Resource Allocation and User Scheduling in Coordinated Multicell MIMO Systems

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1. Introduction

Multiple-input multiple-output (MIMO) systems have become one of the most studied topics in the field of wireless communications because of the well-known potential for increasing spectral efficiency when compared to single-antenna systems. However, in the high interference regime, like in cellular systems, spatial multiplexing MIMO techniques can lose much of their effectiveness. Recently, some techniques have been developed in order to reduce intercell interference in MIMO systems and interference reduction based on base station cooperation seems to be a promising one.

When the multiple base stations can fully cooperate, the multicell downlink system reduces a classical MIMO broadcast channel with per-antenna power constraints. In this case, the optimal strategy to maximize the multicell throughput is the joint dirty-paper coding (DPC) (Caire & Shamai (2003)). Since practical implementation of DPC is still a problem, some sub-optimal solutions have been proposed and some works can be found in the literature (see Andrews et al. (2007) and references therein).

One of the basic requirements for most of the proposed base station cooperation schemes is the need of perfect channel state information at both receivers and network backbone for the joint processing at the central station. Therefore, there is a need of a two-step feedback/training from user terminals to each base station and then from base stations to the backbone. In the end, channel knowledge at the backbone is typically imperfect due to delays in feedback link, imperfect training sequence and etc. Besides, the channel learning at the central station requires a great amount of overhead, which is not desired nor recommended. In the work of Marsch & Fettweis (2008) the authors provide a comprehensive study of the multicell downlink optimization with limited backbone capacity and they analyze the problem of finding the optimal power allocation and beamforming matrices for different scenarios. Since the statistics are expected to change only in long-term basis, Kobayashi et al. (2009) proposed a power allocation strategy based on partial channel state information at the central station, which requires a very small additional backbone capacity.

The power allocation minimizing outage probability strategy for single-carrier with partial channel state information at transmitter for a cooperative downlink transmission system was proposed by Kobayashi et al. (2009). However, it was observed that the solution applied for a single-carrier condition cannot be directly applied to the multicarrier case because the frequency diversity gains were inferior to the loss due to the division of power among the carriers. Hence, Souza et al. (2009b) proposed a power allocation strategy that minimizes the outage probability based on the knowledge of channel statistics for a multicarrier system. The multicarrier power allocation strategy that was proposed exploits the multiplexing gain of cooperative MIMO and the frequency diversity gain provided by the multicarrier transmission scheme.

User scheduling in coordinated multicell MIMO systems is also a problem of a paramount importance. We consider the relevant case of a large number of user terminals and propose a simple scheduling scheme called Distributed Diversity Scheduling (DDS) which efficiently chooses a subset user terminals while limiting the amount of the backhaul communication. More precisely, each base station with local channel state information chooses its best set of user terminals over a predefined partition and reports the corresponding index and value to the central station. The central station then decides and informs the selected set to all base stations. Finally, the selected user terminals are served exactly in the same manner as the previous case of small number of terminals. It is proved that the scheduling algorithm scales optimally in the number of base stations, user terminals and transmit antennas per base station.

Within this context, this chapter aims at presenting some recent advances on adaptive resource allocation strategies for cooperative MIMO-OFDM networks. Such strategies allocate resources as a function of the time-varying channel state information and QoS parameters. In this chapter, we will present our main contributions that were obtained during the last years, indicating some research trends and future directions. Power allocation strategies are presented for single-carrier and multiple carrier networks according to the knowledge of the central station with respect to the wireless channels between the base stations and the terminals (perfect, partial and no channel state information). Additionally, a user scheduling scheme is presented for the case where all user terminals cannot be served at the same time. The so-called Distributed Diversity Scheduling (DDS) scheme selects a subset of user terminals and achieves a diversity gain that scales optimally with the number of cooperative base stations as well as user terminals, even under limited backhaul capacity.

2. Model of multicell MIMO systems

In this section, the multicell MIMO downlink model, where base stations are connected to a central station and communicate simultaneously with K single-antenna user terminals, is presented. It is assumed that the base stations are connected to a common backbone via a possibly error-free wired line, which enables some cooperation between base stations. Furthermore, the base stations do not communicate directly to each other and each one is equipped with M antennas. It is also assumed that each base station knows perfectly the channels while the central station may have perfect, partial or even no channel state information (this will be discussed with more details later on). Figure 1 illustrates an example of the multicell downlink system.

