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Dynamic Neural Field Optimization using the Unscented Kalman Filter

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Abstract—Dynamic neural fields have been proposed as a continuous model of a neural tissue. When dynamic neural fields are used in practical applications, the tuning of their parameters is a challenging issue that most of the time relies on expert knowledge on the influence of each parameter. The methods that have been proposed so far for automatically tuning these parameters rely either on genetic algorithms or on gradient descent. The second category of methods requires to explicitly compute the gradient of a cost function which is not always possible or at least difficult and costly. Here we propose to use unscented Kalman filters, a derivative-free algorithm for parameter estimation, which reveals to efficiently optimize the parameters of a dynamic neural field.

I. INTRODUCTION

Dynamic neural fields have been proposed as a continuous model of a neural tissue. They are used in computational neuroscience to model biological functions of the primate brain but also as a computational paradigm to control artificial agents. In both contexts, tuning the parameters of a neural field is a challenging issue that most of the time relies on the knowledge of an expert and on trials and errors. The main difficulty behind tuning these parameters relies on the difficulty to obtain analytical results on the evolution of such a high-dimensional non-linear dynamical system. The first property observed on dynamic neural fields is the existence of stable localized patterns of activity [1]. Dynamical patterns such as traveling waves have also been obtained. These two properties are understood analytically in a restricted context, for a specific transfer function. Other dynamical properties are more tricky to handle analytically, in particular competition and working memory. These two behaviors are important sub-functions of decision making, for instance in cognitive robotics [2]–[6]. The former endows an artificial agent with the ability to settle a competition between, for example, alternative motor programs that cannot be executed simultaneously. It may also allow targets of one motor action to compete. The latter allows an agent keeping a trace of a transiently presented information relevant for guiding future behaviors, or temporarily store the spatial position of a set of stimuli when several have to be investigated (see for example [7]).

Using dynamic neural fields to design cognitive architectures is a challenging problem, especially because the functions one wants to introduce in the system result from distributed computations within the fields. These interactions are themselves ruled by a set of parameters that have a non-linear influence on a neural field’s behavior. Such a hard problem motivates finding optimization algorithms to automatically tune neural field parameters.

So far, two optimization techniques have been applied for automatically tuning the parameters of a neural field: genetic algorithms [8] and genetic algorithms combined with a gradient descent [9]. In [8], the authors apply genetic algorithms to tune the parameters of a two-dimensional neural field performing selection and dynamic tracking of a moving input stimulus within noise or surrounded by distractors. In [9], the authors combine a genetic algorithm with a gradient-descent based method. The later requires to compute locally the gradient of a cost function which may be difficult or at least costly to perform.

Kalman filters are a popular collection of algorithms in the optimal control and machine learning community for estimating a hidden state from noisy observations, knowing a model of the underlying process and of the observation apparatus. There are several algorithms that have been developed since the seminal work of Kalman [10] on systems involving linear evolution and observation processes. For example the Extended Kalman Filter (EKF) and the Unscented Kalman filter (UKF) are efficient methods that can be used for three applications. They can be used for state estimation, i.e. estimating a hidden state observed through noisy measurements. They can also be used for parameter estimation, i.e. determining the parameters of a non-linear system given noisy observations. These two algorithms can finally be combined for performing so-called dual estimation, i.e. estimating both a hidden state and the parameters governing a non-linear system. The Extended Kalman Filter requires to explicitly compute the derivatives of the process’ equations. In some applications (like the one we address in this article), analytically computing these derivatives can be difficult or even impossible in some cases. The Unscented Kalman Filter does not need to explicitly compute these derivatives. Neural networks are a particular case of non-linear processes and the techniques mentioned above can be applied to tune their parameters. The Kalman Filters reveal to be quite efficient compared to the more
The popular backpropagation algorithm [11]. The derivative-free aspect of UKF combined with its efficiency in optimizing the parameters of neural networks motivate the study presented in this article, where we seek to optimize the parameters of dynamic neural fields.

