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► **To cite this version:**

Laurent Bourgois, Jérôme Juillard. A Recursive Nonlinear System Identification Method Based on Binary Measurements. 5th International Conference on Integrated Modelling and Analysis Applied Control and Automation (IMAACA'11), Sep 2011, Rome, Italy. pp.15-20. hal-00643156

**HAL Id: hal-00643156**

**<https://centralesupelec.hal.science/hal-00643156>**

Submitted on 21 Nov 2011

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# A RECURSIVE NONLINEAR SYSTEM IDENTIFICATION METHOD BASED ON BINARY MEASUREMENTS\*

*IMAACA 2011*

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## Abstract

An online approach to nonlinear system identification based on binary observations is presented in this paper. This recursive method is a nonlinear extension of the LMS-like (least-mean-squares) basic identification method using binary observations (LIMBO). It can be applied in the case of weakly nonlinear Duffing oscillator coupled with a linear system characterized by a finite impulse response. It is then possible to estimate both Duffing and impulse response coefficients knowing only the system input and the sign of the system output. The impulse response is identified up to a positive multiplicative constant. The proposed method is compared in terms of convergence speed and estimation quality with the usual LMS approach, which is not based on binary observations.

**Keywords:** Nonlinear system identification, parameter estimation, self-test, binary data processing, microsystems

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\*This work was supported by funding from the Catrene project TOETS (Towards One European test Solution).

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# 1 INTRODUCTION

Microfabrication of electronic components such as micro-electro-mechanical systems (MEMS) has known an increasing interest over the past two decades. The most notable innovation emanating from these systems is the possibility to massively integrate sensors with self-test features on the same piece of silicon. Indeed, it is well-known that, as characteristic dimensions become smaller, the dispersions afflicting electronic devices tend to become larger. Typical sources of dispersions and uncertainties are variations in the fabrication process or environmental variations such as temperature, pressure or humidity. Consequently, it is usually impossible to guarantee *a priori* that a given device will work properly. Moreover, expensive tests must then be run after fabrication to ensure that only suitable devices are commercialized. An alternative consists in implementing self-test (and self-tuning) features such as parameter estimation routines, so that devices can adapt to changing conditions.

However, traditional identification methods [1, 2] are often tricky to ‘straightforwardly’ adapt from macroscopic scale to microscopic scale. Their integration requires the implementation of a high-resolution analog-to-digital converter (ADC) which results in longer design times as well as larger silicon areas. Thus, parameter estimation routines based on binary observations are very attractive because they only involve the integration of a 1-bit ADC. Some important contributions that keep the added cost of testing as small as possible are available in the literature.

In [3], Wigren has developed a least-mean-squares (LMS) approach to the problem of online parameter estimation from quantized observations. The principle is to estimate the gradient of the least-squares criterion by approximating the quantizer. Under some hypothesis, it is then possible to guarantee the asymptotic convergence of this method to the nominal parameters. In [4], Negreiros suggested to use a white Gaussian input to excite the unknown linear system and to estimate the power spectral density (PSD) of the binary output. From this estimated PSD, the modulus of the unknown

system transfer function can be analytically derived. However, it is not possible to obtain any information concerning the phase of this transfer function. This limitation has been overcome by deriving an analytical relationship between the coefficients of the impulse response of the system and the cross-covariance of its binary input and output. Although this approach is fairly simple to implement, it relies on the mixing properties of the linear system which may not be guaranteed *a priori*. Recently, a basic identification method using binary observations (BIMBO) has been introduced in [5]. The theoretical framework of this offline weighted-least-squares (WLS) approach is based on the minimization of a criterion where the parameter-dependent weights are chosen in order to smooth out the discontinuities of the unweighted least-squares criterion. It is then possible to guarantee the consistency of this approach even in the presence of measurement noise, provided that the signal at the input of the quantizer is Gaussian and centered. Furthermore, the estimation quality of BIMBO has been investigated in the sense of correlation coefficient between the nominal system parameters and the estimated system parameters. An alternative WLS criterion has also been presented in [6] which is easier to implement than the first one in the context of microelectronics. This approach is as efficient as the one proposed in [5] without measurement noise, but leads to a systematic error otherwise. Finally, an online LMS-like method for estimating system parameters based on binary observations (LIMBO) has been derived from the offline WLS approach presented in [6, 5]. Simulations have provided similar results than those obtained with the Wigren's method in terms of convergence speed and estimation quality, and those with a lesser computational complexity [7].

