Energy Efficient Communications in MIMO Wireless Channels: Information Theoretical Limits

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Half Title

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Symbol Description

α	To solve the generator maintenance scheduling, in the past, several mathematical techniques have been applied.
σ^2	These include integer programming, integer linear programming, dynamic programming, branch and bound etc.
$\sum$	Several heuristic search algorithms have also been developed. In recent years expert systems,
abc	fuzzy approaches, simulated annealing and genetic algorithms have also been tested.
$\theta\sqrt{abc}$	This paper presents a survey of the literature
ζ	over the past fifteen years in the generator
∂	maintenance scheduling. The objective is to
sdf	present a clear picture of the available recent literature
ewq	of the problem, the constraints and the other aspects of
bvcn	the generator maintenance schedule.

Part I

This is the first Part

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Energy Efficient Communications in MIMO Wireless Channels

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CONTENTS

Abstract: This chapter is focused on defining and optimizing an energy-efficiency metric for MIMO systems. This metric, which expresses in bit per Joule, allows one to measure how much information is effectively transferred to the transmitter per unit cost of energy consumed at the transmitter. For a MIMO point-to-point communication (single user MIMO channels) this metric can be useful to determine what power level, precoding scheme, training length, or number of antennas have to be used for obtaining the maximum information that is effectively transferred per unit energy spent. Then, we move from a physical layer-type approach to a cross-layer design of energy-efficient power control by including the effects a queue with finite size at the transmitter. As a last step we study a distributed multiple user scenario (MIMO multiple access channels) where each user selfishly maximizes its energy-efficiency by choosing its best individual power allocation policy. Here, we present the most relevant results in this field in a concise and comprehensible manner.

1.1 Introduction

For a long time, the problem of energy consumption mainly concerned autonomous, embarked, or mobile communication terminals. Over the past two decades, designing energy-efficient communication terminals has become a more and more important issue. Concepts such as "green communications" have recently emerged in the literature, e.g., [9], [15] etc. Nowadays, with the existence of large networks involving both fixed and mobile terminals, the energy consumed by the fixed infrastructure has also become a central issue for communications engineers [5]. This chapter presents some of the literature in this framework. More specifically, this is a guide for researchers and engineers on how to devise power control and power allocation schemes in green wireless networks. Among pioneering works on energy-efficient power control there are the works of Goodman [32], [13] and [8] and others like [50]. Therein, the authors define the energy-efficiency of a communication as the ratio between the net data rate (called goodput) and the radiated power; the corresponding quantity is a measure of the average number of bits successfully received per joule of energy consumed at the transmitter. This metric has been used in many works. For example, in [37] it is applied to the problem of distributed power allocation in multi-carrier CDMA (code division multiple access systems) systems. In [38], it is used to model the users delay requirements in energy-efficient systems. In [4], it is re-interpreted as a *capacity per unit cost*[1] measure in MIMO (multiple input multiple output) systems for static and fast fading channels.

In this chapter, we analyze the energy-efficiency metric defined as the ratio between the benefit of the transmission, i.e., the information rate, and the cost of the transmission, i.e., the consumed power to achieve this rate.

The information rate can be formulated through an information theoretical approach as studied in [4] or through a more pragmatic approach as in [50]. However, both of these approaches can be unified via the general concept of the channel capacity per unit cost if the channel is properly defined. The rate defined [32] can be interpreted as the capacity of a binary erasure channel. In a binary channel the signal $X \in \{0, 1\}$ is recieved as $Y \in \{0, 1, \epsilon\}$. Where ϵ represents the erasure of the signal. A binary erasure channel is illustrated in Figure 1.1.

Our analysis is important from both practical and theoretical perspectives. From an engineering point of view, our study helps one to design energy-efficient systems by specifying the optimal transmit power and power allocation policy that maximizes the defined energy-efficiency metric. While from a mathematical point of view, both the information rate and the efficiency can have several interesting properties like concavity, sigmoidal shape or quasi-concavity. This allows the study of interesting optimization problems of the energy-efficiency with respect to the transmit power.

This chapter is structured into two major parts. In the first part, we consider single user MIMO channels, highlighting the impact of imperfect channel state information (CSI). In this case, we study the optimization over not only the transmit power and the power allocation policy, but also over the training sequence length. It turns out that, while using all the available transmit antennas is optimal under perfect CSI, using only a subset of the available antennas is optimal when the channel has to be estimated.

In the second part of the chapter, we focus on the more challenging case of distributed multi-user networks. We analyse the energy-efficiency of networks that in which several users compete for the best energy efficiency by choosing

[1] The capacity per unit cost is an information theoretical notion introduced by Verdu in [50] and a measure of the amount of reliably transmitted information bits over the channel per unit cost.

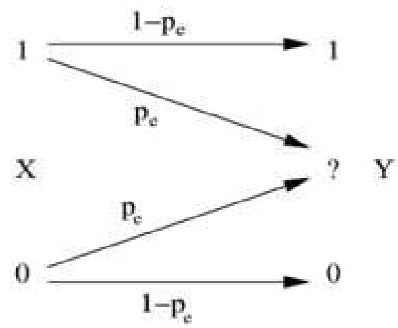

FIGURE 1.1
Definition of the binary erasure channel with a probability of failure (erasure) p_e. The capacity of this channel when $p_e = 1 - f(\text{Power})$ corresponds to the definition in [32]

their power allocation and transmit power policies. Our results indicate that the optimization methodology for maximizing best data rates is different from that of maximizing energy efficiencies.

1.2 On the design of energy-efficient MIMO single-user communications

The major focus of this section is on point-to-point communications. We present a theoretical and numerical analysis for this case. The focus on single user communication systems may be surprising considering that power control is one of the primary problems of interest. However, there are two important reasons for this choice. First of all, the single-user case has most of the main effects we want to emphasize and allows us to describe the proposed approach in a clear and concise manner. Secondly, once the single-user case problem is solved, the multi-user case is tractable provided some conditions are met. One of these conditions being that the performance metric possesses some desirable properties like quasi-concavity (which is verified for the proposed metric) and reasonably complex multi-user channel models are considered (the multiple access channel is one of them).

The energy-efficiency metric is defined at first in a very general way. Then, we specialize it depending on the specific situation under consideration and study it as a function of various model parameters such as the transmit power.

1.2.1 A general definition of the energy-efficiency performance criterion

In what follows, we define and justify the energy-efficiency metric that is suitable to communication systems. Energy-efficiency in general usually refers to how much you gain per unit cost of energy. For example in thermodynamics it refers to the mechanical energy gained per cost of heat energy. In this case, the gain is a measure of how much data was successfully transferred. This leads to the definition of the energy-efficiency metric as the ratio between the data rate and the consumed power. In a MIMO system, the data rate depends on the precoding matrix $\mathbf{Q}$ [22]. The available transmit power P is related to the precoding matrix as $P\mathrm{Trace}(\mathbf{Q}) \leq P$.

$$\eta(\mathbf{Q}) = \frac{\mathrm{Rate}(\mathbf{Q})}{\mathrm{Net\ Power}[\mathrm{Trace}(\mathbf{Q})]}, \tag{1.1}$$

where Net Power[Trace($\mathbf{Q}$)] is the total cost in terms of energy (this may depend on the computation cost, **losses**) and the Rate($\mathbf{Q}$) represents the corresponding effective data rate.

It is interesting to note that this definition can be applied to almost all situations. This observation is shown to be true in the following sections.

1.2.2 Optimizing the total transmit power and power allocation in MIMO channels with perfect CSI

Consider a point-to-point communication with multiple antenna terminals. The signal at the receiver is modeled by:

$$\underline{y}(\tau) = \mathbf{H}(\tau)\underline{x}(\tau) + \underline{z}(\tau), \tag{1.2}$$

Where $\mathbf{H}$ is the $n_r \times n_t$ channel transfer matrix and n_t (n_r) the number of transmit (receive) antennas. The vector $\underline{x}$ is the n_t-dimensional column vector of transmitted symbols and $\underline{z}$ is an n_r-dimensional complex white Gaussian noise distributed as $\mathcal{N}(\underline{0}, \sigma^2\mathbf{I}_{n_r})$. Denoted by $\mathbf{Q} = \mathbb{E}[\underline{xx}^H]$ is the input precoding matrix. The corresponding total power constraint is Trace($\mathbf{Q}$) $\leq P$.

The matrix $\mathbf{H}$ is assumed to be perfectly known at the receiver (coherent communication assumption) whereas only the statistics of $\mathbf{H}$ are available at the transmitter. Three cases will be studied depending on the channel coherence time: i) the static links; ii) fast fading links; iii) slow fading links. For the first two cases, the benefit of the transmission will be measured in terms of Shannon transmission rates. For slow fading channels,outage events exist, while makes reliable communication impossible, and the length of the codeword used determines the success rate. Observe that in the first two cases, the solution is trivial and corresponds to the transmitters remaining silent. However, this is no longer the case for slow fading channels. In this case, the solution to the optimization problem is provided only for the particular case of

MISO (the receiver is equipped with a single antenna). For the MIMO case, the optimal solution is conjectured and validated through numerical simulations.

1.2.2.1 Static Links

By definition, in the static links case, the frequency at which the channel matrix varies is strictly zero. In other words, $\mathbf{H}$ is a constant matrix. In this particular context, both the transmitter and receiver are assumed to know this matrix. This is the same framework as [22]. Thus, for a given precoding scheme $\mathbf{Q}$, the transmitter can reliably send to the receiver $\log_2 \left|\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{HQH}^H\right|$ bits per channel use (bpcu). Let us define the energy-efficiency of this communication by:

$$\eta_{\text{static}}(\mathbf{Q}) = \frac{\log_2 \left|\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{HQH}^H\right|}{\text{Trace}(\mathbf{Q})}. \tag{1.3}$$

The energy-efficiency function $\eta_{\text{static}}(\mathbf{Q})$ corresponds to an achievable rate per unit cost for the MIMO channel as defined in [50] under the assumption that the input alphabet does not contain any zero-cost symbols (i.e., silence at the transmitter does not convey information). It turns out that the result obtained in [50] for the single-input single-output channel extends to the MIMO channel.

