On Imperfect CSI for the Downlink of a Two-Tier Network

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***Abstract*—In this paper, we consider a hierarchical two-tier cellular network where a macrocell is overlaid with a tier of randomly distributed femtocells. We evaluate the combined effect of uncoordinated cross-tier interference, feedback delay, and quantization errors on the achievable rate of transmit beamforming with imperfect channel state information (CSI). We model the femtocell spatial distribution as a Poisson point process (PPP) and the temporal correlation of the channel according to a Gauss-Markov model. Using stochastic geometry tools, we derive the probability of outage at the macrocell users as a function of the temporal correlation, the femtocell density, and the feedback rate. We compute the maximum average achievable rate on the downlink of the macrocell network using a properly designed rate backoff scheme. We show that transmit beamforming with imperfect CSI is a viable option for the downlink of a two-tier cellular network, and that rate backoff recovers the loss in rate due to packet outage.**

I. Introduction

Transmit beamforming uses CSI at the transmitter to achieve capacity performance for multiple input single output (MISO) communication systems. Since CSI is difficult to obtain perfectly at the transmitter, limited feedback is a practical option that provides quantized CSI at the transmitter [1]. Limited feedback yields asymptotically optimal transmit beamforming and multiplexing gain performance [2], [3]. It suffers, however, from performance degradation due to feedback delay and quantization error. Moreover, when considered in an interference limited environment, the performance of transmit beamforming depends on the amount of interference knowledge at the transmitter and the receiver. For a hierarchical two-tier cellular systems, where the macrocells are overlaid with a randomly deployed tier of femtocells, interference is the main bottleneck for area spectral efficiency [4]. Femtocells are low-power, short-range home base stations, deployed to provide better indoor capacity [5]. They cause, when operated in closed access mode, significant cross-tier interference to the macrocell users. Interference from the femtocells is difficult to mitigate as the transmissions in the two cellular tiers are not coordinated.

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Prior results on transmit beamforming with imperfect CSI have focused on codebook design and performance evaluation over block fading channels [6]–[8]. Feedback delay and quantization error have been shown to decrease the achievable rate and to cause interference between spatial data streams in multiple input multiple output (MIMO) systems [9]. Research on tiered femtocell networks with multiple antennas is limited. The benefits of multiple antennas, in terms of coverage and spatial reuse, were evaluated in [10]. The analysis assumed perfect transmit CSI and considered zero forcing precoding for multi-user transmission. The transmission capacity for ad hoc networks with multiple antenna communication was considered in [11], [12] in a one-tier network scenario. An expression for the asymptotic spectral efficiency in the presence of spatially distributed interferers was derived in [13] with full transmit CSI.

In this paper, we analyze transmit beamforming with imperfect CSI for the downlink of a macrocell network with a tier of randomly deployed femtocells. We model the distribution of the femtocells as a homogeneous PPP. Using mathematical tools from stochastic geometry, we derive the effect of feedback delay and quantization error on the probability of outage and achievable rate of the macrocell transceiver in the presence of uncoordinated cross-tier interference. We adopt a system goodput metric [14], [15] to capture the effect of CSI mismatch between the base station and the user terminal due to feedback delay and quantization error. The system goodput is defined as the amount of information successfully received at the user terminal, without retransmissions. To maximize the achievable goodput, we propose a suboptimal transmission strategy based on rate backoff [16] to decrease the CSI mismatch in the system. We show that the maximum density of femtocells increases exponentially with the number of antennas at the macro base station and the quantization size for the limited feedback beamforming system. We further show that rate backoff reverses the adverse effects of packet outage and greatly improves the achievable rate of the system.

II. System Model and Assumptions

Consider a central macrocell overlaid with a tier of uncoordinated femtocells. The macro base station B_0 is located at the center of the macrocell, and is equipped with N_b antennas. Each femtocell is equipped with N_f antennas. The user terminals in the macrocell and the femtocells have a single receive antenna. We assume that each base station serves one active user, M_0, at a time, using intra-cell orthogonal access.

We use a stochastic geometry framework to model the distribution of the femtocells. The femtocells are assumed to be distributed according to a stationary homogeneous spatial Poisson point process Π_f with a fixed average density of λ_f femtocells per meter squared. The average number of actively transmitting femtocells is hence $No_f = \lambda_f C$, where $C = \pi R_c^2$ is the area of the macrocell, R_c is the radius of the macrocell.