Unfortunately, the methods listed above deal with nonlinear systems. Now, in many engineering applications, and especially in microfabricated devices, the dynamic may significantly be affected by nonlinear effects, which must be accounted for in order to robustly model the system. In [8], the authors have studied identification of Wiener and Hammerstein systems, which are particular nonlinear structures, with binary-valued

output observations. In this paper, we propose to extend the LMS-like method introduced in [7] under the name of LIMBO in order to estimate online the parameters of a nonlinear system from binary observations. We consider a weakly nonlinear Duffing oscillator that is coupled with a linear system characterized by a finite impulse response. The convergence of this recursive method is illustrated by simulations and our results are compared with those obtained by the conventional LMS algorithm (*i.e.* without quantization).

The structure of the article is the following. In section 2, the nonlinear system and its model are introduced. In section 3, the LMS-like algorithm is derived. In section 4, the proposed method is compared with a traditional online method, which is not based on binary observations, in terms of convergence speed and estimation quality. Concluding remarks and perspectives are given in section 5.

## 2 FRAMEWORK AND NOTATIONS

Let us consider a nonlinear system illustrated in figure 1 below. The first branch corre-

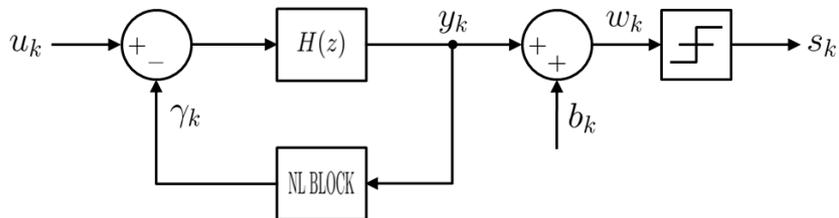


Figure 1: Block diagram of the system model.

sponds to a discrete-time linear time-invariant system  $H$ . We assume that this transfer function has a finite impulse response of length  $L$ , *i.e.* the impulse response can be represented by a column vector  $\theta = (\theta_l)_{l=1}^L$ . A cubic nonlinearity (the so-called Duffing nonlinearity) is then introduced at the level of the negative feedback branch such that  $\gamma_k = \alpha y_k^3$  with  $\alpha \in \mathbb{R}_+$ . Obviously, the subscript indices  $k$  denotes the discrete time.

Let  $\mathbf{b}$  an unknown additive noise and  $\mathbf{w} = \mathbf{y} + \mathbf{b}$  the noisy output. The system output is measured via an 1-bit analog-to-digital converter such that only the sign  $s_k = S(w_k)$  is known. Here, the function  $S$  of a real number  $x$  is defined as follows:

$$S(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{otherwise} \end{cases} \quad (1)$$

By supposing the system not highly nonlinear, the following approximation can be done [9]:

$$\begin{aligned} y_k &= h_k * (u_k - \alpha y_k^3) \\ &= h_k * (u_k - \alpha (h_k * (u_k - \alpha y_k^3))^3) \\ &\approx h_k * (u_k - \alpha (h_k * u_k)^3) \end{aligned} \quad (2)$$

The new block diagram of the system model is then illustrated graphically in figure 2. Consequently, the scalar value of the system output at time  $k$  is given by:

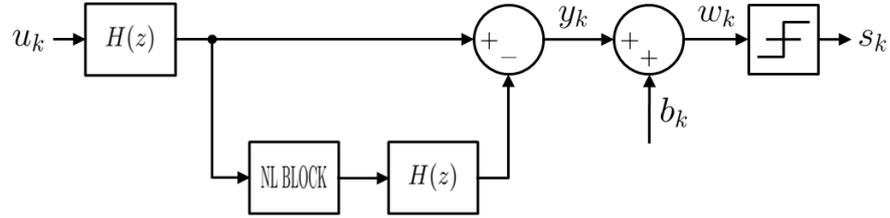


Figure 2: New block diagram of the system model.

$$y_k = \boldsymbol{\theta}^T \boldsymbol{\varphi}_{k,L} - \alpha \boldsymbol{\theta}^T \boldsymbol{\psi}_{k,L} \quad (3)$$