Proposition 1.1 (Optimal precoding matrix for static MIMO channels)**.** *The energy-efficiency of a MIMO communication over a static channel, measured by η_{static}, is maximized when $\mathbf{Q} = \mathbf{0}$ and this maximum is*

$$\eta^*_{\text{static}} = \frac{1}{\ln 2}\frac{\lambda_{\max}(\mathbf{HH}^H)}{\sigma^2}. \tag{1.4}$$

where $\lambda_{\max}(\mathbf{HH}^H)$ represents the maximum eigenvalue of the matrix $\mathbf{HH}^H$. The proof can be found in [4]. It can be seen that, for static MIMO channels, the energy-efficiency defined in Eq. (1.3) is maximized by transmitting at very low powers. This kind of scenario occurs for example, when deploying sensors in the ocean to measure a temperature field (which varies very slowly). In some applications however, the rate obtained by using such a scheme can be insufficient. In these cases, the benefit to cost ratio can turn out to be an irrelevant measure and other performance metrics have to be considered (e.g., minimize the transmit power under a rate constraint).

1.2.2.2 Fast fading Links

In this section, the frequency with which the channel matrix varies is the reciprocal of the symbol duration ($\underline{x}(\tau)$ being a symbol). This means that it can be different for each channel use. Therefore, the channel varies over a transmitted codeword (or packet) and, more precisely, each codeword sees as many channel realizations as the number of symbols per codeword. In this framework, let us define energy-efficiency by:

$$\eta_{\text{fast}}(\mathbf{Q}) = \frac{\mathbb{E}_{\mathbf{H}}\left[\log\left|\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right|\right]}{\text{Trace}(\mathbf{Q})}. \tag{1.5}$$

The proof for the static links case can be applied for any channel realization and thus the trivial solution is obtained irrespective of the channel distribution.

Proposition 1.2 (Optimal precoding matrix for fast fading MIMO channels)**.** *The energy-efficiency of a MIMO communication over a fast fading channel, measured by* η_{fast}*, is maximized when* $\mathbf{Q} = \mathbf{0}$ *and this maximum is*

$$\eta^*_{\text{fast}} = \frac{1}{\ln 2}\frac{\text{Trace}(\mathbb{E}\left[\mathbf{H}\mathbf{H}^H\right])}{n_t\sigma^2}. \tag{1.6}$$

For fast fading MIMO channels, it can be seen that maximizing energy-efficiency also amounts to transmitting at low power. Interestingly, in slow fading MIMO channels, where outage events are unavoidable, note that the answer can be different.

1.2.2.3 Slow fading Links

In this section, the channel remains constant over a codeword and varies from block to block. As a consequence, the Shannon achievable rate is equal to zero. A suitable performance metric that measures the benefit of the transmission in slow-fading channels is the probability of an outage for a given transmission rate target R given in [3]. This metric allows one to quantify the probability that the rate target R is not reached by using a good channel coding scheme and is defined as follows:

$$\text{P}_{\text{out}}(\mathbf{Q}, R) = \Pr\left[\log_2\left|\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right| < \xi\right], \tag{1.7}$$

where $\xi = R/R_0$ and R_0 bpcu (bits per channel use) is used to represent the bandwidth.

For the sake of simplicity, the entries of $\mathbf{H}$ are assumed i.i.d. zero-mean unit-variance complex Gaussian random variables. In terms of information assumptions, here again, it can be checked that only the second-order statistics of $\mathbf{H}$ are required to optimize the precoding matrix $\mathbf{Q}$. In this framework, [4] defines the energy-efficiency as follows:

$$\eta_{\text{slow}}(\mathbf{Q}, R) = \frac{R[1 - \text{P}_{\text{out}}(\mathbf{Q}, R)]}{\text{Trace}(\mathbf{Q})}. \tag{1.8}$$

In other words, the energy-efficiency or goodput-to-power ratio (GPR) is defined as the ratio between the expected throughput (see [32] for details) and the average transmit power. The expected throughput can be seen as the

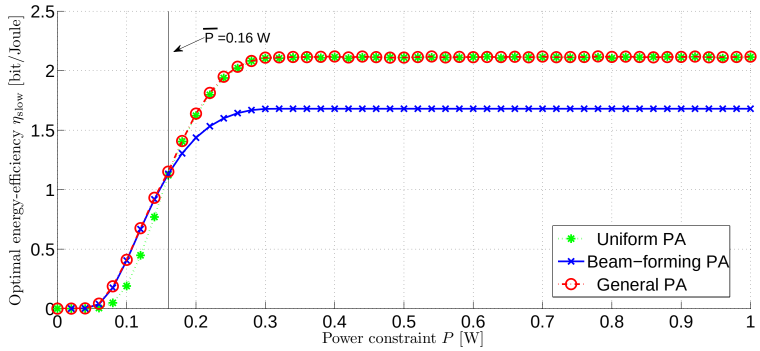

FIGURE 1.2
Optimal energy-efficiency vs. power constraint p, comparison between beamforming PA, UPA (Uniform power allocation) and General PA.

average system throughput over many transmissions. In contrast with static and fast fading channels, energy-efficiency is not necessarily maximized at low transmit powers. Thus, a non-trivial solution may exist to the optimization of GPR.

Finding the optimal covariance matrix is not trivial. Indeed, even the outage probability minimization problem w.r.t. $\mathbf{Q}$ is still an open problem [22], [49]. The general solution is conjectured as follows.

Conjecture 1.1 (Optimal precoding matrix for slow fading MIMO channels)**.** *There exists a power threshold $\overline{p}$ such that:*

- *if $p \leq \overline{p}$ then $\mathbf{Q}^* \in \arg\min_{\mathbf{Q}} P_{\rm out}(\mathbf{Q}, R) \;\; \Rightarrow \;\; \mathbf{Q}^* \in \arg\max_{\mathbf{Q}} \;\; \eta_{\rm slow}(\mathbf{Q}, R)$;*
- *if $p > \overline{P}$ then $\eta(\mathbf{Q}, R)$ has a unique maximum in $\mathbf{Q}^* = \frac{p^*}{n_t}\mathbf{I}_{n_t}$ where $p^* \leq P$.*

This conjecture states that, if the available transmit power is less than a threshold, maximizing the GPR is equivalent to minimizing the outage probability. If it is above the threshold, the uniform power allocation is optimal. However using all the available power is generally suboptimal in terms of energy-efficiency. This conjecture is validated in Fig. 1.2 for the MIMO scenario: $n_t = n_r = 2$, $\xi = 1$, $1/\sigma^2 = 3$ dB. For the exact same threshold $= 0.16$ W, we have that, for $p \leq \overline{p}$ the beamforming PA structure optimal and above it, UPA structure is optimal.

Regarding the optimization problem associated with (1.8) several comments are in order. First, there is no loss of optimality by restricting the search for optimal precoding matrices to diagonal matrices: for any eigenvalue decomposition $\mathbf{Q} = \mathbf{U}\mathbf{D}\mathbf{U}^H$ with $\mathbf{U}$ unitary and $\mathbf{D} = \mathbf{Diag}(\underline{p})$ with

$\underline{p} = (p_1, \ldots, p_{n_t})$, both the outage and trace are invariant w.r.t. the choice of $\mathbf{U}$. The energy-efficiency can be written as:

$$\eta_{\text{slow}}(\mathbf{D}, R) = \frac{R[1 - \mathrm{P}_{\text{out}}(\mathbf{D}, R)]}{\sum_{i=1}^{n_t} p_i}. \tag{1.9}$$

Second, the GPR is generally not quasi-concave w.r.t. $\mathbf{D}$. In [4], a counter-example for which the GPR is proven not to be quasi-concave is provided. Third, the conjecture was validated using Monte-Carlo numerical simulations for the 2×2 case where both the transmitter and receiver are equipped with two antennas. Fourth, the conjecture 1.1 was rigorously solved for MISO channels where the receiver is equipped with a single antenna (see [4] for details).

Proposition 1.3 (Optimal precoding matrix for slow fading MISO channels)**.** *For all $\ell \in \{1, ..., n_t - 1\}$, let c_ℓ be the unique solution of the equation (in x) $\Pr\left[\frac{1}{\ell+1}\sum_{i=1}^{\ell+1}|X_i|^2 \leq x\right] - \Pr\left[\frac{1}{\ell}\sum_{i=1}^{\ell}|X_i|^2 \leq x\right] = 0$ where X_i are i.i.d. zero-mean Gaussian random variables with unit variance. By convention $c_0 = +\infty$, $c_{n_t} = 0$. Let ν_{n_t} be the unique solution of the equation (in y) $\frac{y^{n_t}}{(n_t-1)!} - \sum_{i=0}^{n_t-1}\frac{y^i}{i!} = 0$. Then the optimum precoding matrices have the following form:*

$$\mathbf{D}^* = \left| \begin{array}{ll} \frac{p}{\ell}\boldsymbol{Diag}(\underline{e}_\ell) & \text{if } p \in \left[\frac{c}{c_{\ell-1}}, \frac{c}{c_\ell}\right) \\ \min\left\{\frac{\sigma^2(2^\xi-1)}{\nu_{n_t}}, \frac{p}{n_t}\right\}\mathbf{I}_{n_t} & \text{if } p \geq \frac{c}{c_{n_t-1}} \end{array} \right. \tag{1.10}$$

where $c = \sigma^2(2^R - 1)$ and $\underline{e}_\ell \in \mathcal{S}_\ell$.