In this paper, we assume that the femtocell interference is the main source of interference. We treat the aggregate co-channel interference from neighboring macro base stations as noise, in the interest of evaluating the effect of the interference from the femtocells on the downlink of the macrocell. Both the macrocell and the femtocells employ a limited feedback system [1], [3]. The user terminal estimates the downlink CSI sequence using pilot symbols sent by its serving base station. The CSI is quantized by means of a codebook and the quantization index is sent to the base station via a finite rate channel. The system is discrete time, where the continuous time signals are sampled at the symbol rate $\frac{1}{T_s}$, with T_s being the symbol duration. Each signal is represented by a sequence of samples with n denoting the sample index. For a narrowband channel, the n-th received data sample at M_0 is

$$\begin{aligned} y_0[n] &= \sqrt{\rho_m}\, \mathrm{D}^{\frac{-\alpha_m}{2}} \mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]s_0[n] \\ &+\sqrt{\rho_f}\sum_{F_i\in\Pi_f}|\mathrm{D}_i|^{\frac{-\alpha_f}{2}}\mathbf{g}_i^*[n]\mathbf{w}_i[n]r_i[n]+v_0[n], \end{aligned} \quad (1)$$

where $\mathbf{h}_0[n] \in \mathbb{C}^{N_b\times 1}$ is the channel between M_0 and B_0, with $\mathcal{CN}(0,1)$ i.i.d entries, and $\mathbf{h}_0^*$ is used to denote the conjugate transpose of the vector $\mathbf{h}_0$. The vector $\mathbf{g}_i[n] \in \mathbb{C}^{N_f\times 1}$ is the downlink interference channel from the femtocell, located at $F_i \in \Pi_f$, to M_0. The feedback delay is denoted by d. The vector $\mathbf{f}_0[n-\mathrm{d}] \in \mathbb{C}^{N_b\times 1}$ is the transmit beamforming vector at B_0 for M_0. The vector $\mathbf{w}_i[n] \in \mathbb{C}^{N_f\times 1}$ is the transmit beamforming vector used by the femtocell at F_i to maximize the received power at its user terminal, for the transmitted signal r_i. The signals s_0 and r_i are such that the maximum transmit power at the base station and the femtocells is satisfied. ρ_m and ρ_f are functions of the base stations transmit power, the carrier frequency and the penetration loss from indoor-to-outdoor propagation. D is the distance between B_0 and M_0, and D_i is the distance between F_i and M_0. $\alpha_m > 2$ and $\alpha_f > 2$ denote, respectively, the pathloss exponent of the outdoor channel between B_0 and M_0, and the indoor to outdoor channel between F_i and M_0. The scalar $v_0[n] \sim \mathcal{CN}(0,\sigma^2)$ is the additive white Gaussian noise (AWGN) at M_0. We consider hereafter an interference limited system, and ignore the effect of thermal noise.

We assume that M_0 estimates the channel $\mathbf{h}_0$ perfectly. We ignore the channel estimation error at the receiver, and leave it for further study. The feedback channel is assumed to be error free, but with a fixed feedback delay d caused by signal processing, scheduling protocol and propagation channel.

III. The Goodput Model with imperfect CSI

For the beamforming system with imperfect CSI, channel direction information is fed back using a quantization codebook $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_N\}$ of size $N = 2^B$. The codebook is known to both the transmitter and the receiver. The mobile station M_0 quantizes its desired channel $\mathbf{h}_0$ to the closest codeword, determined by the inner product

$$\begin{aligned} \mathbf{f}_0[n] = \mathcal{Q}\{\mathbf{h}_0[n]\} &= \arg\max_{\mathbf{v}_k\in\mathcal{F}} |\tilde{\mathbf{h}}_0^*\mathbf{v}_k|^2 \quad 1\le k\le N \\ &= \arg\max_{\mathbf{v}_k\in\mathcal{F}} \cos^2\left(\angle\Big(\tilde{\mathbf{h}}_0[n],\mathbf{v}_k\Big)\right), \end{aligned} \quad (2)$$

where $\tilde{\mathbf{h}}_0 = \frac{\mathbf{h}_0}{\|\mathbf{h}_0\|}$ is the channel direction. To facilitate the analysis, the quantization codebook is generated using random vector quantization (RVQ), shown to be asymptotically optimal as $N_b, B \to \infty$, $\frac{N_b}{B} \to \mathrm{constant}$ [2].