In the previous relation,  $\varphi_{k,L} = (u_l)_{l=k}^{k-L+1}$  is the  $L$  sample-long column regression vector at time  $k$  and  $\psi_{k,L}$  is defined in the same way by:

$$\psi_{k,L} = \left( (\boldsymbol{\theta}^T \boldsymbol{\varphi}_{l,L})^3 \right)_{l=k}^{k-L+1} \quad (4)$$

Our goal is to develop a recursive estimation method to simultaneously find good estimates of both the parameter vector  $\boldsymbol{\theta}$  and the Duffing coefficient  $\alpha$  starting from  $N$  observations of the binary output  $\mathbf{s}$  knowing the input  $\mathbf{u}$ . Let  $\hat{\boldsymbol{\theta}}_k$  be the estimated vector of parameters and  $\hat{\alpha}_k$  be the estimated Duffing coefficient at time  $k$ . Let us also introduce  $\hat{y}_k$  the estimated system output at time  $k$  and  $\hat{s}_k = S(\hat{y}_k)$ .

### 3 PROPOSED LMS APPROACH

In [7], the LIMBO method is carried out in order to estimate online the parameters of a linear system from binary observations. Since only  $s_k$  is available at time  $k$ , the authors judiciously defined the following instantaneous error:

$$\boldsymbol{\varepsilon}_k = |s_k - \hat{s}_k| \hat{y}_k \quad (5)$$

This suitable formulation has been specified to ensure the derivability with respect to  $\hat{\boldsymbol{\theta}}_k$  [5, 6]. Without loss of generality, we adopt this criterion to deal with nonlinear constraints. Obviously, (5) is also differentiable with respect to  $\hat{\alpha}_k$ . By inspiring from the general LMS algorithm, we can write:

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k - \mu_k \frac{\partial \boldsymbol{\varepsilon}_k^2}{\partial \hat{\boldsymbol{\theta}}_k} = \hat{\boldsymbol{\theta}}_k - 2\mu_k \boldsymbol{\varepsilon}_k \frac{\partial \boldsymbol{\varepsilon}_k}{\partial \hat{\boldsymbol{\theta}}_k} \quad (6)$$

In the same way, the following relation can be established:

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k - \lambda_k \frac{\partial \varepsilon_k^2}{\partial \hat{\alpha}_k} = \hat{\alpha}_k - 2\lambda_k \varepsilon_k \frac{\partial \varepsilon_k}{\partial \hat{\alpha}_k} \quad (7)$$

The dynamic LMS steps  $\mu_k$  and  $\lambda_k$  must satisfy some conditions to guarantee stability and convergence of the algorithm. In order to determine the previous derivatives, let us first develop the instantaneous error by introducing (3) into (5):

$$\varepsilon_k = |s_k - \hat{s}_k| (\hat{\theta}_k^T \varphi_{k,L} - \hat{\alpha}_k \hat{\theta}_k^T \hat{\psi}_{k,L}) \quad (8)$$

Thus, we can express the derivative with respect to  $\hat{\theta}_k$ :

$$\frac{\partial \varepsilon_k}{\partial \hat{\theta}_k} = |s_k - \hat{s}_k| \left( \varphi_{k,L} - \hat{\alpha}_k \left( \hat{\psi}_{k,L} + \hat{\theta}_k^T \frac{\partial \hat{\psi}_{k,L}}{\partial \hat{\theta}_k} \right) \right) \quad (9)$$

By using relation (4), we have:

$$\frac{\partial \hat{\psi}_{k,L}}{\partial \hat{\theta}_k} = 3 \left( \varphi_{l,L} (\hat{\theta}_k^T \varphi_{l,L})^2 \right)_{l=k}^{k-L+1} \quad (10)$$

This yields:

$$\hat{\theta}_k^T \frac{\partial \hat{\psi}_{k,L}}{\partial \hat{\theta}_k} = 3 \left( (\hat{\theta}_k^T \varphi_{l,L})^3 \right)_{l=k}^{k-L+1} = 3 \hat{\psi}_{k,L} \quad (11)$$