Similarly to the optimal precoding scheme for the outage probability minimization [49], the solution maximizing the GPR consists in sharing the available power uniformly among a subset of $\ell \leq n_t$ antennas. As i.i.d entries are assumed for $\mathbf{H}$, the choice of these antennas does not matter. What matters is the number of antennas selected, which depends on the available transmit power p: the higher the transmit power, the higher the number of used antennas. The difference between the outage probability minimization and GPR maximization problems appears when the transmit power is greater than the threshold $\frac{c}{c_{n_t-1}}$. In this regime, saturating the power constraint is suboptimal for the GPR optimization. The conjecture 1.1 has also been solved for the SIMO channel where the transmitter is equipped with a single antenna, and also for the MIMO channel assuming the extreme SNR regimes (low and high SNR regimes).

A special case of interest is the case of uniform power allocation (UPA): $\mathbf{D} = \frac{p}{n_t}\mathbf{I}_{n_t}$ where $p \in [0, P]$ and $\eta_{\text{UPA}}(p, R) \triangleq \eta_{\text{slow}}\left(\frac{p}{n_t}\mathbf{I}_{n_t}, R\right)$. One of the reasons for studying this case is the famous conjecture of Telatar in [22].

This conjecture states that, depending on the channel parameters and target rate (i.e., σ^2, R), the power allocation (PA) policy minimizing the outage probability is to spread all the available power uniformly over a subset of $\ell^* \in \{1, \ldots, n_t\}$ antennas. If this can be proved, then it is straightforward to show that the covariance matrix $\mathbf{D}^*$ that maximizes the GPR is $\frac{p^*}{\ell^*}\mathbf{Diag}(\underline{e}_{\ell^*})$, where $\underline{e}_{\ell^*} \in \mathcal{V}_{\ell^*}$[2]. Thus, $\mathbf{D}^*$ has the same structure as the covariance matrix minimizing the outage probability except that using all the available power is not necessarily optimal, $p^* \in [0, P]$. In conclusion, solving Conjecture 1.1 reduces to solving Telatar's conjecture and also the UPA case.

The main difficulty in studying the outage probability and/or the energy-efficiency function is the fact that the probability distribution function of the mutual information is generally intractable. In the literature, the outage probability is often studied by assuming an UPA policy over all the antennas and also using the Gaussian approximation of the p.d.f. of the mutual information. This approximation is valid in the asymptotic regime of large number of antennas. However, simulations show that it also quite accurate for reasonable small MIMO systems [12] (for e.g., assuming four-antenna terminals, the approximation is very good and assuming eight-antenna terminals, the error is negligible).

Under the UPA policy assumption, the GPR $\Gamma_{\mathrm{UPA}}(p, R)$ is conjectured to be quasi-concave w.r.t. p.

Conjecture 1.2. *[Quasi-concavity of the energy-efficiency function] Assume that* $\mathbf{D} = \frac{p}{n_t}\mathbf{I}_{n_t}$. *Then* $\eta_{\mathrm{UPA}}(p, R)$ *is quasi-concave w.r.t.* $p \in [0, P]$.

This conjecture was proved for the special cases of MISO and SIMO. Furthermore, it was proved for the general MIMO case assuming the large system approach for three cases: $n_t < +\infty$ and $n_r \to +\infty$; $n_t \to +\infty$ and $n_r < +\infty$; $n_t \to +\infty$, $n_r \to +\infty$ with $\lim\limits_{n_i \to +\infty, i \in \{t,r\}} \dfrac{n_r}{n_t} = \beta < +\infty$. Numerical simulations were provided to validate the conjecture for finite number of antennas. In Fig. 1.3, we plot the energy-efficiency function as a function of power $p \in [0, 1]$ W for MIMO channels where $n_r = n_t = n \in \{1, 2, 4, 8\}$, UPA $\mathbf{D} = \frac{p}{n_t}\mathbf{I}_{n_t}$, $\rho = 10$ dB, $R = 1$ bpcu.

Furthermore, the numerical simulations show that the optimal value of the energy-efficiency metric is increasing with the number of antennas. So far, only the case of a MIMO single user system with perfect CSI at the receiver was studied. In practice, the channel has to be estimated at the receiver. In the following, we analyse the effect of channel estimation on the system energy-efficiency. This analysis is best suited for the case of slow fading channels considering that the time to estimate the channel and send data is finite.

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[2]The set $\mathcal{V}_\ell = \left\{\underline{v} \in \{0,1\}^{n_t} | \sum_{i=1}^{n_t} v_i = \ell\right\}$ represents the set of n_t-dimensional vectors containing ℓ ones and $n_t - \ell$ zeros, for all $\ell \in \{1, \ldots, n_t\}$.

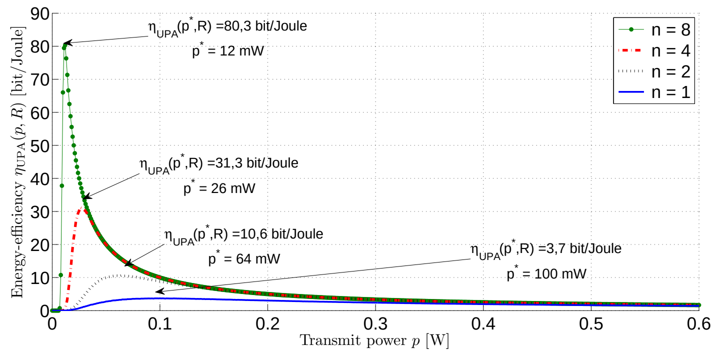

FIGURE 1.3
Energy-efficiency (GPR) vs. transmit power assuming UPA. The energy-efficiency function is a quasi-concave function w.r.t. p. The optimal point p^* is decreasing and $\eta_{\text{UPA}}\,(p^*, R)$ is increasing with n.

1.2.3 Energy efficient communication in MIMO channels with imperfect CSI

In this section, we consider the effect of constant power consumption by the transmitter which is independent on the radio power transmitted. The main goal of this section is to introduce and justify a definition of energy-efficiency of a communication system with multiple input, multiple output, slow fading links, no CSI at the transmitter, and imperfect CSI at the receiver. Slow fading channels are considered here because if the channel is fast fading then the channel can not be estimated faster than the alteration of the channel making learning a non-viable option. On the other hand if the channel is static, once the channel is estimated, it will never change and so the training does not have a significant cost that has to be optimized in the long run.

Therefore we consider slow fading channels. Since the channel matrix is not known at the transmitter, we assume the UPA, i.e., $\mathbf{Q} = p\frac{\mathbf{I}_{n_t}}{n_t}$. However, similarly to the previous cases, using all the antennas may not always be optimal.

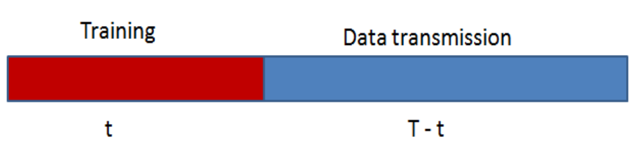

FIGURE 1.4
The coherence time T is divided among training and data transmission

Assume that the channel follows the simple discrete-time block-fading law, where the channel is constant for some time interval, after which it changes to an independent value that it holds for the next interval [16]. This model is appropriate for the slow-fading case where the time with which $\mathbf{H}$ changes is much larger than the symbol duration. Each transmitted block of data is assumed to comprise a training sequence for the receiver to be able to estimate the channel. The training sequence length in symbols is represented by t_s and the block length in symbols by T_s. Continuous counterparts of the latter quantities are defined by $t = t_s \times S_d$ and $T = T_s \times S_d$, where S_d is the symbol duration in seconds. In the training phase, all n_t transmitting antennas broadcast orthogonal sequences of known pilot/training symbols of equal power on all antennas. The receiver estimates the channel, based on the observation of the training sequence, denot! ed by $\widehat{\mathbf{H}}$. The estimation error is denoted by $\delta\mathbf{H} = \mathbf{H} - \widehat{\mathbf{H}}$. Concerning the number of observations needed to estimate the channel, note that typical estimators such as maximum likelihood estimators generally require at least as many measurements as unknowns [17] that is to say:

$$t_s \geq n_t. \tag{1.11}$$

The channel estimate normalized to variance one is denoted by $\widetilde{\mathbf{H}}$. From [16], we have

$$\underline{\widetilde{y}} = \sqrt{\frac{\rho_{eff}(\rho, t)}{M}} \widetilde{\mathbf{H}} \underline{x} + \underline{\widetilde{z}} \tag{1.12}$$

provided that the effective SNR $\rho_{eff}(\rho, t)$ and equivalent observation noise $\underline{\widetilde{z}}$ are defined properly that is,

$$\left\{ \begin{array}{ccc} \underline{\widetilde{z}} & = & \rho \times \delta\mathbf{H} \times \underline{x} + \underline{z} \\ \rho_{eff}(\rho, t) & = & \frac{\frac{t_s}{n_t}\rho^2}{1+\rho+\rho\frac{t_s}{n_t}} \end{array} \right. . \tag{1.13}$$

Note that the above equation does not correspond to any real signal, the observation equation simply denotes a mathematical equivalence of the SNR received due to channel estimation [16]. The worst case scenario for the estimation noise is assumed. Thus, in all formulas derived in the following are lower bounds on the mutual information and success rates. Since the channel matrix is unknown at the transmitter, we assume that $\mathbf{Q} = \mathbf{I}_{n_t}$, meaning that the transmit power is allocated uniformly over all the transmit antennas i.e $p = \frac{\mathbf{I}_{n_t}}{n_t}$. Under this assumption, the energy-efficiency is defined in [28] as:

$$\eta_{n_t}(p, t) = \frac{R \times \left(1 - \frac{t}{T}\right) \times \Pr\left[\log\left|\mathbf{I}_{n_t} + \frac{1}{n_t}\rho_{eff}\left(\frac{Lp}{\sigma^2}, t\right)\mathbf{H}\mathbf{H}^H\right| \geq \xi\right]}{ap + b} \tag{1.14}$$

where L is a term for the path loss, $a > 0$, $b \geq 0$ are parameters representing the radio-frequency power efficiency and a constant load due to cooling, coding etc., R is the transmission rate in bit/s, R_0 is a parameter which expresses in

Hz (e.g., the system bandwidth). The numerator represents the benefit associated with transmitting namely, the net transmission rate (called the goodput in [15]) of the communication and is measured in bits/s. The goodput comprises a term $1-\frac{t}{T}$ which represents the loss in terms of information rate due to the presence of a training mechanism and a term representing the transmission success probability. Note that a packet is received successfully only if the associated mutual information (which is obtained from the equivalent observation equation (1.12)) is above a certain target. The denominator of (1.14) represents the cost of transmission in terms of power. The proposed form for the denominator of 1.14 is inspired from [18] where the! authors propose to relate the average power consumption of a transmitter (base stations in their case) to the average radiated or radio-frequency power by a linear model. Note that, without any loss of generality, we can choose $a = 1$ to simplify the analysis. In what b is concened, two different regimes are identified:

- The regime where b is small allows one to study not only communication systems where the power consumed by the transmitter is determined by the radiated power but also those which have to been green in terms of electromagnetic pollution or due to wireless signal restrictions [27].