The model assumes multi-carrier transmission with Orthogonal Frequency Division Multiplexing (OFDM) in the wireless channels between the base stations and the user terminals. For each carrier n, these frequency flat fading channels are mutually independent and distributed as $\mathbf{h}_{bk}[n] \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2_{bk}[n]\mathbf{I}_M)$. For an OFDM system model with N carriers, let $\{\mathbf{g}_{bk}[n]\}$ be the precoding vectors, $\{s_{bk}[n]\}$ be the transmit symbols and $\{p_{bk}[n]\}$ be the transmit powers from base station b to user terminal k at the n-th carrier. Base station b forms

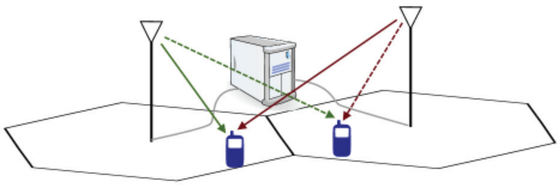

Fig. 1. System model.

its transmit vector at the n-th carrier as follows:

$$\mathbf{x}_b[n] = \sum_{k=1}^{K} \sqrt{p_{bk}[n]}\mathbf{g}_{bk}[n]s_{bk}[n], \tag{1}$$

and is subject to the power constraint $\sum_k \sum_n p_{bk}[n] \leq P_b$, where P_b is maximum transmit power.

If a distributed zero-forcing beamforming scheme is applied for a small number of terminals ($K \leq M$), then it can be shown that the model is equivalent to a system with K parallel MISO channels. In this case, the k-th user terminal achieves a diversity gain of $B(M - K + 1)$. Let $n_k[n]$ be the equivalent zero-mean white Gaussian noise, then the received signal at the n-th carrier is given by:

$$y_k[n] = \sum_{b=1}^{B} \sqrt{p_{bk}[n]}a_{bk}[n]s_{bk}[n] + n_k[n], \tag{2}$$

where $a_{bk}[n] = \mathbf{h}_{bk}^H[n]\mathbf{g}_{bk}[n]$ represents the channel gain between base station b and terminal k at carrier n. The unitary precoding vector $\mathbf{g}_{bk}[n]$ is orthogonal to $\mathbf{h}_{bj}[n]$ for $j \neq k$ and the random variable $|a_{bk}[n]|^2$ is chi-squared distributed with $2(M - K + 1)$ degrees of freedom.

Assuming that each user terminal k perfectly knows the channel state $\mathbf{a}_n^k = (a_{1k}[n], \ldots, a_{Bk}[n])$, it decodes the space-time code and achieves the following rate:

$$R_k = \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \sum_{b=1}^{B} |a_{bk}[n]|^2 \, p_{bk}[n]\right). \tag{3}$$

The capacity region of the K parallel MISO channels in Equation (2) for a fixed set of power $\mathbf{p}_n^k = (p_{1k}[n], \ldots, p_{Bk}[n])$ and channel state $\mathbf{a}_n^k = (a_{1k}[n], \ldots, a_{Bk}[n])$ for all k and n is given by:

$$\mathcal{R}(\mathbf{a}; \mathbf{p}) = \left\{ \mathbf{R} \in \mathbb{R}_+^K : R_k \leq \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \sum_{b=1}^{B} |a_{bk}[n]|^2 \, p_{bk}[n]\right) \forall k \right\}, \tag{4}$$

where $\mathbf{a} = \{\mathbf{a}_n^k\}$ and $\mathbf{p} = \{\mathbf{p}_n^k\}$ for notation simplicity. The above region is convex (rectangular for $K = 2$ and $N = 1$). Let $\mathcal{P}$ denote a power allocation policy $\mathbf{a} \mapsto \mathbf{p}$ that maps the channel state $\mathbf{a}$ into the power vector $\mathbf{p}$ with component $\mathcal{P}_{bk}(\mathbf{a}) = p_{bk}$ and $\mathcal{F}$ denote the feasibility set satisfying $\sum_k \sum_n \mathcal{P}_{bk}(\mathbf{a}) \leq P_b, \forall b$ for any channel realization $\mathbf{a}$. Then, the capacity region of the K parallel MISO channels (2) under the individual base station power constraints $\mathbf{P} = (P_1, \ldots, P_B)$ for a fixed channel state $\mathbf{a}$ is given by:

$$\mathcal{C}(\mathbf{a}; \mathbf{p}) = \bigcup_{\mathcal{P} \in \mathcal{F}} \mathcal{R}(\mathbf{a}; \mathbf{p}), \tag{5}$$

The capacity region (5) is convex and its boundary can be explicitly characterized by solving the weighted sum rate maximization as specified in the next section.