We find:

$$\begin{aligned} \hat{\theta}_{k+1} &= \hat{\theta}_k - 2\mu_k \varepsilon_k |s_k - \hat{s}_k| (\varphi_{k,L} - 4\hat{\alpha}_k \hat{\psi}_{k,L}) \\ &= \hat{\theta}_k - 2\mu_k (s_k - \hat{s}_k)^2 \hat{y}_k (\varphi_{k,L} - 4\hat{\alpha}_k \hat{\psi}_{k,L}) \end{aligned} \quad (12)$$

Following the same reasoning, the derivative of the instantaneous error with respect to  $\hat{\alpha}_k$  is easily obtained:

$$\frac{\partial \varepsilon_k}{\partial \hat{\alpha}_k} = |s_k - \hat{s}_k| (-\hat{\theta}_k^T \hat{\psi}_{k,L}) \quad (13)$$

And we finally have:

$$\hat{\alpha}_{k+1} = \hat{\alpha}_k + 2\lambda_k (s_k - \hat{s}_k)^2 \hat{y}_k \hat{\theta}_k^T \hat{\psi}_{k,L} \quad (14)$$

The algorithm 1 synthetize all the previous equations. The normalization step on line

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**Algorithm 1** LIMBO NL

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**Require:**  $\mathbf{u}, \mathbf{s}, L, N$

- 1:  $\hat{\chi}_1 \leftarrow [1 \ 0 \ \dots \ 0]^T$
  - 2:  $\hat{\theta}_1 \leftarrow \frac{\hat{\chi}_1}{\|\hat{\chi}_1\|_2}$
  - 3:  $\hat{\alpha}_1 \leftarrow 0$
  - 4: **for**  $k = 1$  to  $N$  **do**
  - 5:    $\varphi_{k,L} \leftarrow (u_l)_{l=k}^{k-L+1}$
  - 6:    $\hat{\psi}_{k,L} \leftarrow \left( (\hat{\theta}_k^T \varphi_{l,L})^3 \right)_{l=k}^{k-L+1}$
  - 7:    $\hat{y}_k \leftarrow \hat{\theta}_k^T \varphi_{k,L} - \hat{\alpha}_k \hat{\theta}_k^T \hat{\psi}_{k,L}$
  - 8:    $\hat{s}_k \leftarrow S(\hat{y}_k)$
  - 9:    $\hat{\chi}_{k+1} \leftarrow \hat{\theta}_k - 2\mu_k (s_k - \hat{s}_k)^2 \hat{y}_k (\varphi_{k,L} - 4\hat{\alpha}_k \hat{\psi}_{k,L})$
  - 10:    $\hat{\theta}_{k+1} \leftarrow \frac{\hat{\chi}_{k+1}}{\|\hat{\chi}_{k+1}\|_2}$
  - 11:    $\hat{\eta}_{k+1} \leftarrow \hat{\alpha}_k + 2\lambda_k (s_k - \hat{s}_k)^2 \hat{y}_k \hat{\theta}_k^T \hat{\psi}_{k,L}$
  - 12:    $\hat{\alpha}_{k+1} \leftarrow \hat{\eta}_{k+1} \|\hat{\chi}_{k+1}\|_2^3$
  - 13: **end for**
  - 14: **return**  $\hat{\theta}_{k+1}, \hat{\alpha}_{k+1}$
- 

10 ensures that the norm of  $\hat{\theta}_k$  is equal to unity and the line 12 is added to guarantee the homogeneity. Finally, the full operating model is illustrated graphically in figure 3.

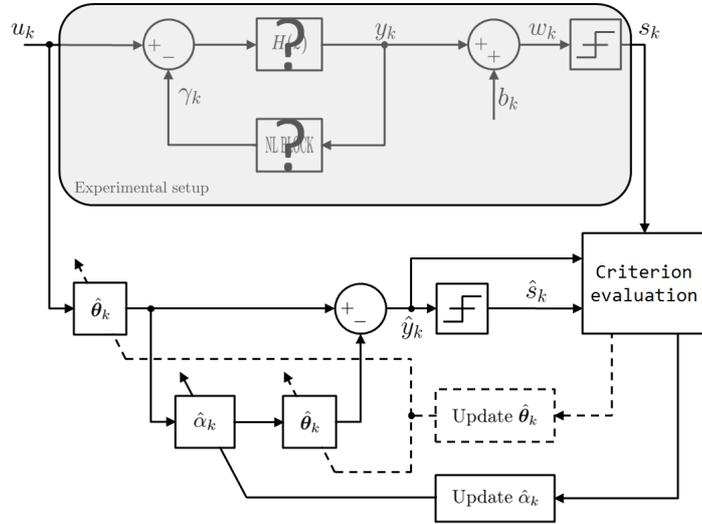


Figure 3: Block diagram of nonlinear LIMBO.