- The regime where b is large allows one to study not only communication systems where the consumed power is almost independent of the radiated power but also those where the performance criterion is the goodput. Note that when $b = 0$, $t \to +\infty$, $\frac{t}{T} \to 0$, the the framework of [4] can be retrieved as a special case.

Table 1.1 presents a summary of the results available in this framework. In the following, we will present some of these results in detail.

Quasi-concavity of η_{n_t} with respect to p for UPA with imperfect CSI	
SISO, MISO and SIMO Large MIMO Very low or high SNR	Proven
General MIMO	Conjecture
Concavity of η_{n_t} with respect to t when η_{n_t} is optimized for p	Proven
Quasiconcavity of η_{n_t} with respect to n_t	Conjecture

TABLE 1.1
Summary of known results from the current state of literature on energy efficiency of MIMO systems with imperfect CSI

1.2.3.1 Power control

By inspecting (1.14), we see that using all the available transmit power can be suboptimal. For instance, if the available power is large and all of it is used, then $\eta_{n_t}(p,t)$ tends to zero. Since $\eta_{n_t}(p,t)$ also tends to zero when p

goes to zero, there must be at least one maximum at which energy-efficiency is maximized, showing the importance not to exploit all the available power in certain regimes. The objective of this section is to study those aspects namely, to show that η_{n_t} has a unique maximum for a fixed training time fraction and provide the equation determining the optimum value of the transmit power.

From [26] we know that a sufficient condition for the function $\frac{f(x)}{x}$ to have a unique maximum is that the function $f(x)$ be sigmoidal/S-shaped and possess some mild properties (which are verified in our setup); a function f is sigmoidal if it is convex up to a point and then becomes concave. To apply this result in our context, [28] defines the function f by

$$f(\rho_{eff}) = \Pr\left[\log\left|\mathbf{I}_{n_t} + \frac{1}{n_t}\rho_{eff}\mathbf{H}\mathbf{H}^H\right| \geq \xi\right]. \tag{1.15}$$

It turns out that proving that f is sigmoidal in the general case of MIMO is a non-trivial problem, as advocated by the current state of relevant literature [4], [49], [7]. This is why we provide here a conjecture and a proposition concerning relevant special cases of MIMO systems [4].

Conjecture 1.3 (Optimization w.r.t. p for general MIMO systems)**.** *For a fixed t and any pair (n_t, n_r), the energy-efficiency function $\eta_{n_t}(p,t)$ is a quasi-concave function with respect to p and has a unique maximum.*

This conjecture becomes a theorem in all classical special cases of interest, which is stated next. It is also supported by an intensive campaign of simulations.

Proposition 1.4 (Optimization w.r.t. p for special cases of MIMO systems)**.** *If one of the following conditions is met:*

(a) $n_t \geq 1$, $n_r = 1$;

(b) $n_t \to +\infty$, $n_r < +\infty$;

(c) $n_t < +\infty$, $n_r \to +\infty$;

(d) $n_t \to +\infty$, $n_r \to +\infty$, $\lim_{n_t\to+\infty, n_r\to+\infty} \frac{n_t}{n_r} = \beta < +\infty$;

(e) $p \to 0$;

(f) $p \to +\infty$;

then $\eta_{n_t}(p,t)$ is a quasi-concave function w.r.t. p and has a unique maximum.

This proposition is proved in [28]. Under one of the assumptions of the above proposition, it is relevant to characterize the unique solution of $\frac{\partial \eta_{n_t}}{\partial p}(p,t) = 0$ that is, the root (ρ^*_{eff}) of

$$\frac{L}{\sigma^2}(p+b)\frac{\tau\rho\left[(\tau+1)\rho+2\right]}{\left[(\tau+1)^2+1\right]^2}f'(\rho_{eff}) - f(\rho_{eff}) = 0 \tag{1.16}$$

with $\tau = \frac{t_s}{n_t}$. Note that p is related to ρ through $p = \frac{\sigma^2 \rho}{L}$ and ρ is related to ρ_{eff} through equation (1.13) and can be expressed as

$$\rho = \frac{1}{2\tau}\rho_{eff}\sqrt{(1+\tau)^2 + \frac{4\tau}{\rho_{eff}}}. \tag{1.17}$$

Therefore (1.16) can be expressed as a function of ρ_{eff} and solved numerically; once ρ^*_{eff} has been determined, ρ^* follows by 1.17, and $P^* = \rho^*\sigma^2$ follows. A special case is where $b = 0$ and $\tau \to +\infty$; this amounts to finding the unique root of $\rho_{eff} f'(\rho_{eff}) - f(\rho_{eff}) = 0$ which corresponds to the optimal operating SNR in terms of energy-efficiency of a channel with perfect CSI.

Fig. 1.5 illustrates the energy-efficiency as a function of the transmit power (p) for different values of b and illustrates the quasi-concavity of the energy efficiency function w.r.t p, i.e conjecture 1.3. The parameters used are $R = 1600$, $\xi = \frac{R}{R_0} = 16$, $T_s = 55$ and $n_t = n_r = 4$, $t = 4$ ms.

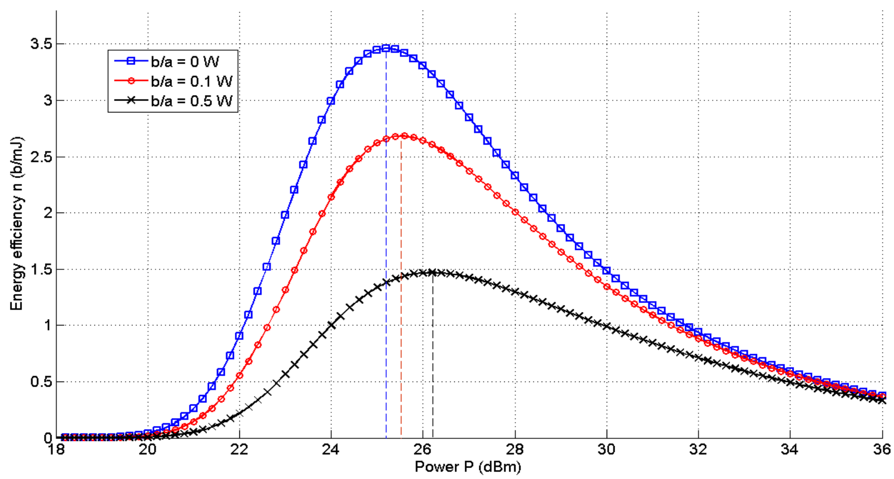

FIGURE 1.5
Energy-efficiency (η_{n_t}) in bits per Joule (bpJ) v.s transmit power (p) for a MIMO system with $t_s = 4$, $n_t = n_r = 4$,$R = 1600$bps (bits per second), $\xi = \frac{R}{R_0}$=16 and $T_s = 55$ symbols. Observe that η_{n_t} is quasi-concave and has a unique maximum for each value of b.

Now we have presented on how to tune the transmit power in a MIMO system, however there is are additional optimizations than can be performed in this case of imperfect CSI. As we learn the channel within a certain time interval t, this time can be reduced to allow for more time for data transmission but at the cost of a worse channel estimate. This trade-off is studied in the following section.

1.2.3.2 Optimizing training sequence length

The expression of $\eta_{n_t}(p,t)$ shows that only the numerator depends on the fraction of training time. Choosing $t = 0$ maximizes $1 - \frac{t}{T}$ but the packet success rate vanishes. Choosing $t = T$ maximizes the latter but makes the former term go to zero. Again, there is an optimal tradeoff to be found. Interestingly, it is possible to show that the function $\eta_{n_t}(p^*, t)$ is strictly concave w.r.t. t for any MIMO channels in terms of (n_t, n_r), where p^* is a maximum of η_{n_t} w.r.t p. This is stated in the following proposition.

Proposition 1.5 (Optimization w.r.t. t for general MIMO systems)**.** *The energy-efficiency function $\eta_{n_t}(p,t)$ is a strictly concave function with respect to t for any p satisfying $\frac{\partial \eta_{n_t}}{\partial p}(p,t) = 0$ and $\frac{\partial^2 \eta_{n_t}}{\partial p^2}(p,t) < 0$, i.e, at the maxima of η_{n_t} w.r.t. p.*

The proof of this proposition is provided in [28]. The proposition means that once energy-efficiency has been maximized w.r.t. the transmit power, the uniqueness of the solution to the optimization problem w.r.t. t, the optimal training time follows. Based on this, the optimal fraction of training time is obtained by setting $\frac{\partial \eta_{n_t}}{\partial t}(p,t)$ to zero which can be written as:

$$\left(\frac{T_s}{n_t} - \tau\right) \frac{\rho^2(\rho+1)}{\left[\tau\rho + \rho + 1\right]^2} f'(\rho_{eff}) - f(\rho_{eff}) = 0 \tag{1.18}$$

again with $\tau = \frac{t_s}{n_t}$. Here again, following the same reasoning as for optimizing the η_{n_t} w.r.t. p, it is possible to solve numerically the equation w.r.t. ρ_{eff} and find the optimal t_s, which is denoted by t_s^*.