The quantization process at M_0 does not take into account the interference from the femtocells, as the macro base station and the femtocells transmissions are not synchronized. M_0 has no knowledge of the effective channels $\mathbf{g}_i\mathbf{w}_i$ from $F_i \in \Pi_f$. To modulate the transmitted signals, the base station estimates the signal to interference ratio, $\mathsf{SIR}^{\mathsf{t}}[n,\mathrm{d}]$, based on the delayed and quantized CSI as well as statistical knowledge of the interference. The estimated $\mathsf{SIR}^{\mathsf{t}}[n,\mathrm{d}]$ might differ from the signal to interference ratio, $\mathsf{SIR}[n,\mathrm{d}]$, estimated at the receiver. When the transmitted rate exceeds the supported rate of the channel, a rate or a packet outage occurs. To account for the rate outage, we use the system goodput as the performance metric. The goodput gives the amount of information successfully received (without retransmissions) at the user terminal. Denoting the transmitted rate by $\mathrm{R}^{\mathsf{t}}[n-\mathrm{d}]$ and the rate supported by the channel by $\mathrm{R}[n,\mathrm{d}]$, the goodput is defined as

$$\Lambda[n] = \mathrm{R}^{\mathsf{t}}[n-\mathrm{d}]\mathcal{I}\left(\mathrm{R}^{\mathsf{t}}[n-\mathrm{d}] \le \mathrm{R}[n,\mathrm{d}]\right), \quad (3)$$

where $\mathcal{I}(A)$ is the indicator function, which evaluates to 1 if the event A is true, and 0 otherwise. While hybrid automatic repeat request (HARQ) has been recently used in cellular systems to combat the CSI mismatch problem, we assume in this paper that HARQ is not present. For systems employing HARQ, the long term goodput achieved depends on the number of successfully received packets after the retransmissions [17], [18].

The rate $\mathrm{R}[n,\mathrm{d}]$ at M_0, assuming Gaussian distributed transmit symbols $s_0[n]$ and Shannon capacity achieving codes, is written as a function of $\mathsf{SIR}[n,\mathrm{d}]$,

$$\mathrm{R}[n,\mathrm{d}] = \log_2\left(1+\mathsf{SIR}[n,\mathrm{d}]\right),$$

where

$$\begin{aligned} \mathsf{SIR}[n,\mathrm{d}] &= \frac{\rho_m\,\mathrm{D}^{-\alpha_m}|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]|^2}{\rho_f\sum_{F_i\in\Pi_f}|\mathrm{D}_i|^{-\alpha_f}|\mathbf{g}_i^*[n]\mathbf{w}_i[n]|^2} \\ &= \frac{|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]|^2}{\mathrm{Q}_{\mathrm{D}} I_f}. \end{aligned} \quad (4)$$

The interference function $I_f = \sum_{F_i \in \Pi_f} |\mathrm{D}_i|^{-\alpha_f} |\mathbf{g}_i^*[n]\mathbf{w}_i[n]|^2$ is a shot noise process, and the pathloss ratio $\mathrm{Q_D} = \frac{\rho_f}{\rho_m \mathrm{D}^{-\alpha_m}}$.

The transmitted rate, $\mathrm{R^t}[n-\mathrm{d}]$, is written in terms of $\mathsf{SIR}^{\mathsf{t}}[n, \mathrm{d}]$ estimated at B_0,

$$\mathrm{R^t}[n-\mathrm{d}] = \log_2\left(1 + \mathsf{SIR}^{\mathsf{t}}[n, \mathrm{d}]\right), \tag{5}$$

where

$$\mathsf{SIR}^{\mathsf{t}}[n, \mathrm{d}] = \bar{\rho}|\mathbf{h}_0^*[n-\mathrm{d}]\mathbf{f}_0[n-\mathrm{d}]|^2 \tag{6}$$

and $\bar{\rho} = \frac{1}{\mathbb{E}[Q_\mathrm{D} I_f]}$.

