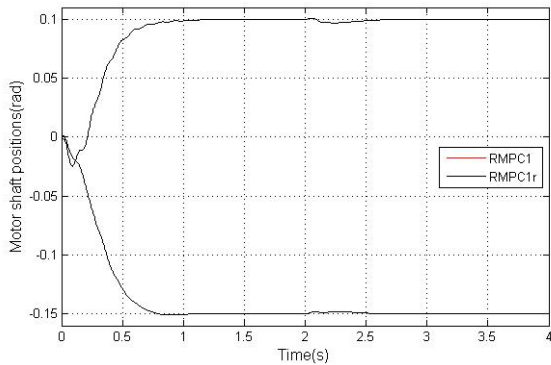


Figure 12: Output  $q_m$ . Nonlinear model with uncertainties of  $\mathbf{J}_m - 20\%\mathbf{J}_m$ .

## 6 CONCLUSIONS

This paper proposes a comparison between advanced control techniques for the control of the angular position of a two axes model of a cardiovascular robot, which is a strongly nonlinear multivariable system. In order to improve the controllers' robustness, several layers of robustification are further considered.

A linear quadratic controller (LQG) and a Model Predictiv Control (MPC) law are first designed to achieve similar level of performance for the time-domain response. In a first step, additional measurements of the joints accelerations are used in order to increase the initial level of robustness of the two controllers. Robust stability under unstructured uncertainties is explicitly considered in the synthesis of the robustified MPC controllers, while, for the LQG controller, the robust stability under unstructured uncertainties is verified a posteriori. Simulation results show a trade-off between robust stability and disturbances rejection.

The robustness towards the variation of some parameters (i.e. structured uncertainties) is verified a posteriori for all the considered controllers. An interesting perspective is to take into account these structured uncertainties during the synthesis of the robustified MPC. A possibility is to consider a polytopic uncertain domain around the nominal model as in (Stoica et al., 2009) and to guarantee the stability over the specified uncertain polytopic domain solving a BMI (Bilinear Matrix Inequality) optimisation problem.

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