3. Resource allocation strategies for single carrier systems

The purpose of the resource allocation problems is to optimize the power distribution over the carriers of all user terminals for a given target rate tuple γ. According to the system model that was presented in the previous section, equations (3) and (4) provide generic expressions for the rate achieved by user terminal k and capacity region of the parallel MISO channels rate, respectively. For the special case of single carrier networks, these equations reduce to:

$$R_k = \log\left(1 + \sum_{b=1}^{B} |a_{bk}|^2 \, p_{bk}\right), \tag{6}$$

$$\mathcal{R}(\mathbf{a};\mathbf{p}) = \left\{\mathbf{R} \in \mathbb{R}_+^K : R_k \le \log\left(1 + \sum_{b=1}^{B} |a_{bk}|^2 \, p_{bk}\right) \forall k\right\}. \tag{7}$$

The next subsections present resource allocation strategies for single carrier systems. The strategies depend on the assumptions regarding the channel state information that is available at the transmitters. Strategies for three different assumptions will be presented in this section. First we consider the case of perfect channel state information (CSI) at the central stations. Then, we consider the case of partial channel state information at the central station. Finally, we review the case when the central station has absolutely no channel state information. Each assumption requires a different approach and, at the end, we compare the performance of the resource allocation strategies with respect to the outage probability of the system.

3.1 Perfect channel state information

When perfect (or full) channel state information, i.e. knowledge of all channel realizations is available at the central station, the optimal power allocation is found by an iterative algorithm which is a generalization of the classical waterfilling algorithm (see Lee & Jindal (2007) and Yu et al. (2004)). Ideally, the transmitter could adjust its powers such that the outage of the system is reduced to zero. In this case, the problem is focused on the search for power allocation policies that provide the rate tuple proportional to the target rate tuple (rate-balancing). It is remarked that the zero-outage performance is not achieved when the transmitter is under limited power constraints. This policy equalizes the individual outage probability of all user terminals and thus provides the strict fairness among them. It is worthy to note that fairness is one of the most desired properties.
The objective is to find the set of powers $\{p_{bk}\}$ that satisfy the following condition:

$$\frac{R_k(\mathbf{p})}{R_1(\mathbf{p})} = \frac{\gamma_k}{\gamma_1} \triangleq \alpha_k, \quad k = 2, \dots, K \tag{8}$$

where $\frac{\gamma_k}{\gamma_1} = \alpha_k$ and $\alpha_1 = 1$. More precisely, according to Lee & Jindal (2007) the optimal power allocation is a solution of:

$$\min_{\sum_k \theta_k = 1} \max_{\mathbf{R} \in \mathcal{C}(\mathbf{a},\mathbf{P})} \sum_{k=1}^{K} \theta_k \frac{R_k}{\alpha_k}. \tag{9}$$

Notice that the inner problem for a fixed set $\{\theta_k\}$ is convex since the objective function (weighted sum rate) is concave and the constraints are linear. As discussed by Boyd & Vandenberghe (2004), it is necessary and sufficient to solve the Karush-Kuhn-Tucker (KKT) conditions given by:

$$\frac{\theta_k}{\alpha_k} \frac{|a_{bk}|^2}{1 + \sum_{b=1}^{B} |a_{bk}|^2 p_{bk}} = \frac{1}{\mu_b}, \quad k = 1, \dots, K, \quad b = 1, \dots, B \tag{10}$$

where μ_b denotes the Lagrangian variable to be determined such that $\sum_k p_{bk} \leq P_b$. Although a closed-form solution does not exist, the multiuser waterfilling algorithm for the MIMO multiple access channel proposed by Yu et al. (2004) can be easily modified to solve the KKT conditions (10) iteratively. Then, at each iteration it is found a new set of K powers $(p_{b1}, \ldots, p_{bK})$ related to base station b by treating the powers of the other base stations constant and computing:

$$p_{bk} = \left[\frac{\theta_k \mu_b}{\alpha_k} - \frac{1 + \sum_{j \neq b} |a_{jk}|^2 p_{jk}}{|a_{bk}|^2} \right]_+ , \quad \forall k. \tag{11}$$

The outer problem consists of minimizing the solution of the inner problem with respect to $\theta_2, \ldots, \theta_K$. Since the problem at hand is convex, a subgradient method can be suitably applied as suggested by Bertsekas (1999). The overall algorithm implements the rate-balancing by allocating the rates proportional to the target rate tuple and it is summarized as follows.