The average goodput thus follows in terms of $\mathrm{R^t}[n-\mathrm{d}]$ and $\mathrm{R}[n, \mathrm{d}]$ as

$$\begin{aligned}\bar{\Lambda} &= \mathbb{E}\left[\mathrm{R^t}[n-\mathrm{d}]\mathcal{I}\left(\mathrm{R^t}[n-\mathrm{d}] \le \mathrm{R}[n,\mathrm{d}]\right)\right] \\ &= \mathbb{E}\left[\mathrm{R^t}[n-\mathrm{d}]\mathbb{P}\left[\mathrm{R^t}[n-\mathrm{d}] \le \mathrm{R}[n,\mathrm{d}] \mid \mathrm{R^t}[n-\mathrm{d}]\right]\right] \\ &= \int_0^\infty \mathrm{R^t}(\Upsilon)\mathbb{P}\left[\mathrm{SIR} \ge \Upsilon \mid \Upsilon\right] f_\Upsilon(\Upsilon) d\Upsilon. \end{aligned} \tag{7}$$

As the log function is monotone increasing in SIR, the probability of successful reception is expressed in terms of SIR. The integration to compute the average is taken over $\Upsilon = \mathsf{SIR}^{\mathsf{t}}[n, \mathrm{d}]$, and $f_\Upsilon(\Upsilon)$ denotes the probability density function of $\mathsf{SIR}^{\mathsf{t}}[n, \mathrm{d}]$.

IV. The Achievable Goodput with Imperfect CSI

To compute the maximum achievable goodput without CSI exchange between the two tiers, we first compute the probability of successful reception at the receiver, then use it to derive the achievable rate in the system, through rate backoff.

A. The Probability of Successful Reception

Rate outage occurs due to the CSI mismatch at the transmitter, in addition to the spatial randomness. The probability of successful reception is evaluated as the complementary probability of outage. It is the complementary cumulative distribution function of the desired channel power $|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]|^2$ given the interference shot noise I_f, for a given threshold Υ,

$$\mathbb{P}\left[\mathsf{SIR} \ge \Upsilon\right] = \mathbb{P}\left[|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]|^2 \ge \mathrm{Q_D} I_f \Upsilon\right]. \tag{8}$$

We consider a Gauss-Markov autoregressive model to account for the temporal correlation in the system. This model has been shown in the literature to be accurate for small delays on the communication links [19]. The effective channel at the receiver is thus expressed as

$$\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}] = \left(\eta \mathbf{h}_0^*[n-\mathrm{d}] + \sqrt{1-\eta^2}\mathbf{e}^*[n]\right)\mathbf{f}_0[n-\mathrm{d}]. \tag{9}$$

where $\mathbf{e}[n]$ is a Gaussian vector with $\mathcal{CN}(0,1)$ entries, independent of $\mathbf{h}_0$. For the numerical analysis, the correlation coefficient η is determined using Clarke's isotropic scattering model as $\eta = \mathcal{J}_0(2\pi \mathrm{d} f_d T_s)$, where f_d is the maximum Doppler spread, T_s is the symbol duration, and $\mathcal{J}_0(.)$ is the zero-th order Bessel function of the first kind. The desired instantaneous channel power is written as a function of the first term of the Gauss-Markov model $\mathbf{h}_0[n-\mathrm{d}]$, for large values of η corresponding to low mobility.

$$\begin{aligned}|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]|^2 &= \eta^2 \left|\mathbf{h}_0^*[n-\mathrm{d}]\mathbf{f}_0[n-\mathrm{d}]\right|^2 \\ &= \eta^2 \left|\mathbf{h}_0\right|^2 \cos^2 \angle(\mathbf{h}_0[n], \mathbf{f}_0[n-\mathrm{d}]), \end{aligned} \tag{10}$$

where $\cos^2\left(\angle(\mathbf{h}_0[n], \mathbf{v})\right)$ and $\left|\mathbf{h}_0[n]\right|^2$ are independent.

Lemma 1: The cumulative distribution function (CDF) of the effective channel power $Z = \eta^2 \left|\mathbf{h}_0^*[n]\mathbf{f}_0[n-\mathrm{d}]\right|^2$ is

$$\begin{aligned}F_Z(z) &= 1 - c_2 \exp\left(-\frac{z}{\kappa_2}\right) \\ &+ c_1 \exp\left(-\frac{z}{\kappa_1}\right) \sum_{i=0}^{N_b-2}\sum_{\ell=0}^{i} \frac{\delta^i}{(i-\ell)!}\left(\frac{z}{\kappa_1}\right)^{i-\ell}, \end{aligned} \tag{11}$$

where $\delta = 2^{-\frac{B}{N_b-1}}$, $c_1 = (1-\delta)\left(\frac{1}{\delta}\right)^{N_b-1}$, $\kappa_1 = 2\eta^2(1-\delta)$, $c_2 = \left(\frac{1}{\delta}\right)^{N_b-1}$, and $\kappa_2 = 2\eta^2$.