Fig. 1.6 studies the optimized energy efficiency $\eta_{n_t}^*$ as a function of the transmit power with various values of t_s. The figure illustrates that beyond a certain threshold on the available transmit power, there is an optimal training sequence length that has to be used to maximize the efficiency, when the optimization w.r.t p has been done, which has been proven analytically in proposition 1.5. The parameters are $R = 1$Mbps, $\xi = 16$, $b = 0$, $n_t = n_r = 4$, $b = 0$ and $T_s = 55$.

It should be noted that a solution to equation (1.18) necessarily exists only if η has been optimized w.r.t p. However, in many practical situations, this optimization might not be possible. Even in such situation, it is favorable to optimize the training time. The following conjecture describes how the optimal training time behaves as the transmit power is varied.

Conjecture 1.4 (Optimal training sequence length)**.** *For a given number of transmit antennas n_t, $\eta_{n_t}(p,t)$ is maximized for $t_s^* = T_s - 1$ in the limit of $p \to 0$. As p increases, t_s^* decreases monotonically until for some P_+, $t_s^* = M$ and then for all $p \geq P_+$, $t_s^* = M$. Where P_+ is simply the smallest p for which $t_s^* = n_t$.*

This shows that the optimal training sequence length clearly depends on

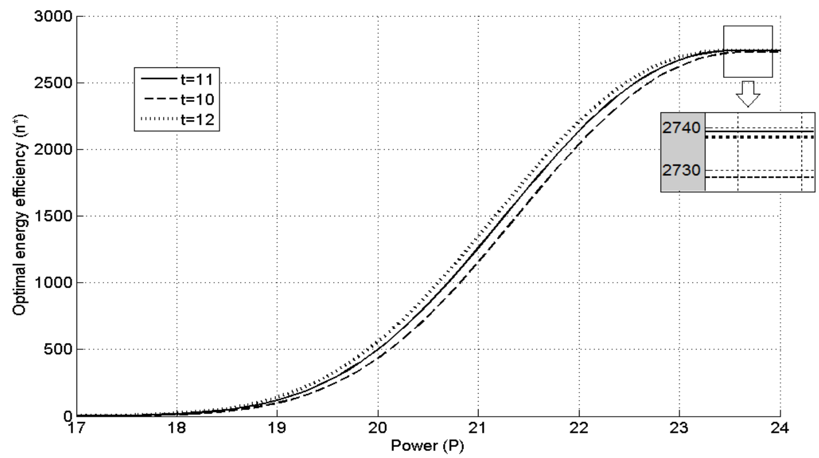

FIGURE 1.6
Optimized efficiency (ν^*) vs. transmit power (p) for a MIMO system with $n_r = 4$, $R = 1$ Mbps, $\xi = \frac{R}{R_0}$=16, $M = 4$, $T_s = 55$ and $b = 0$W. As proved in the proposition, t is concave for optimal p and it can be seen that beyond a certain threshold on the available power, $t_s = 11$ is always optimal.

the number of antennas used. Note that (1.18) can be easily exploited to prove some parts of the conjecture. This is what the following proposition is about.

Proposition 1.6 (Optimal fraction of training time in extreme SNR regimes)**.** *It can be shown that:* $\lim_{p\to+\infty} t_s^* = n_t$ *for all MIMO systems in general.*

The proof for this can be found in [28].

Fig. 1.7 studies the optimal training sequence length t_s as a function of the transmit power p. Note that in this case, we are not optimizing the efficiency with respect to P and so this figure illustrates conjecture 1.4 and proposition 1.6. With p large enough $t_s = n_t$ becomes the optimal training time and for p small enough $t_s = T_s - 1$ as seen from the figure. The parameters are $R = 1600$, $b = 0$ W, $\xi = 16$ and $T_s = 10$. (We use $T_s = 10$, as if the coherence time is too large, the outage probabilities for low powers that maximize the training time, such that $t_s^* = T_s - 1$, become too small for any realistic computation.)

1.2.3.3 Optimizing number of transmit antennas

So far the assumption was that the precoding matrix was chosen to be the identity matrix i.e., $\mathbf{Q} = \mathbf{I}_{n_t}$. Clearly, if nothing is known about the channel, the choice $\mathbf{Q} = \mathbf{I}_{n_t}$ is relevant (and may be shown to be optimal by formulating the problem as an inference problem). On the other hand, if some information

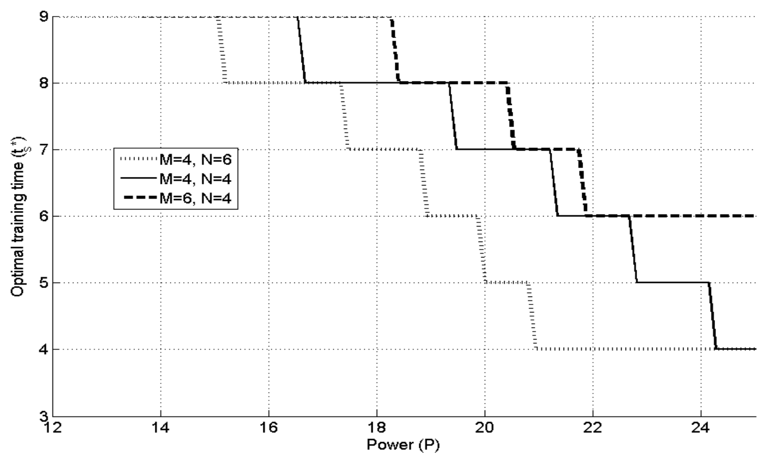

FIGURE 1.7
Optimal training sequence length (t_s) vs. Power (P) MIMO system with $\xi = \frac{R}{R_0} = 16$,$R = 1$ Mbps $T_s = 10$ symbols. Observe that t_s^* decreases monotonically from $T_s - 1$ to n_t.

about the channel is available (the channel statistics as far as this paper is concerned), it is possible to find a better precoding matrix. As conjectured in [22] and proved in some special cases (see e.g., [49]), the outage probability is minimized by choosing a diagonal precoding matrix and a certain number of 1's on the diagonal. The position of the 1's on the diagonal does not matter since channel matrices with i.i.d. entries are assumed. However, the optimal number of 1's depends on the operating SNR. The knowledge of the channel statistics can be used to compare the operating SNR with some thresholds and lead to th! is optimal number. Although we consider equation (1.14) as a performance metric instead of the outage probability, we are in a similar situation to [4], meaning that the optimal precoding matrix in terms of energy-efficiency is conjectured to have the same form and that the number of used antennas have to be optimized. In the setting of this paper, as the channel is estimated, an additional constraint has to be taken into account that is, the number of optimal antennas n_t^* cannot exceed the number of training symbols t_s. This leads us to the following conjecture.

Conjecture 1.5 (Optimal number of antennas)**.** *For a given number of training symbols t_s, η_{n_t} is maximized for $n_t^* = 1$ in the limit of $p \to 0$. As p increases, n_t^* also increases monotonically until for some P_+, $n_t^* = t_s$ and then for all $p > P_+$, $n_t^* = t_s$.*

This conjecture can be understood intuitively by noting that the only influence of n_t on η_{n_t} is through the success rate. Therefore, optimizing n_t for any given p and t amounts to minimizing outage. Based on the conjecture in [22], which has been proven for several special cases, we can conclude that the optimal number of antennas is one in the very low SNR regime and that it increments as the SNR increases. However, the effective SNR decreases by increasing n_t, this will result in the optimal n_t for each p with training time lower than or equal to the optimal n_t obtained with perfect CSI. Concerning special cases, it can be easily checked that the optimal number of antennas is 1 at low SNR, and is t_s at high SNR.

Finally, a possible refinement of the definition 1.14 regarding to n_t is possible. Indeed, by creating a dependency of the parameter b towards n_t one can better model the energy consumption of a wireless device. For instance, if the transmitter architecture is such that one radio-frequency transmitter is used per antenna, then, each antenna will contribute to a separate fixed cost. In such a situation the total power can written as $ap + n_t b_0$ where b_0 is the fixed energy consumption per antenna. It can be trivially seen that this does not affect the goodput in any manner and only brings in a constant change to the total power as long as n_t is kept a constant. So the optimization w.r.t p and t will not change but it will cause a significant impact on the optimal number of antennas to use.

Fig. 1.8 studies the optimized energy efficiency η^* as a function of the transmit power with various values of n_t. The figure illustrates that beyond a certain threshold on the available transmit power, there is an optimal number of antennas that has to be used to maximize the efficiency. The parameters are $R = 1$Mbps, $\xi = 16$, $b = 0$W, $n_r = 4$, $t = 15$ and $T_s = 55$. We observe that for p beyond a certain threshold, the optimal number of antennas is 10. This result is interesting because we are also optimizing the energy efficiency w.r.t p simultaneously and we find that the optimal strategy is to use only a limited number of antennas.

1.2.4 Cross layer design in energy efficiency communication

In the previous subsections, the energy-efficiency of systems where data is continuously transmitted has been studied. In practice, this is not the case and the data traffic is typically random and depends on several factors such as: the used protocol, the location of the user, the time of the day e.t.c. In the remaining of this section, we focus on the situation where data packets arrive from an upper layer (based on the transfer protocol) randomly into a buffer from which transmission is performed. The energy efficiency of this system is studied in [29].