The Unscented Kalman Filter [12] belongs to the family of Sigma-Point Kalman filter (SPKF) which can be used to estimate hidden variables producing observations through a non-linear function (therefore extending the original Kalman filtering framework [10] applicable on linear functions). The challenge is to estimate the parameters of this non-linear model only from partial and noisy observations. The Unscented Kalman Filter has already been applied on several machine learning problems. For example, in [13], UKF is used for supervised learning and in [14], the authors use UKF in the context of reinforcement learning to approximate the value function given a reward scheme which provides the indirect noisy observations.

The paper is organized as follow: the unscented Kalman filter algorithm is provided in section II-A, the dynamic neural field equation and parameters are described in section II-B and two example scenarios are studied in section III. Extensions of the presented framework and the implications of such a method in the context of dynamic neural fields applied to cognitive robotics are then discussed.

II. METHODS

A. Unscented Kalman Filter (UKF)

The Unscented Kalman filter (UKF) is an efficient derivative-free method for nonlinear filtering problems. In particular, it can be applied to estimate the parameters of a non-linear function. It relies on the unscented transform (UT) [12] which allows to compute a local linear approximation of the non-linear function. To do so, the unscented transform introduces a set of so-called sigma-points from which statistics of interest are computed. We briefly sketch below the different steps involved in the UKF for parameter estimation but a more detailed explanation of this algorithm can be found in [13].

Basically, the parameter estimation problem is stated as: given a non-linear function $f_\theta$ parametrized by the vector $\theta$, given a set of input-output samples $(x_i,y_i)$, find the parameter vector $\theta^*$ that best accounts for the mapping $\forall i, y_i = f_\theta(x_i) + v_i$ in a least square sense, where $v_i$ is a white observation noise (zero-mean and independent). Therefore, at step $i$, UKF seeks at minimizing the following cost function:

$$\theta_i = \arg\min_{\theta} \sum_{j=1}^{i} \frac{1}{P_{vij}} (y_j - f_\theta(x_j))^2$$  \hspace{1cm} (1)

where $P_{vij}$ is the variance of the noise $v_j$. This cost function is defined on the samples presented so far. The general purpose of UKF, like Kalman filtering, is to track the hidden state of some system given sequential noisy observations. Here, the state is the hidden parameter vector. The associated state-space formulation of the parameter estimation problem is given by:

$$\begin{cases} \theta_i = \theta_{i-1} + n_i \\ y_i = f_\theta(x_i) + v_i \end{cases}$$

Using the vocabulary of Kalman filtering, the first equation is the evolution equation: it specifies that the true parameter vector follows a random walk, the expectation of which corresponds to the optimal estimate of the parameters (in a least-square sense). The evolution noise $n_i$ is white (independent and centered) and of variance $P_{nn_i}$ (to be chosen by the practitioner). This allows handling non-stationary parameters (without harming the stationary case), but it also allows avoiding local minima of the cost-function (1) (its effects can roughly be understood as simulated annealing). The second equation is the observation equation: it links the parameters to the observations through the parametrized function $f_\theta$. The observation noise $v_i$ is also white and of variance $P_{vvi}$ (to be chosen by the practitioner, possibly on the basis of some physical considerations). UKF is an incremental algorithm that improves step by step the parameters. The idea is to obtain a Widrow-Hoff-like linear update rule of the form $\theta_i = \theta_{i-1} + K_i(y_i - \bar{y}_i)$ where $\bar{y}_i$ is a prediction of what should be the next observation $y_i$ given the current estimate of the parameters and $K_i$ is the so-called Kalman gain. The UKF algorithm aims at incrementally compute the best gain $K_i$, that is the one that minimizes the cost function (1).

The algorithm we present below\(^1\) and depicted schematically on figure 1 uses the Unscented Transform for clarity of presentation. However, it is the Scaled Unscented Transform [13] that is used in practice for efficiency reasons. Both rely on the same ideas, the Unscented Transform being shorter to introduce, and slight variations lead to the scaled transform which is provided in more details in [13]. In addition, the algorithm provided below is the vectorial algorithm while we used scalar notations previously for introducing it. The algorithm is initialized with random parameters $\theta_0$ and a prior on the variance of these parameters $P_0$ (in practice, a diagonal matrix). At step $i$, it first updates the covariance of the parameters :

$$P_{i|i-1} = P_{i-1} + P_{nn_i}$$

Then, a set of $2p + 1$ so-called sigma-points $\theta^j_i$, $j \in [0..2p]$ ($p$ is the number of parameters to estimate) is introduced :

$$\begin{align*} \theta^0_i &= \theta_{i-1} \\ \theta^j_i &= \theta_{i-1} + (\sqrt{(p+\kappa)P_{i|i-1}})_j, 1 \leq j \leq p \\ \theta^{j+p}_i &= \theta_{i-1} - (\sqrt{(p+\kappa)P_{i|i-1}})_{j-p}, p + 1 \leq j \leq 2p \end{align*}$$