## 4 RESULTS AND DISCUSSION

In this section, the results obtained with nonlinear LIMBO method are compared with those obtained by applying a typical LMS method. Let us underline that contrary to our approach, the standard LMS is not based on quantized output measurements and is manifestly not well adapted to the context of micro electronics. The idea is to compare the performance in terms of convergence speed of our method with one which not suffers from a lack of *a priori* information.

The input signal is a Gaussian white noise with zero mean and unit standard deviation. We consider an impulse response of length  $L = 50$  and the Duffing coefficient  $\alpha$  is set to 0.01. In relations (12) and (14), the dynamic LMS steps  $\mu_k$  and  $\lambda_k$  must be chosen in order to guarantee stability and convergence of the algorithm. Concretely, these regulative coefficients can be determined by following an adaptive step size strategy. Unfortunately, their expression is often not perfectly adjusted to the integration context of micro devices. Hence, we prefer using constant regulative coefficients which are empirically determined. In the present case, we impose  $\mu = 0.0092$  and  $\lambda = 0.000079$

for LMS and  $\mu = 0.0049$  and  $\lambda = 0.0000165$  for LIMBO. Obviously, the two methods are compared on the same test case. The quality of the online estimation  $\hat{\theta}_k$  is defined as  $1 - v_k$  where  $v_k$  is the cosine of the angle made by  $\hat{\theta}_k$  and  $\theta$ . Since both vectors are normalized, we have  $v_k = \theta^T \hat{\theta}_k$  and the following equivalence relation :

$$\lim_{k \rightarrow \infty} (1 - v_k) = 0 \Leftrightarrow \lim_{k \rightarrow \infty} v_k = 1 \Leftrightarrow \lim_{k \rightarrow \infty} \hat{\theta}_k = \theta \quad (15)$$

Without measurement noise, both methods present encouraging results in terms of estimation quality concerning the impulse response. Indeed, the fifty coefficients of the column vector  $\theta$  have been successfully estimated. Without surprise, the Duffing coefficient identification also yields reasonable results for both methods, in terms of convergence speed and estimation quality, but with a notable advantage for the nonlinear LMS approach in terms of convergence speed. This difference is shown in figure 4 and is an immediate consequence of the quantized data. The same behavior is dis-

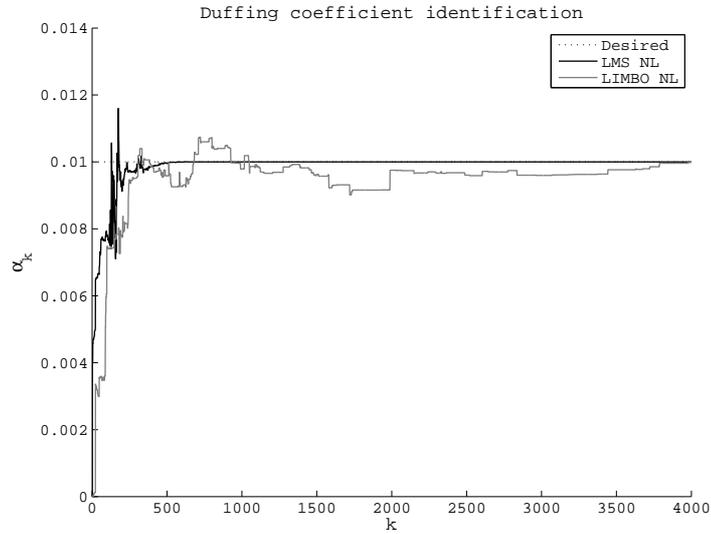


Figure 4: Comparison of nonlinear LMS and LIMBO methods for Duffing coefficient identification.

tinctly observable in figure 5 where the nonlinear LIMBO approach stops converging

after reaching an error level approximately equal to  $10^{-6}$  and where the nonlinear LMS approach converges to the nominal parameters within the limits of finite machine precision. In order to perturb the data we consider an additive Gaussian noise such that the