Proof: Under the assumption of spatially uncorrelated Rayleigh fading, and using Gersho's conjecture on asymptotic quantization [6], [20], it was shown in [21], Lemma 2, that $|\mathbf{h}_0[n]|^2 \cos^2\left(\angle(\mathbf{h}_0[n], \mathbf{v})\right)$, can be expressed as the sum of two Gamma distributed random variables with parameters $(N_b - 1, 1-\delta)$ and $(1,1)$, respectively. Gersho's conjecture assumes that each quantization cell is a Voronoi region of a spherical cap with a surface area equal 2^{-B} of the total surface area of the unit sphere [3], [21]. It has been shown to closely model the performance of codebook design techniques such as RVQ. It is used in the literature to analyze the performance of limited feedback [3], [21]. The distribution of the sum of two independent Gamma random variables follows from [22]. ■

Using the CDF of the effective desired channel power, the probability of successful reception at M_0 is given by the following theorem.

Theorem 1: The probability of successful reception at a mobile cellular user, in the presence of ad-hoc cross tier interference, for a limited feedback MISO system, over temporally correlated channels, is given by

$$\mathbb{P}\left[\mathsf{SIR} \ge \Upsilon\right] = A_1 \omega_1(\Upsilon) e^{-\omega_1(\Upsilon)} + A_2 e^{-\omega_1(\Upsilon)} + c_2 e^{-\omega_2(\Upsilon)}, \tag{12}$$

where

$$A_1 = c_1 \sum_{i=1}^{N_b-2}\sum_{\ell=0}^{i-1} \frac{\delta^i}{(i-\ell)!}(-1)^{i-\ell} \prod_{m=0}^{i-\ell-1}\left(\frac{2}{\alpha_f} - m\right),$$

$$\omega_1(\Upsilon) = \lambda_f C_f \left(\frac{\Upsilon \mathrm{Q_D}}{\kappa_1}\right)^{2/\alpha_f}, \quad A_2 = -c_1 \sum_{i=0}^{N_b-2} \delta^i,$$

$\omega_2(\Upsilon) = \lambda_f C_f \left(\frac{\Upsilon \mathrm{Q_D}}{\kappa_2}\right)^{2/\alpha_f}$, and $C_f = \frac{2\pi}{\alpha_f}\Gamma\left(\frac{2}{\alpha_f}\right)\Gamma\left(1 - \frac{2}{\alpha_f}\right)$, with Γ being the Euler-Gamma function.

Proof: The proof is provided in [23]. ■

Using Theorem 1, we derive an estimate of the maximum contention density λ_f of femtocells in the network, given a probability of outage constraint ϵ.

Corollary 1: The maximum femtocell density $\lambda_f(\mathrm{D})$ for which the probability of successful reception satisfies the maximum outage probability constraint $\mathbb{P}\left[\mathsf{SIR} \ge \Upsilon\right] \ge 1 - \epsilon$, given Υ, is the solution to

$$A_1\omega_1(\lambda_f)e^{-\omega_1(\lambda_f)}+A_2e^{-\omega_1(\lambda_f)}+c_2e^{-\omega_2(\lambda_f)}\geq 1-\epsilon. \quad (13)$$

In the high resolution regime, for small values of δ corresponding to a large codebook size 2^B, λ_f is given by

$$\lambda_f \leq \frac{\text{LambertW}\left(-\frac{1-\epsilon}{A_1\exp(\frac{A_2+c_2}{A_1})}\right)+\frac{A_2}{A_1}+\frac{c_2}{A_1}}{-C_f\left(\frac{\Upsilon \mathrm{Q_D}}{\kappa_1}\right)^{\frac{2}{\alpha_f}}}, \quad (14)$$

where LambertW denotes the Lambert W function that solves the equation $W\exp(W)=X$ for W as a function of X.

Proof: The proof is provided in [23]. ■

The number of femtocells increases exponentially with the number of antennas at the macro base station and the quantization size for the limited feedback beamforming system.