\(^1\)In the description of the algorithm, a matrix is written with a capital bold letter (e.g. $P_i$), a vector with a bold letter (e.g. $x_i$) and a scalar with a normal letter (e.g. $x_i$).
where \( \sqrt{(p + \kappa)P_{i|i-1}} \) is the j-th column of the Cholesky decomposition of the matrix \((p + \kappa)P_{i|i-1}\), and \( \kappa \geq 0 \) a parameter of the algorithm which controls the spread of the sigma-points sampling. The image of each of the sigma-points through the non-linear function \( f_0 \) is then computed, each sigma-point defining a specific parametrization:

\[
y_i = f_{\theta_i}(x_i), \forall j
\]

A set of weights, used to compute different statistics of interest is then defined as:

\[
w_0 = \frac{\kappa}{p + \kappa} \quad \text{and} \quad w_j = \frac{1}{2(p + \kappa)}, \forall j > 0
\]

From the image of the sigma-points, the set of weights previously defined and the current sigma-points, one can compute the following statistics:

\[
y_i = \sum_j w_j y_i^j
\]

\[
P_{\theta y_i} = \sum_j w_j (\theta_i^j - \theta_{i-1})(y_i^j - y_i)\]

\[
P_{y y_i} = \sum_j w_j (y_i^j - y_i)(y_i^j - y_i)^T
\]

The last step is to update the Kalman gain \( K_i \), the parameters \( \theta_i \) as well as the covariance matrix of the parameters \( P_i \):

\[
K_i = P_{\theta y_i} \cdot (P_{\theta \nu i} + P_{y y_i})^{-1}
\]

\[
\theta_i = \theta_{i-1} + K_i(y_i - \bar{y}_i)
\]

\[
P_i = P_{i|i-1} - K_i(P_{\theta \nu i} + P_{y y_i})K_i^T
\]

**B. Dynamic neural fields**

Dynamic neural fields have been introduced as a model of a continuous neural tissue [1], [15], [16]. The neural field equation (3) states that the evolution of the membrane potential at position \( x \) and time \( t \), depends on an input at that position \( i(x, t) \), on a baseline \( h \) and on the lateral influence within the field characterized by a weight function \( w(x, y) \) and a transfer function \( f \).

\[
\tau \frac{\partial u}{\partial t}(x, t) = -u(x, t) + \int_y w(x, y)f(u(y, t))dy + i(x, t) + h
\]

To be properly defined, the previous equation also requires an initial condition \( u_0 = \{u(x, 0), \forall x\} \). In order to cast the optimization of the parameters of a neural field into a Kalman filtering problem, one needs to parametrize the neural field equation and specify how it is sampled. To parametrize the neural field, we first discretize equation (3) both in space and time using an Euler scheme. The following equations are therefore specific for the Euler scheme but could be easily adjusted to other schemes. The discrete neural field equation therefore reads:

\[
u(x, t + \Delta t) = (1 - \frac{\Delta t}{\tau})u(x, t) + \frac{\Delta t}{\tau} \sum_y w(x, y)f(u(y, t)) + \frac{\Delta t}{\tau}(i(x, t) + h)
\]
and a specific weight function. The transfer function is defined as a sigmoidal function parametrized by \((a, b, x_0)\):

\[
f(x) = \frac{a}{1 + \exp(b(x - x_0))}
\]

(5)

The weight function is defined as a difference of gaussians parametrized by \((A_+, \sigma_+, A_-, \sigma_-)\):

\[
w(x, y) = A_+ \exp \left( -\frac{(x - y)^2}{2\sigma_+^2} \right) + A_- \exp \left( -\frac{(x - y)^2}{2\sigma_-^2} \right)
\]

(6)

This symmetric weight function is suitable for the neural field scenarios we consider in the following. However, we may have easily considered other weight functions such as the asymmetrical one used in [17] in order to obtain traveling patterns of activity. Given the previous parametrization, our problem is to optimize 9 parameters: \(\theta = (\tau, h, a, b, x_0, A_+, \sigma_+, A_-, \sigma_-)\). Now, we need to specify the samples that are presented to the algorithm (inputs \(x_i\)) and the constraints that specify the optimization problem (outputs \(y_i\)).