Consider a buffer of size K at the transmitter. The packets arrival follows a Bernoulli process with probability q for entering the queue (this corresponds to classical ON/OFF sources). All packets in this model are assumed to be of the same size. The throughput (rate) on the radio interface equals to R (bit/s) and this depends on several parameters like modulation and the coding

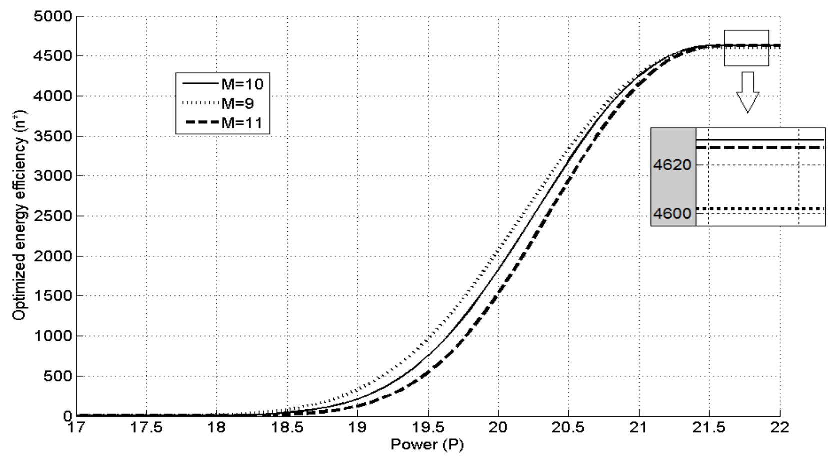

FIGURE 1.8
Optimized efficiency (η^*) vs. transmit power (p) for a MIMO system with N=4, R = 1 Mbps, $\xi = \frac{R}{R_0}$=16, t = 15, T_s = 55 and b = 0W. It can be seen that beyond a certain threshold, $n_t = 10$ is always optimal while $n_t = 4$ or 9 were optimal for lower powers.

scheme. We consider the case when the transmitter is always active, meaning that it always transmits its packet while the buffer is not empty. Each packet transmitted on the channel is received without any errors with a probability $f(p)$ which depends on the quality of the channel and transmission power p. If the channel fading due to path loss is represented by g, the success probability depends on the SNR $= \frac{gp}{\sigma^2}$. However, based on the block fading channel assumption, we make a slight abuse of notations by using the notation $f(p)$ instead of $f(\text{SNR})$. ! In some places in this paper, we even remove the variable p for the sake of clarity and use the notation f. We denote by α_t the size of the queue at the transmitter at time slot t. The size of the queue α_t is a Markov process on the state space $\alpha = \{0, \ldots, K\}$. We have the following transition probabilities $\forall i, j \in \alpha$, $P_{i,j} := I\!\!P(\alpha(t+1) = i | \alpha(t) = j)$ given by:

1. $P_{0,0} = 1 - q + qf$,
2. $P_{K,K} = (1 - q)(1 - f) + q$,
3. for any state $i \in \{0, \ldots, K-1\}$, $P_{i,i+1} = q(1 - f)$,
4. for any state $i \in \{1, \ldots, K\}$, $P_{i,i-1} = (1 - q)f$,
5. for any state $i \in \{1, \ldots, K-1\}$, $P_{i,i} = (1 - q)(1 - f) + fq$.

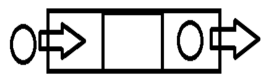

FIGURE 1.9
Packet arrival and transmission from a queue, the boxes represent the available buffers while the circles represent packets of data

A new packet is lost if the queue is full when it comes in and the transmission of the packet currently on the radio interface failed on the same time slot. Indeed, we consider that a packet is in service (occupying the radio interface) until it is transmitted successfully. Thus, a packet in service blocks the queue during $\frac{1}{f(p)}$ time slots on the average. We assume that an arrival of a packet in the queue and a departure (successful transmission) at the same time slot can occur.

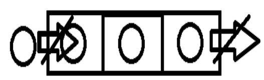

FIGURE 1.10
Packet arrival halted when the buffers are full and the system has failed in transmitting a packet. The boxes represent the buffers while the circles represent packets of data

Given the transition probabilities above, the stationary probability of each state is given by (see e.g., [53]):

$$\forall s \in \alpha, \quad \Pi_s = \frac{\rho^s}{1 + \rho + \ldots + \rho^K}, \tag{1.19}$$

with

$$r = \frac{q(1-f)}{(1-q)f}. \tag{1.20}$$

When a packet arrives and finds the buffer full (meaning that the packet currently on the radio interface is not transmitted successfully), it is blocked and this event is considered as a packet loss. The queue is full in the stationary regime with probability Π_K :

$$\Pi_K = \frac{r^K}{1 + r + \ldots + r^K} = \frac{r^K(r-1)}{r^{K+1} - 1}. \tag{1.21}$$

In order to evaluate the performance of this system, we first determine the expression for the packet loss probability. A packet is lost (blocked) only if a new packet arrives when the queue is full and, on the same time slot, transmission of the packet on the radio interface failed. Note that these two events are independent because the event of "transmit or not" for the current packet on the radio interface, does not impact the current size of the queue, but only the one for the next time slot. This amounts to considering that a packet coming at time slot t, is rejected at the end of time slot t, the packet of the radio interface having not been successfully transmitted. We consider the stationary regime of the queue and then, the fraction of lost packets, Φ, can be expressed as follows:

$$\Phi(p) = [1 - f(p)]\Pi_K(p). \tag{1.22}$$

Thus the average data transmission rate is $q[1 - \Phi(p)]R$. Now, let us consider the cost of transmitting. For each packet successfully transmitted, there have been $\frac{1}{f(p)}$ attempts on an average [32]. $f(p)$ typically depends on the system and for example, in [32] $f(p) = 1 - \exp(-kp)$, where k is a constant. For each time slot, irrespective of whether transmissions occur, we assume that the transmitter consumes energy. A simple model which allows one to relate the radiated power to the total device consumed power is provided in [18] (see also [5] is given by $p_{\text{device}} = ap + b$, where $a \geq 0, b \geq 0$ are some parameters; b precisely represents the consumed power when the transmit power is zero. The average power consumption is in our case $b + \frac{pq(1-\Phi)}{f(p)}$ (we assume without loss of generality that $a = 1$). We are now able to define the energy-efficiency metric $\eta(p)$ as the ratio between the average net data transmission rate and the average power consumption, which gives:

$$\eta(p) = \frac{q[1 - \Phi(p)]R}{b + \frac{pq[1-\Phi(p)]}{f(p)}}. \tag{1.23}$$

The above expression shows that the cross-layer design approach of power control is fully relevant when the transmitter has a cost which is independent on the radiated power; otherwise (when $b = 0$), one falls into the original framework of [32].

In this part, we prove that there exists a unique power where the energy efficiency function is maximized when the transmission rate is a sigmoidal or "S"-shaped function of p. In [26], it was shown that having a sigmoidal success rate $f(p)$ implies quasi-concavity and a unique maximum for $\frac{f(p)}{p}$. This assumption was shown to be highly relevant from a practical viewpoint in [32] as well as from an information theoretical viewpoint in [4].

Theorem 1.1. *The energy efficiency function η_{n_t} is quasi-concave with respective to p and has a unique maximum denoted by $\eta_{n_t}(p^*)$ if the efficiency function $f(p)$ has a sigmoidal shape.*

The proof of this is provided in [29]. We are then able to determine the optimal power p^* which maximize the energy efficiency function, by solving the following equation:

$$0 = \frac{-\mathrm{d}\Phi}{\mathrm{d}p}\{b + \frac{pq(1-\Phi)}{f(p)}\} + (1-\Phi)\{\frac{\mathrm{d}\Phi}{\mathrm{d}p}\frac{p}{f(p)} + \frac{\mathrm{d}(p/f(p))}{\mathrm{d}p}\}. \tag{1.24}$$

In Fig. 1.11, we study the energy efficiency of a system with $\frac{b}{\sigma^2} = 100$. Here we see that as q decreases p^* decreases. Also seen from the same figure is the quasi-concavity of the energy efficiency function and the asymptotic behavior.

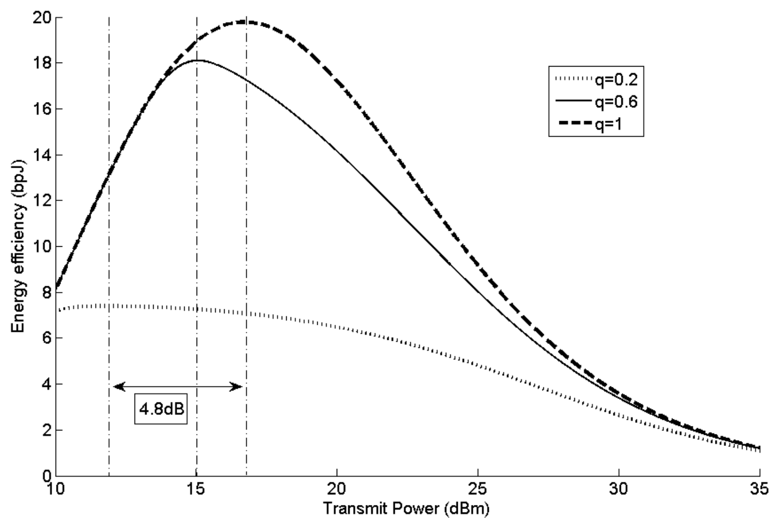

FIGURE 1.11
η_{n_t} in bits per Joule (bpJ)vs p of a system with $\frac{b}{\sigma^2} = 100$ (20dB). Observe that the function is quasi-concave for all q and that p^* decreases as q decreases.

1.3 On the design of energy-efficient MIMO multi-user communications

The focus of this section will be on the more practical case of multi-user communications. Furthermore, we will consider distributed multi-user networks, i.e., networks that operate with little or no intervention from a central authority. Other advantages of having a distributed system are the Independence of each user or base station, non-reliance on cooperation, etc.

In reality, there are several factors and layers to be considered while studying the energy efficiency of a network like the MAC layer consisting of random arrival and departure of users, the arrival of packets from each user based on the protocol etc and the physical layer dealing with channel estimation, coding and finally transmission and reception from multiple base stations and antennas. However, research is still in progress in this field and the results presented here are limited to physical layer systems with uniform power allocation and perfect CSI.