B. Rate Backoff

To decrease the probability of outage, we implement rate backoff at the transmitter. Instead of transmitting at a rate corresponding to $\mathsf{SIR}^{\mathsf{t}}$, the base station computes a backoff signal to interference ratio $\mathsf{SIR}^{\mathsf{b}}$, reducing $\mathsf{SIR}^{\mathsf{t}}$ by a multiplicative factor β,

$$\mathsf{SIR}^{\mathsf{b}}=\beta\,\mathsf{SIR}^{\mathsf{t}}. \quad (15)$$

The backoff $\mathsf{SIR}^{\mathsf{b}}$ is chosen such that the average achievable goodput is maximized. Under the backoff model, the average goodput is written as

$$\begin{aligned}\bar{\Lambda}^{\mathsf{b}} &= \mathbb{E}\Big[\log_2\left(1+\mathsf{SIR}^{\mathsf{b}}\right)\mathbb{P}\big[\mathsf{SIR}^{\mathsf{b}}\leq\mathsf{SIR}\big]\Big]\\ &= \int_0^\infty \mathrm{R}^{\mathrm{t}}(\beta\Upsilon)\mathbb{P}\left(\mathsf{SIR}\geq\beta\Upsilon\right)f_\Upsilon(\Upsilon)d\Upsilon. \quad (16)\end{aligned}$$

Setting the backoff factor, β, to a value close to 1 provides a goodput rate close to that achieved without rate adaptation. A smaller β value, however, might be conservative and leads to a low average achievable rate. We are interested in computing the optimal $\beta\in[0\ 1]$ such that $\bar{\Lambda}^{\mathsf{b}}$ is maximized. The average goodput $\bar{\Lambda}^{b}$ is a continuous and differential function in β. Thus, there exists an optimal backoff factor β^* such that

$$\beta^*=\arg\max_\beta \bar{\Lambda}^b. \quad (17)$$

Theorem 2: The backoff factor β, that maximizes the average goodput for a limited feedback beamforming MISO system, in the presence of cross tier interference with maximum density $\lambda_f(\mathrm{D})$ is the non-trivial solution of

$$\begin{aligned}&\left(\frac{1}{1+\frac{\beta\Upsilon}{\mathrm{Q_D}}}\right)\left[\left(A_1\omega_1(\beta\Upsilon)+A_2\right)e^{-\omega_1(\beta\Upsilon)}+c_2e^{-\omega_2(\beta\Upsilon)}\right]=\\ &-\log_2\left(1+\frac{\beta\Upsilon}{\mathrm{Q_D}}\right)\left[\frac{\frac{2}{\alpha_f}}{\beta}e^{-\omega_1(\beta\Upsilon)}\left((A_1-A_2)\omega_1(\beta\Upsilon)\right.\right.\\ &\left.\left.-\ A_1\omega_1^2(\beta\Upsilon)\right)-\frac{c_2\frac{2}{\alpha_f}}{\beta}\omega_2(\beta\Upsilon)e^{-\omega_2(\beta\Upsilon)}\right].\end{aligned}$$

For large codebook sizes, the backoff factor β^* is computed as the $\left(\frac{2}{\alpha_f}\right)$th square root of the roots of the polynomial in $\beta^{\frac{2}{\alpha_f}}$ of order 2,

$$\begin{aligned}&A_1\frac{2}{\alpha_f}\omega_1^2(\beta\Upsilon)+\frac{2}{\alpha_f}\left(c_2-A_1(1+\log(2))+A_2\right)\omega_1(\beta\Upsilon)\\ &\qquad -\frac{A_2+c_2}{\log(2)}=0, \quad (18)\end{aligned}$$

such that $\beta^*\in[0\ 1]$.

Proof: The proof is provided in [23]. It follows from the claim that in (16), maximizing the argument of the integration maximizes the integration. The problem thus becomes that of finding

$$\beta^* = \arg\max_\beta\left[\log_2\left(1+\beta\Upsilon\right)\mathbb{P}\left(\mathsf{SIR}\geq\beta\Upsilon\right)\right]. \quad (19)$$

■

The backoff factor β^* is a decreasing function of the velocity of M_0 and thus an increasing function of the correlation coefficient η. It is further a decreasing function of the femtocell contention density λ_f, and the distance D of M_0 from B_0. It is, however, an increasing function of δ.

V. Numerical Results: Verification of Analysis and Key Observations

We first verify the analysis of the achievable rate with and without rate backoff. We consider a single macrocell of radius $R_c=1$ km, with an average femtocell density of λ_f femtocells per cell. The users are uniformly distributed inside each cell.