The optimization problem is defined by providing both the input feeding the neural field and a rough estimate of the desired mean firing rate (called desired output in the following). We use the mean firing rate \(f(u(x, t))\) instead of the membrane potential \(u(x, t)\) since it is bounded and easier to specify a desired firing rate than a desired membrane potential. This constraint is just introduced for practical reasons but could be dropped if necessary. A given input and desired output define a neural field scenario. Two example scenarios are shown on figures 3a,b and 4a,b. These figures illustrate space/time representations of two scenarios: a competition scenario and a working memory scenario. Given these scenarios, a sample (the pairs \((x_i, y_i)\)) we introduced in section II-A) can be defined in several ways. We consider in this article two sampling methods, illustrated on figure 2. The first sampling method selects randomly a time and the whole neural field activity at that time is presented to the algorithm. The second sampling method selects randomly pairs of times and positions and presents the neural field activity at these times and positions. The performance of the algorithm with these two sampling methods will be compared in the Results section and others will be discussed in conclusion. The first sampling method is probably the most natural of the two for sampling the state of a dynamical system but the second makes the algorithm faster to converge. Given we used an Euler scheme for discretizing in time equation (3), we consider only temporal samples that are multiple of the time-step \(\Delta t\).

To state the optimization of neural fields’ parameters as a Kalman filtering problem, we must define the evolution and observation equations. The evolution equation states that the parameters follow a random walk, with a white independent noise \(n_t\). The observation equation links the parameters with the observations. To define this equation, we need to introduce an additional notation. Let’s denote \(u(., t)\) the activity of the whole neural field at a given time \(t\) and denote \(g_0\) the parametric function linking \(u(., t + \Delta t)\) with \(u(., t)\):

\[
u(., t + \Delta t) = g_0(u(., t))
\]

\[
g_0(u) = (1 - \frac{\Delta t}{\tau})u + \frac{\Delta t}{\tau} W \cdot f(u) + \frac{\Delta t}{\tau} (i + h)
\]

(7)

where \(W\) denotes the weight matrix, \(f(u)\) denotes the vector whose components correspond to the components of \(u\) on which \(f\) is applied. The state of the neural field at a certain time \(t\), multiple of \(\Delta t\), is defined as the repeated composition of \(g_0\) applied on the initial condition \(u_0\):

\[
u(., j\Delta t) = g_0(g_0(...g_0(u_0)...)) = g_0^j(u_0)
\]

We can now define the so-called state-space description of the system, for a certain input \(x_i = (k, j\Delta t), k \in [0, n]\):

\[
\begin{align*}
\theta_i &= \theta_{i-1} + n_i \\
y_i &= g_{b_i}(u_0)|_{(k + v_i)}
\end{align*}
\]

The previous state-space description is given for a pair of spatial/temporal position and should be slightly adjusted to correspond to the previous definition of a sample. We do not specify it further to avoid unreadable equations. One should note that the previous definition of the Kalman filtering problem is not dependent on the specific choice of the weight function and transfer function. Given that UKF is a derivative-free algorithm, we do not even need to use derivable functions. This is in contrast to gradient-descent-based methods such as in [9] which require to compute derivatives. The simulations of the neural field and the UKF algorithm were written in C++/GSL.