1.3.1 A discussion on the performance criterion

In multi-user networks, it is not straightforward to define an energy-efficiency metric. Several different possible notions arise: (i) the ratio of the total utility over the total cost; (ii) the minimal individual energy-efficiency; (iii) individual user energy-efficiency. The first two goals are rather social ones applying to a centralized scenario, whereas the last one applies to distributed multi-user networks where each user tries to optimize its own energy-efficiency. In a network, there are primarily two modes of information transfer i.e the uplink and the downlink. The uplink case is when mobile terminals send information to the corresponding base station and the downlink case is when the base stations send information to the mobile terminals.

For the downlink case, the base station tries to optimize the global efficiency. This problem has been analysed in several works. In [10], the authors consider energy-efficient transmission schemes in cooperative cellular systems with unbalanced traffic between uplink and downlink, and derive the optimal transmission data rate, which minimizes the total energy consumption of battery powered terminals per bit of information. In [11], the quality of service (QoS) constrained radio resource allocation problem at the downlink of multi-user multi-carrier systems is studied based on the trade-off between energy consumption and transmit power within a cross physical and link layer system model, jointly considering power allocation, adaptive modulation and coding and ARQ/HARQ retransmission protocols.

For the uplink, one of the key issues in wireless system design is energy consumption at users terminals. Since in many scenarios, the users' terminals are battery-powered, efficient energy management schemes are required in order to prolong the battery life. Hence, power control plays an even more crucial role in such systems. For the uplink case, for every user, the purpose of power control is to transmit at the optimal power to achieve the required quality of service (QoS) at the uplink receiver without causing unnecessary interference to other users in the system. This motivates the application of game theory, and in this section, we will focus on the uplink case and present some of the novel results in this field.

For the SISO case, several works have analysed this problem. In [32], the authors study power control algorithms for the multi-user distributed case using a non-cooperative game theoretical framework and propose algorithms that achieve the Nash equilibrium. A Nash equilibrium is defined as a set of strategy choices and the corresponding payoffs in which each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged. The same authors, in [13], propose a pricing scheme for transmit power levels to improve the efficiency of the system, as the Nash equilibrium is not the most optimal scenario in terms of the sum of the utilities.

In the next section we present the results available for the power control game in the case of multi-carrier systems based on [37]. These results can also be interpreted as a power control game for a diagonal MIMO system.

1.3.2 Power control in multi-carrier CDMA systems

As explained before, due to the competitive nature of the users' interaction, the natural framework for modelling and studying a power control problem in CDMA systems is game theory. Consider a non-cooperative power control game, in which each user seeks to maximize its overall utility by choosing the optimal transmit power over each carrier. The utility function is defined as the ratio of any user's total throughput to its total transmits power over all the carriers. This utility function has units of bits/Joule as before and is suitable for applications where saving power (preserving battery life) is critical. The non-cooperative nature of the proposed game implies that no coordination among the users exists. There are two difficulties to the problem studied in section, being that, firstly, users' strategies in the multi-carrier case are vectors (rather than scalars) and this leads to an exponentially larger strategy set for each user. Secondly, the utility function considered here is not quasi-concave. This means that many of the standard theorems from game theory as well as convex optimization cannot be applied here.

In this section, the Nash equilibrium for the proposed power control game is derived and its existence and uniqueness are studied. Some of the questions answered in this section are the following. The existence of a Nash equilibrium, and the possibility of users reaching it, the kind of carrier allocations and the spread of usage in the carriers at a Nash equilibrium,and the performance of this joint maximization of utility over all the carriers compared with that of an approach where utility is maximized independently over each carrier are some of issues studied in [37] and are presented here.

Consider the uplink of a synchronous multi-carrier DS-CDMA data network with N users, M carriers and processing gain L (for each carrier). The carriers are assumed to be sufficiently far apart so that the (spread-spectrum) signal transmitted over each carrier does not interfere with the signals transmitted over other carriers [11]. We also assume that the delay spread and Doppler spread are negligible for each individual carrier. At the transmitter, the incoming bits for user n are divided into M parallel streams and each stream is spread using the spreading code of user n. The M parallel streams are then sent over the M (orthogonal) carriers. For the m-th carrier, the received signal at the uplink receiver (after chip-matched filtering and sampling) can be represented by an $L \times 1$ vector as

$$\underline{y}_m = \Sigma_{n=1}^{N} \sqrt{P_{n,m} h_{n,m}} x_n + \underline{z}_m \tag{1.25}$$

Where b_n, P_n, h_n are the n-th user transmitted bit, transmit power and path gain, respectively, for the m-th frequency channel (carrier); x_n is the spreading sequence for user n which is assumed to be random with unit norm; and $\underline{z}_m$ is the noise vector each element of which is assumed to be Gaussian with mean 0 and covariance σ^2.

We study the non-cooperative game in which each user chooses their transmit powers over the D carriers to maximize its overall utility. In other words, each user (selfishly) decides how much power to transmit over each frequency channel (carrier) to achieve the highest overall utility. Let $G_M = [\mathcal{N}, \{A_n\}, \{\eta_n\}]$ denote the proposed non-cooperative game where $\mathcal{N} = \{1, ..., N\}$, and $A_n = [0, P]^M$ is the strategy set for the n-th user. Here, P is the maximum transmit power on each carrier. Each strategy in A_n can be written as $\underline{p}_n = [p_{n,1}, ..., p_{n,M}]$. The utility function (the energy efficiency) for user n is defined as the ratio of the total throughput to the total transmit power for the M carriers, i.e

$$\eta_n(\underline{p}_1, \ldots, \underline{p}_N) = R \frac{\Sigma_{m=1}^{M} f(\gamma_{n,m})}{\Sigma_{m=1}^{M} p_{n,m}} \tag{1.26}$$

Where R is the target rate (assumed to be the same for all users without any loss in generality), $f()$ is the success rate, $\gamma_{n,m}$ is the SINR of user n on carrier m, hence, the resulting non-cooperative game can be expressed as the following maximization problem:

$$\max_{p_{n,1}, \ldots, p_{n,M}} \frac{\Sigma_{m=1}^{M} f(\gamma_{n,m})}{\Sigma_{m=1}^{M} p_{n,m}} \tag{1.27}$$

The relationship between $\gamma_{n,m}$ and $p_{n,m}$ is dependent on the uplink receiver. It should be noted that the assumption of equal transmission rates for all users can be made less restrictive. For our analysis, it is sufficient for the users to have equal transmission rates over different carriers but the transmission rate can be different for different users. More generally, the proposed power control game can be extended to allow the users to pick not only their transmit powers but also their transmission rates over the D carriers. While joint power and rate control is important, particularly for data applications, our focus throughout this work is on power control only.

For the non-cooperative power control game, a Nash equilibrium (NE) is a set of power vectors, $(p_1, \ldots, p_N)$, such that no user can unilaterally improve its utility by choosing a different power vector, i.e., $(p_1^*, \ldots, p_N^*)$ is a Nash equilibrium if and only if

$$\eta_n(p_n^*, p_{-n}^*) \geq \eta_n(p_n, p_{-n}^*) \forall n \tag{1.28}$$

Here, p^*_{-n} denotes the set of transmit power vectors of all the users except for user n at the NE. This game is particularly difficult to analyse because users' strategies are vectors (rather than scalars) and the utility function is not a quasi-concave function of the user's strategy. For this utility function, it is shown in [37] that at a Nash equilibrium each user transmits only on the carrier that has the best effective channel for that user. Additionally, the conditions required for the existence and uniqueness of the NE are also detailed.

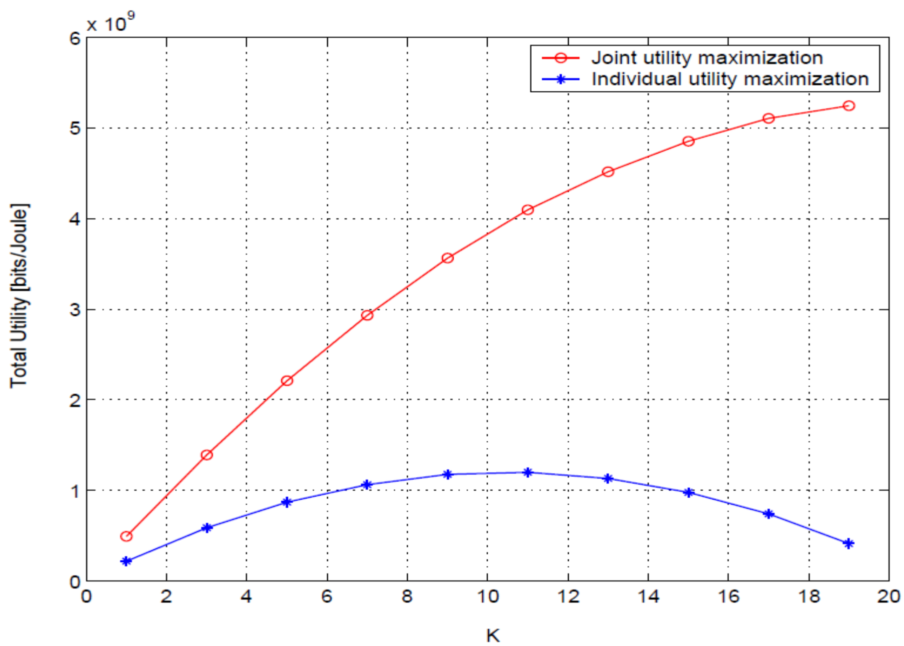

FIGURE 1.12
Energy efficiency at NE (η^*) v.s number of users (N) for a system with 2 carriers using matched filter.

Remark: At this point, it is interesting to note that maximizing the spectral efficiency or the rate will lead to a water filling solution over all the carriers [40]. However, we find that when optimizing energy-efficiency, the best strategy is to transmit with all the power on the best carrier [37]. This means that while deciding on a transmission strategy, a choice has to be made on what to maximize.