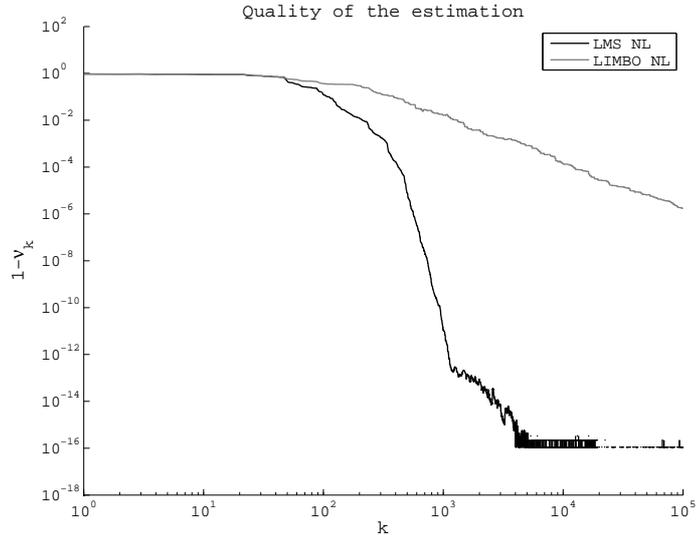


Figure 5: Comparison of nonlinear LMS and LIMBO methods in terms of estimation quality (SNR =  $\infty$  dB).

SNR is set to be 20 dB. The quality of the estimation is illustrated in figure 6. In this experiment, the two approaches stop converging after reaching an error level approximately equal to  $10^{-3}$  for LIMBO and  $10^{-4}$  for LMS. Although measurement noise has induced performance degradation, the estimation quality remains appreciable. Once again, LMS method presents the best results in terms of convergence speed but the gap is slightly reduced.

Finally, let us remember that for LIMBO, unknown parameters are updated only if the instantaneous error defined in (5) is null, *i.e.* only if  $s_k \neq \hat{s}_k$ . This ‘change of sign’ has appeared 458 times in absence of noise and 3715 times with a SNR of 20 dB. Consequently, LIMBO seems to give similar performances with a lesser iteration number than the LMS method, and especially with perturbed data. However, let us bear

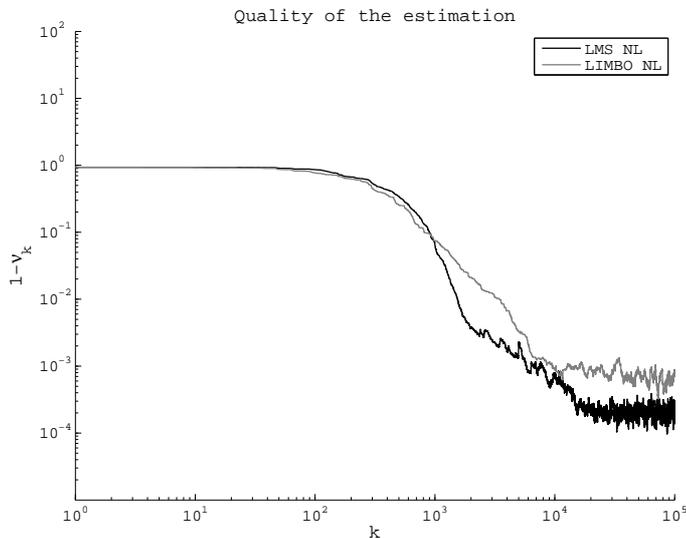


Figure 6: Comparison of nonlinear LMS and LIMBO methods in terms of estimation quality (SNR = 20 dB).

in mind that contrary to LMS approach, it is not possible to obtain any information concerning the amplitude of the impulse response coefficients in LIMBO since  $\theta$  is normalized.

## 5 CONCLUSION

In this paper, we have extended the LIMBO method introduced in [7] in order to estimate online the parameters of a nonlinear system from binary observations. We have studied the identification of a weakly nonlinear Duffing oscillator that is coupled with a linear system characterized by a finite impulse response. Results obtained by simulations are admirable in terms of convergence speed and estimation quality without measurement noise, and nearly similar to those obtained with the LMS method, which is not based on binary observations, in the noisy case. Consequently, nonlinear LIMBO is an inexpensive online test method easily implementable on microfabricated devices since it only requires the integration of a 1-bit ADC.

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