III. RESULTS

We applied the UKF algorithm on two classical scenarios of dynamic neural fields: competition and working memory. In the first scenario (fig. 3), several regions of a 1D neural field are locally excited and we want to find the parameters of the neural field settling a competition between these excited regions so that the field’s activities converge to the representation of only one of the excited regions. Figures 3a,b illustrate the input feeding the neural field as well as the desired firing rate. The input consists in two locally excited regions with relatively close magnitudes (1.0 and 0.75), mixed with a random noise. As an output, we want the neural field to produce a competition between these two regions leading to the selection of the stimulus with the highest amplitude. Intuitively, this competition relies on lateral inhibition where both regions are inhibiting each other. A proper range and amplitude for the lateral inhibition term has to be found in order to get this competition. Figure 3c illustrates the firing rate obtained after 233 iterations of the algorithm showing that
the algorithm has found a proper set of parameters allowing this behavior. To compare the two sampling methods and to get an idea of the time it takes for the algorithm to converge, we repeated 1000 times the optimization procedure. It took on average respectively 260 steps and 52 steps to converge (to reach a RMS error of 0.1) for the two sampling methods. The algorithm reached a local minima 15% of the trials for the sampling in time, and 3.5% of the trials for the sampling in time and space. The evolution of the mean RMS, through the iteration of the algorithm, is shown on figure 5a. In addition, some local minima reached by the algorithm with their respective RMS are shown. Sampling randomly in both time and space clearly leads to a faster convergence.

A more difficult scenario is proposed on figure 4. A working memory behavior: when the amplitude of the input reaches a threshold, the stimulus enters the working memory and stays there until the input is completely suppressed. Despite this decrease in amplitude, the stimulus should stay in the working memory. The input and desired output shown on figures 4a,b illustrate this behavior. There, the neural field is fed with two locally excited regions of initially weak amplitude. A transient increase of the amplitude of the input stimuli, reflecting some kind of selection or gating for the entrance in working memory, triggers the emergence of the stimulus in working memory. Removing the gating signal leads to decrease the amplitude of the selected stimulus back to its initial and weak value. Despite this decrease in amplitude, the stimulus should stay in the working memory. There are mainly three critical parameters allowing to get this

![Fig. 3. Competition scenario. (a) Space/time representation of the input exciting the neural field. The two localized excitations are gaussians with an amplitude of 1.0 and 0.75, a standard deviation of 4.0, mixed with a uniform random noise of a maximal amplitude of 0.1 (b) The desired membrane potential seeks to produce competition between the two excitation regions, the one with the highest amplitude being the winner. (c) Obtained neural field after 233 iterations of the algorithm and a RMS error of 0.05. The algorithm converged to the following parameters: \( a = 0.93, b = -4.99, x_0 = 0.59, A_k = 1.53, \sigma_k = 0.18, A_s = 1.59, \sigma_s = 47.74, \tau = 0.56, h = 0.02 \). The neural field, involving 40 neurons, is simulated synchronously during 40 seconds with a time-step \( \Delta t = 0.1s \). The parameters of the scaled UKF (see [13] section 3.2.2 for the meaning of these parameters) were: \( \alpha = 0.3, \beta = 2.0, \kappa = 0, P_0 = 0.1, P_{\alpha n} = 0.0, P_{\kappa v} = 0.1, I \). The membrane potentials ranging from 0.0 to 1.0 are represented with a graded color from white to black. A video of the optimization algorithm is available at http://jeremy.fix.free.fr/demo.php?demo=CCMB2011.