Figure 1 shows the average rate for an average of 95 femtocells per cell. It compares the average goodput achieved with beamforming with imperfect CSI, with and without rate backoff, to that achieved using open loop random beamforming, for decreasing distance of M_0 away from B_0, given here by the signal to noise ratio (SNR). The averaging is done over 100 spatial Poisson processes realizations, with 100 Rayleigh channels instances per realization, for $N_b=N_f=4$ and $B=5$ for limited feedback. The achievable average throughput, $\mathrm{R}[n,\mathrm{d}]$, supported by the channel when the probability of outage is zero is also shown in the figure. Comparing the goodput and the throughput clearly shows the effect of the probability of outage on the achievable rate of the system. We further observe that beamforming with imperfect CSI achieves an average rate gain of 5 dB over random beamforming when the throughput is considered. For the goodput $\bar{\Lambda}$, this gain is on average 4 dB. This suggests that beamforming with imperfect CSI, although prone to errors due to delay and quantization, is a viable option for use in heterogeneous networks MISO systems. Moreover, Figure 1 shows that applying rate backoff at the transmitter achieves a 10 dB gain over the goodput achieved without backoff. The backoff rate $\bar{\Lambda}(\beta)$ increases as the distance D decreases.

We next consider the maximum number of femtocells obtained in Equation (14) as a function of the SNR at M_0, for fixed thermal noise at the receiver, and with a 10% outage probability requirement, for increasing δ. The density λ_f is averaged over 1000 uniformly distributed users in the macrocell, for an average user velocity of 20 km/h. As the number of feedback bits B increases, for $N_b=N_f=4$

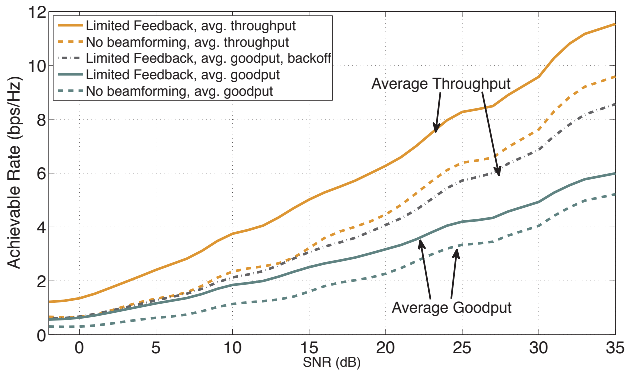

Fig. 1. The average rate as a function of SNR with and without rate backoff for $N_f = N_b = 4$, $N_{of} = 95$ femtocells per cell, and $B = 5$ feedback bits.

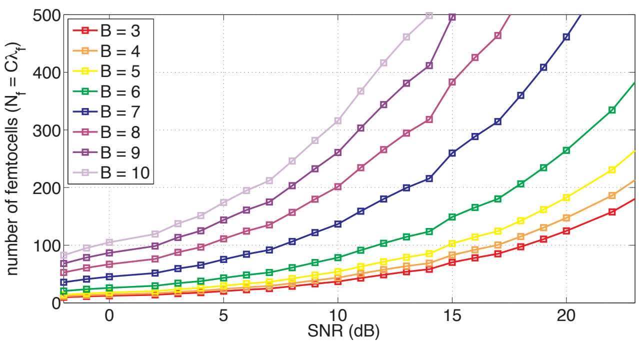

Fig. 2. The maximum number of femtocells interferers in a macrocell of area of $C = \pi 1000^2$ for a 4×1 MISO limited feedback system with increasing number of feedback bits B for $N_b = N_f = 4$, velocity $v = 20$ km/h, and delay $\mathrm{d} = 2$ time frames.

antennas, the number of femtocell interferers that can be allowed in the system without violating the probability of outage requirement increases, this increase is more discernable for higher feedback rate, $B > 6$. Similarly, as N_b increases, for a fixed B, λ_f increases.

VI. Conclusion

In this paper, we derived closed form expressions for the probability of outage of a limited feedback system with feedback delay, in a Poisson field of interferers. We computed the maximum achievable rate for the system using a well designed rate backoff mechanism. Our results indicate that the femtocell contention density increases exponentially with the number of feedback bits and the number of transmit antennas. They also show that rate backoff is beneficial to maximize the achievable goodput for an interference limited feedback system, despite the presence of feedback delay and uncoordinated cross tier interference.

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