Algorithm 1 Resource allocation for single carrier systems with perfect CSI

1: Initialize $\theta_k \in [0, 1]$ for $k > 1$.
2: **repeat**
3: for the fixed set of weights θ_k calculate via waterfilling approach (11):

$$\arg \max_{\mathbf{R} \in \mathcal{C}(\mathbf{a}, \mathbf{P})} \sum_{k=1}^{K} \theta_k \frac{R_k}{\alpha_k} \tag{12}$$

4: Calculate new subgradients
5: Update the weights θ_k
6: **until** convergence of (9)

3.2 Partial channel state information

For the special case of partial (or statistical) channel state information at the central station, the optimal power allocation is proposed by Kobayashi et al. (2009) and is the solution of the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & P_{\text{out}}(\gamma, \mathbf{p}) = 1 - \prod_{k=1}^{K} \Pr\left(R_k > \gamma_k\right) \\ \text{subject to} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} p_{bk}[n] \leq P_b. \end{aligned} \tag{13}$$

The expression for the outage probability of the system for a fixed power allocation $\mathbf{p}$ is given by:

$$P_{\text{out}}(\gamma, \mathbf{p}) = 1 - \prod_{k=1}^{K} \left[1 - \Pr\left(\Delta_k(\mathbf{p}^k) < c_k \right) \right], \tag{14}$$

where $\Delta_k(\mathbf{p}^k) = \sum_b |a_{bk}|^2 p_{bk}$ and $c_k = 2^{\gamma_k} - 1$. It can be shown that, for a fixed set of powers $\mathbf{p}^k = (p_{1k}, \ldots, p_{Bk})$ of user terminal k, that $\Delta_k(\mathbf{p}^k)$ is a Hermitian quadratic form of a Gaussian random variable and its characteristic function is:

$$\Phi_{\Delta_k(\mathbf{p}^k)}(s) = \prod_{b=1}^{B} \frac{1}{(1 + s\alpha_{bk} p_{bk})^{M-K+1}}, \tag{15}$$

where $\alpha_{bk} = \sigma_{bk}/(M-K+1)$. The widely used upper bound is the Chernoff bound and for fixed powers $\mathbf{p}^k$ it is defined as:

$$\Pr\left(\Delta_k(\mathbf{p}^k) < c_k\right) \leq \min_{\lambda \geq 0} e^{\lambda c_k} \Phi_{\Delta_k(\mathbf{p}^k)}(\lambda) \triangleq \overline{F}_{\Delta_k(\mathbf{p}^k)}(c_k, \mathbf{p}^k). \tag{16}$$

Using the expression (16) of the Chernoff upper bound for each user terminal k, the outage probability of the system for a fixed $\mathbf{p}$ is upper-bounded by:

$$P_{\text{out}}(\gamma, \mathbf{p}) \leq 1 - \prod_{k=1}^{K}\left(1 - \overline{F}(c_k, \mathbf{p}^k)\right). \tag{17}$$

The power optimization based on the exact outage probability does not seem to be feasible and the algorithm proposed by Kobayashi et al. (2009) searches for the power allocation that minimizes the Chernoff upperbound. In that work, the authors conjecture that the proposed algorithm converges to the optimal solution that minimizes the exact outage probability, although there is no formal proof.
Hence, the corresponding optimization problem is formed as follows:

$$\begin{aligned} \text{maximize} \quad & f(\{\lambda_k\}, \{p_{bk}\}) \triangleq \prod_{k=1}^{K}(1 - h_k(\lambda_k, \mathbf{p}^k)) \\ \text{subject to} \quad & \sum_{k=1}^{K} p_{bk} \leq P_b, \quad b = 1, \ldots, B \\ & \lambda_k \geq 0, \;\; k = 1, \ldots, K \\ & p_{bk} \geq 0, \quad b = 1, \ldots, B, \; k = 1, \ldots, K \end{aligned} \tag{18}$$

where the function $h_k(\lambda_k, \mathbf{p}_k)$ is defined as:

$$h_k(\lambda_k, \mathbf{p}^k) = \frac{e^{\lambda_k c_k}}{\prod_{b=1}^{B}(1 + \lambda_k \alpha_{bk} p_{bk})^{M-K+1}}. \tag{19}$$