![Fig. 4. Working memory scenario. (a) Space/time representation of the input feeding the neural field. The two localized excitations are gaussians with an initial amplitude of 0.3, transiently increased up to 1.0, and a standard deviation of 2.0. A uniform random noise of amplitude 0.1 is added to the input. The transient increase of the stimuli’s amplitude reflects some kind of selection, allowing a target to enter the working memory. (b) The desired firing rate specifies a working memory behavior; when the amplitude of the input reaches a threshold, the stimulus enters the working memory and stays there until the input is completely suppressed. (c) Obtained neural field after 1000 iterations and a RMS error of 0.077. The algorithm converged to the following parameters: \( a = 0.92, b = -1.93, x_0 = 1.12, A_k = 1.21, \sigma_k = 2.11, A_s = 0.34, \sigma_s = 5.28, \tau = 0.5, h = -0.5 \). The neural field, involving 40 neurons, is simulated synchronously during 60 seconds with a time step \( \Delta t = 0.1s \). The parameters of the scaled UKF (see [13] section 3.2.2 for the meaning of these parameters) were: \( \alpha = 0.3, \beta = 2.0, \kappa = 0, P_0 = 0.1, P_{\alpha n} = 0.0, P_{\kappa v} = 0.1, I \). The firing rates ranging from 0.0 to 1.0 are represented with a graded color from white to black. A video of the optimization algorithm is available at http://jeremy.fix.free.fr/demo.php?demo=CCMB2011. ]
behavior: the baseline, the lateral excitation and the lateral inhibition. The baseline allows to set up the threshold above which a stimulus will enter the working memory. The lateral excitation allows to maintain a stimulus in working memory when the input’s amplitude is set back to its initial value. This lateral excitation compensates the decrease of the input drive. Usually, this lateral excitation is much stronger than in the competition scenario. Given the strength of this recurrent term, it may propagate over the whole field and saturate it completely. Lateral inhibition prevents this spread by somehow constraining the size of the excited regions within the field. Given the strength of the lateral excitation, the neural field may also become insensitive to the input somehow becoming saturated as soon as the input reaches a threshold. One may need that a stimulus gets removed from working memory when it disappears from the input, a behavior that cannot be guaranteed if lateral excitation is too strong. Therefore, a proper balance of lateral excitation/inhibition must be found. This additional constraint is added to the scenario by requiring the firing rate of the neural field to go back to the resting level as soon as the input stimuli disappear, which we see when it disappears from the input, a behavior that cannot be guaranteed if lateral excitation is too strong. Therefore, a proper balance of lateral excitation/inhibition must be found.

The major advantage of UKF compared to the previous works [8], [9] is its derivative-free aspect which does not restrict to the usage of derivable functions (e.g. a Heaviside function is also a classical transfer function). In terms of complexity, the genetic algorithms require to compute the fitness over the whole time horizon of all the individuals, i.e. an order of $O(N_I n_T n_x^2)$ computations, where $N_I$ is the number of individuals, $n_T$ the number of time steps and $n_x$ the number of space positions. They also require to update the parameters but given that there are much less parameters...
than the number of space and time positions (i.e., $p \ll n_x$ and $p \ll n_T$), this step has a low impact on the overall complexity. The gradient-descent-based method used in [9] requires an order of $O(p n_T n_x^2)$ computations. UKF involves different operations such as matrix inversion, Cholesky decomposition, etc. However, for the same reason as before (i.e. $p \ll n_x$ and $p \ll n_T$), the complexity of UKF is only dependent on $2p + 1$ evaluations of a neural field (when computing the image of the sigma-points). Therefore, a single UKF update requires an order of $O(p n_T n_x^2)$ computations. The complexity of UKF and a gradient-descent are similar but one should expect that UKF converges faster since UKF belongs to the family of Sigma-Point Kalman Filters (SPKF) which are an online form of a modified Gauss-Newton method which is a variant of natural gradient descent (see [13], ch. 4.5). More than just estimating the complexity of a single step, one would also need to perform benchmarks of the three algorithms to determine the number of steps they require to converge in order to provide a fair comparison. In addition, we discuss below some improvements of the proposed method to make UKF even faster to converge.

Two improvements of the presented method are currently investigated. The first one is to propose a more efficient sampling method for speeding up the convergence of the algorithm. In particular, one can use the information on the covariance of the output (equation 2) to bias the sampling of the inputs. Instead of the random sampling used so far, we could use a probabilistic draw biased by the covariance of the output of the non-linear function. It would speed up the convergence by presenting the samples where the uncertainty on the predictions (of the observations) is higher. This uncertainty depends both on the uncertainty on the value of the parameters and also on the sensitivity of the non-linear function respect to the parameters. A second improvement is also based on the way the neural field is sampled. In particular, instead of evaluating the whole neural field until a certain time, one could use transitions from, say state $\nu(t)$ to $\nu(t + 1)$ given that $\nu(t)$ is provided by the desired output. Given the desired output is only a rough estimation of the state that the dynamic neural field can reach, or said differently that the desired output is a noisy observation of a reachable output starting from a given initial condition, one can introduce additional equations in the state-space formulation to consider that the neural field state is also part of the evolution equation. These improvements would allow to scale up more easily the algorithm to deal with two or higher dimensional neural fields which are more common in practical applications.

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