1.3.3 Non-cooperative resource allocation in multi-user MIMO systems

Finally in this section we study a power control game for a MIMO system where the power allocation is chosen by each player trying to maximize their energy-efficiency metric. This section is based on [43].

In this subsection, we present some results on the uplink of a multi-user MIMO communication system, wherein both the mobile terminals and the common access point (AP) are equipped with multiple antennas. We are interested in the design of non-cooperative resource allocation policies aiming at energy-efficiency maximization, which is defined here as the number of reliably delivered information symbols per unit of consumed energy from the battery. Energy-efficiency maximization is indeed a crucial problem in mobile wireless communications, wherein mobile users are interested in making a careful and smart use of the energy stored in their battery. This section is an extension of [37] to multi-user MIMO wireless systems. We consider three problems depending on the energy-efficiency optimization variables:

1. The transmit power of each user, assuming matched filtering at the receiver;
2. The transmit power and the choice of the uplink linear receiver for each user;
3. The transmit power, the beam forming vector and the choice of the uplink linear receiver for each user;

Consider the uplink of a N-user synchronous, single-cell, MIMO multi-user flat fading channel. Denote by n_t the number of transmit antennas for each user, and by n_r the number of receive antennas. The received signal can be written as

$$\underline{y} = \sqrt{p_n}\mathbf{H}_n\mathbf{a}_n x_n + \underline{z} \tag{1.29}$$

Where $\mathbf{H}_k$ is the $n_r \times n_t$ channel matrix between the receiver and the nth user, $\mathbf{a}_k$ is the beamforming vector of the n-th user satisfying $\mathbf{a}_n^T\mathbf{a}_n = 1$ and s_k is the symbol transmitted, and p_n is the transmission power of user n.

The energy efficiency function, which is also the utility considered while using a game theoretic approach can be defined for the n-th user as

$$\eta_n = R\frac{f(\gamma_n)}{p_n} \tag{1.30}$$

Where γ_n is the SINR of user n which depends on p_n,$\mathbf{a}_k$ and the receiver used.

1.3.3.1 Optimizing transmit power with a matched filter

For a linear receiver, the SINR can be written as

$$\gamma_n = \frac{p_n(\mathbf{d}_n\mathbf{H}_n\mathbf{a}_n)^2}{\Sigma_{j\neq n}p_j(\mathbf{d}_j\mathbf{H}_j\mathbf{a}_j)^2 + p_n(\mathbf{d}_n\mathbf{H}_n\mathbf{a}_n)^2} \tag{1.31}$$

if the linear receiver detects symbols according to $\mathbf{d}_k^T\mathbf{y}$ where $\mathbf{d}_k$ is the L dimensional vector representing the receive filter.

Consider the non-cooperative game in which each user chooses its power so that their utility is maximized. The game can be expressed mathematically as:

$$\max_{p_n \in [0,P]} \frac{f(\gamma_n)}{p_n}, \text{ for } n = 1, \ldots, N \tag{1.32}$$

where P is the maximum allowed transmit power for the n-th user, it is proved in [43] that the non-cooperative game defined in 1.32 admits a unique NE point p_n^* for all n which satisfies $f(\gamma_n) = \gamma f'(\gamma_n)$, p_n^* being the power at which γ_n^* is achieved.

1.3.3.2 Optimizing the choice of linear receiver

Now consider the following non-cooperative game where each user chooses its transmit power and its linear receiver competing for the best utility defined mathematically as follows:

$$\max_{p_n \in [0,P], \mathbf{d}_n \in \mathbb{R}^{n_r}} \frac{f(\gamma_n)}{P_n}, \text{ for } n = 1, \ldots, N \tag{1.33}$$

We do this by at first finding the optimal receiver and then finding the power. As it is known that among linear receivers the MMSE receiver maximizes the SINR, and so, in [43] it is shown that the game described in 1.33 has a unique NE $(P_n^*, \mathbf{d}_n^*)$.

$$\mathbf{d}_p^* = \sqrt{p_n}\mathbf{F}^{-1}\mathbf{H}_n\mathbf{s}_n \tag{1.34}$$

with $\mathbf{F} = I_{n_t}\sigma^2 + \Sigma_{j=1}^N p_j \mathbf{H}_j \mathbf{s}_j \mathbf{s}_j^T \mathbf{H}^T$ and $p_n^* = \min(\bar{P}_n, P)$, such that the n-th user has a SINR of γ_k^* as in the previous section.

1.3.3.3 Optimizing the beamforming vector

Now we present the most general and challenging optimization considered in this chapter performed on a multi-user MIMO system. Here, the users can chose their total transmit power, their linear receiver as well as their beamforming vector. The game that describes this process can be written down as

$$\max_{p_n \in [0,P], \mathbf{d}_k \in \mathbb{R}^N, \mathbf{a}_n \in \mathbb{R}^N} \frac{f(\gamma_n)}{p_n}, \text{ for } n = 1, \ldots, N \tag{1.35}$$

Given the above equation, we have to consider first the problem of SINR maximization with respect to the vectors $\mathbf{d}_n$ and $\mathbf{a}_n$. Again, the SINR-maximizing linear receiver is the MMSE receiver. The n-th user SINR for MMSE detection is already known and the vector that maximizes this for the MMSE receiver is the eigenvector corresponding to the maximum eigen value of $\mathbf{H}_n^T\mathbf{F}^{-1}\mathbf{H}_n$. Therefore the optimal strategy for each player would be to cyclically update their beam forming vectors satisfying the afore mentioned criteria. The following result holds, from [43] (the proof can be found in the paper).

Theorem 1.2. *Assume that the active users cyclically update their beam forming vectors in order to maximize their own achieved SINR at the output of a linear MMSE receiver. This procedure converges to a fixed point.*

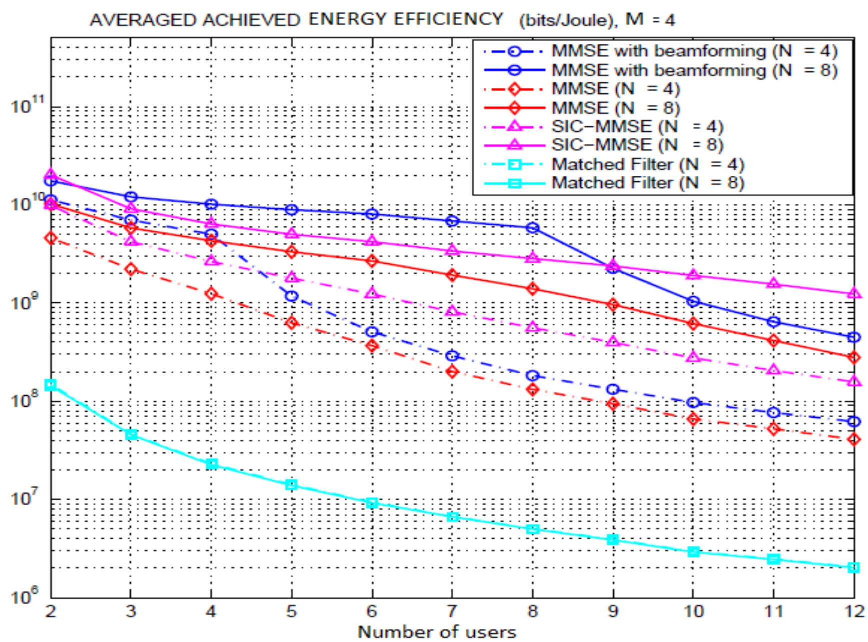

FIGURE 1.13
Energy efficiency at NE (η^*) v.s number of users (N)

Figure 1.13 (taken from [43]) show the achieved average utility (energy efficiency) at the receiver output versus the number of users, for the considered games, and for a 4 and 4×8 MIMO system. Inspecting the curves, it can be observed that a smart resource allocation algorithm brings very remarkable performance improvements.

1.4 Concluding remarks and open issues

Having studied the energy efficiency in all kinds of situations, it is clear that using zero power is not always optimal. The fraction of power to be used depends on various factors like the number of antennas, the fixed power consumption.

The available results w.r.t. the energy-efficiency of single-user channels are the following:

1. There exists a unique optimal transmit power that achieves the maximum energy-efficiency.

2. There is an optimal number of antennas to be used based on the coherence time.

3. When considering a cross layer approach, the optimal power to use is smaller than when transmission occurs the time, depending on the packet arrival rate.

For the multi-user networks the results are:

1. In a multi-user multi-carrier SISO or diagonal MIMO system, users maximizing their energy efficiency will converge under specific conditions to an equilibrium where they transmit on their best "effective" carrier.

2. In a multi-user MIMO power allocation game, if users cyclically update their beamforming vectors to maximize their own energy efficiency, this process converges to a fixed point.

We have seen that here are several unsolved and open problems left in this field:

1. Proof of the quasi-concavity of the energy-efficiency function for a MIMO system where an uniform power allocation policy is assumed.

2. Finding the pre-coding matrix that optimizes the energy-efficiency metric for the general MIMO case.

3. Considering the power consumption of each individual antenna due to additional energy costs of coding, infrastructure.

4. The energy-efficiency study of multi-user MIMO networks with imperfect CSI and finite coherence time.

5. Considering multi-user cross layer models for the energy-efficiency.

1.5 Glossary

SISO: single-input single-output

MIMO: multiple-input multiple-output

CSIT: channel state information at the transmitter

CDIT: channel distribution information at the transmitter

PA: power allocation

RF: radio frequency

SNR: signal-to-noise ratio

SINR: signal-to-interference plus noise ratio

CDMA: code division multiple access

BER: bit error rate

FSK: frequency shift keying

bpcu: bits per channel use

NE: Nash equilibrium

OFDMA: orthogonal frequency-division multiple access

STBC: space-time block coding

AWGN: additive white Gaussian noise

UPA: uniform power allocation

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