The solution of (18) is found by a two-step approach that explores the structure of the objective function. It is observed that the maximization of f with respect to $\{\lambda_k\}$ can be decoupled into the minimization of h_k over λ_k for each k, where h_k is convex in λ_k. In addition, since f is concave in $\{p_{bk}\}$, the overall problem is convex.
The first step consists in minimizing the monotonically decreasing function h_k with respect to λ_k. When the transmission powers are fixed, the optimal value of λ_k is given by the solution of the following polynomial of degree B:

$$\frac{c_k}{M-K+1} = \sum_{b=1}^{B} \frac{\alpha_{bk} p_{bk}}{1 + \lambda_k \alpha_{bk} p_{bk}}. \tag{20}$$

The second step is the maximization of the concave function $f(\lambda, \mathbf{p})$ with respect to p_{bk}. The Lagrangian function associated with $f(\lambda, \mathbf{p})$ is formed with the introduction of the Lagrangian multipliers $\{\mu_b\}$ and the following Karush-Kuhn-Tucker (KKT) conditions conditions for $k = 1, \ldots, K$ are obtained:

$$\frac{h_k(\lambda_k, \mathbf{p}_k)}{1 - h_k(\lambda_k, \mathbf{p}_k)} \frac{\alpha_{bk} \lambda_k}{1 + \lambda_k \alpha_{bk} p_{bk}} = \mu_b. \tag{21}$$

Finally, the iterative algorithm which solves the optimization problem (18), i.e. minimizes the Chernoff upperbound, is listed below. Although a formal proof has not been provided yet, it is conjectured that Algorithm 2 converges to its optimal solution. At each iteration, λ_k is determined as a unique solution for all k and a fixed set of powers. Regarding the power iteration, since the objective function (21) is concave in p_{bk} when fixing all other powers, a sequential update of the powers $\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_B, \mathbf{p}_1 \ldots$ shall converge under individual base station power constraints.

Algorithm 2 Resource allocation for single carrier systems with partial CSI

1: Initialize $\mathbf{p}$
2: **repeat**
3: **for** each base station **do**
4: Update λ with the solution of the polynomial (20)
5: Update $\mathbf{p}$ by evaluating the KKT conditions (21)
6: **end for**
7: **until** convergence of $f(\lambda, \mathbf{p})$

3.3 No channel state information

When there is no channel state information at the central station, the strategy is to equally divide the total available power of each base station among all user terminals. Thus, there is no optimization problem here. Assuming that the maximum power of each base station is equal to P and defining $p \triangleq p_{bk} = P/K$, then:

$$\begin{aligned} \Delta_k &= \sum_{b=1}^{B} |a_{bk}|^2 \, p_{bk} \\ &= p \sum_{b=1}^{B} |a_{bk}|^2 \triangleq \Delta_k^e, \end{aligned} \tag{22}$$

where Δ_k^e is a chi-squared random variable with $4(M-K+1)$ degrees of freedom. If it is assumed that $\sigma_{bk} = 1$ for all links, then the cumulative distribution function of Δ_k^e is given by the following expression:

$$F_{\Delta_k^e}(y) = 1 - \exp\left[-\left(\frac{M-K+1}{p}y\right)\right] \sum_{k=0}^{2(M-K+1)-1} \frac{1}{k!}\left(\frac{M-K+1}{p}y\right)^k. \tag{23}$$

Let $c_k = 2^{\gamma_k} - 1$. Under these assumptions, the outage probability of the system is given by:

$$\begin{aligned} P_{\text{out}}(\gamma, \mathbf{p}) &= 1 - \prod_{k=1}^{K} \Pr\left(\log\left(1 + \sum_{b=1}^{B} |a_{bk}|^2 \, p_{bk}\right) > \gamma_k\right) \\ &= 1 - \prod_{k=1}^{K} \Pr\left(p \sum_{b=1}^{B} |a_{bk}|^2 > 2^{\gamma_k} - 1\right) \\ &= 1 - \prod_{k=1}^{K} \Pr\left(\Delta_k^e > c_k\right) \\ &= 1 - \prod_{k=1}^{K} \left(1 - F_{\Delta_k^e}(c_k)\right). \end{aligned} \tag{24}$$

3.4 Performance of resource allocation strategies

Figure 2 shows the outage probability performance versus signal-to-noise ratio for $K = 2$ user terminals and $M \in \{2, 4\}$ antennas. The target rate is fixed to $\gamma = [1, 3]$ bpcu (bits per channel use). The three different power allocation strategies are compared and the baseline case without network MIMO, where each base station sends a message to its corresponding user terminal in a distributed fashion, is also shown. The base station cooperation schemes provides a diversity gain of $2(M - K + 1)$, i.e. 2 and 6 with 2 and 4 antennas, respectively. These gains are twice as large as the case without network MIMO. Moreover, the schemes provide a additional power gains compared to equal power allocation.

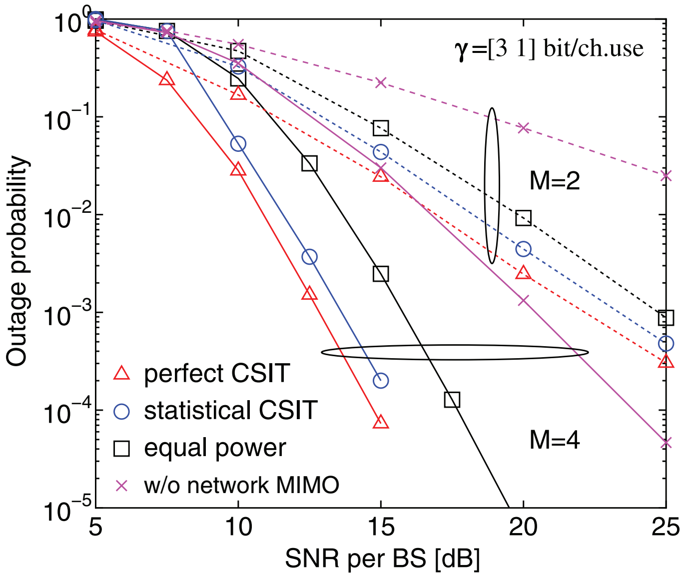

Fig. 2. Outage probability versus signal-to-noise ratio for $M \in \{2, 4\}$ antennas per base station.

In Figure 3, it is plotted the individual outage probability under the same setting as Figure 2 only for $M = 2$. Assuming perfect channel state information at the central station, the proposed waterfilling allocation algorithm guarantees identical outage probability for both user terminals by offering the strict fairness. Under partial channel state information, the algorithm provides a better outage probability to user terminal 1 but keeps the gap between two user terminals smaller than the equal power allocation.

In real networks, there is a need to identify the best situations for the use of coordinated multicell MIMO. In order to identify the situations where coordinated transmission provides higher gains, a simulation campaign similar to the one done by Souza et al. (2009a) was configured. The basic simulation scenario consists of two cells, which contain a two-antenna base station each. Single-antenna user terminals are uniformly distributed in the cells. At each simulation step, the base stations transmit the signal to two randomly chosen user terminals, one terminal at each cell. The channel model that was adopted in these simulations is based on the sum-of-rays concept and it is described by IST-WINNER II (2007).

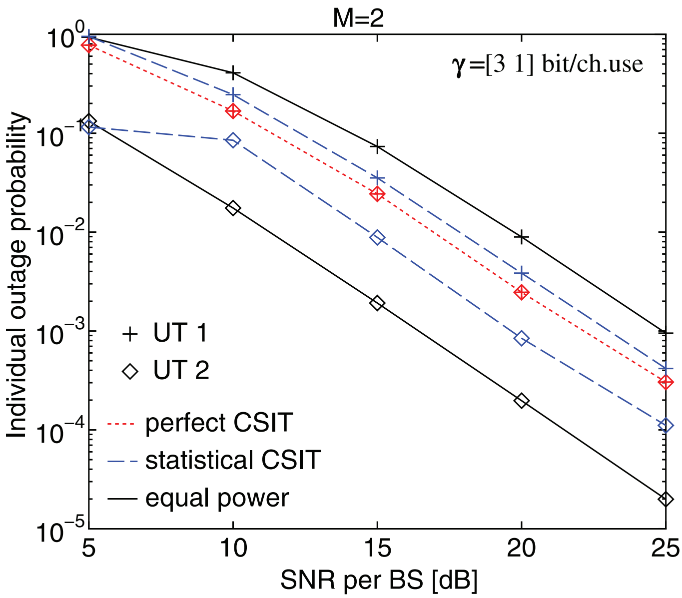

Fig. 3. Individual outage probabilities of each user terminal (UT) vs. SNR with $B = K = 2$ and $M = 2$.

Basically, the system has two transmit modes. In normal mode, each base station transmits to only one user terminal by performing spatial multiplexing. In coordinated mode, signal is transmitted according to the model that is described in Section 2. Let r_c be the cell radius. The transmit mode of the system is chosen by the function $r : [0, 1] \to \mathbb{R}^+$ which is given by the following expression:

$$r(\xi) = (1 - \xi)r_c. \tag{25}$$

The system operates in coordinated mode if and only if the chosen user terminals are inside the shadowed area of Figure 4; otherwise, the system operates in normal mode. The size of the shadowed area is controlled by the variable ξ in equation (25): if $\xi = 0$ the system operates in normal mode; if $\xi = 1$ the system operates in coordinated mode regardless the position of the user terminals; for other values of ξ it is possible to control the size of the shadowed area. Hence, the coordinated transmit mode may be enabled for the user terminals that are on the cell edges and, consequently, the normal transmit mode is enabled for the user terminals that are in the inner part of the cells.
Figure 5 shows the performance of the system when $\gamma = [1, 1]$ bpcu for given values of ξ. It is observed that the system performs best when $\xi = 1$, because under this configuration the coordinated mode provides more significant gains for all user terminals. In addition, it is seen that the gains of the coordinated mode are not significant when transmit powers are low. Under this power conditions, it is better for the system to operate in normal transmit mode because it would reduce the load of the feedback channels and signaling between the central and the base stations. The coordinated transmit mode outperforms the normal mode only when transmit powers are higher.
The distance between base stations and user terminals impacts the performance of the system and this is shown in Figure 6. The results in this figure refer to normal and coordinated

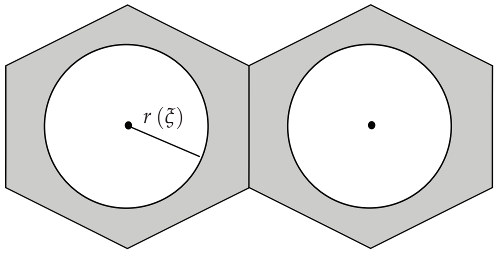

Fig. 4. Simulation scenario.

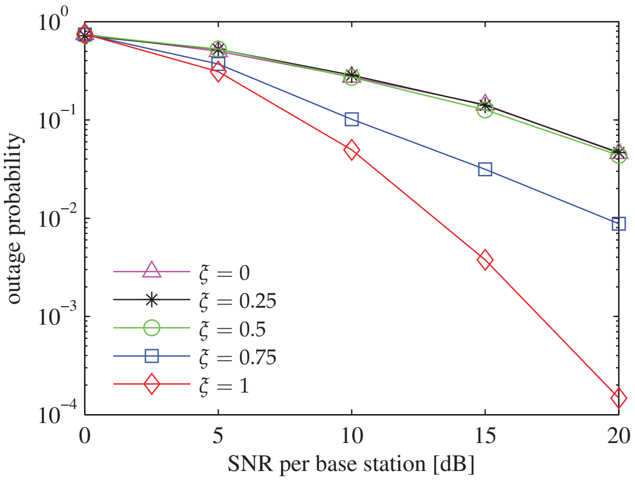

Fig. 5. Outage probability versus SNR per base station for $\gamma = [1, 1]$ bpcu

transmit modes for three given value of base stations' transmit powers. In all cases it is seen that gains of the coordinated mode decrease when distances increase. It is evident that the user terminals which are in the cell edges and experience bad propagation conditions cannot squeeze similar gains from the coordinated mode as the user terminals which are in the inner area of the cells.

For example, if the user terminals of the communication system are required to operate at a fixed outage probability of 10^{-3}, the results such as the ones in Figure 6 may provide systems' administrators with insights into the choice of the transmit mode and transmit powers of each base stations. In this example, if the system operates in normal transmit mode, signal-to-noise ratio would have to be equal or greater than 20 dB for the system to provide the performance which is required by the user terminals and this would be achieved only for distances less than 350 meters. However, the coordinated mode allows the system to serve the same set of user terminals in lower signal-to-noise ratio (around 10 dB in this case). On the other hand, if the base stations transmit with the same power and the system operates in coordinated mode, then it would be possible to serve all terminals with this required outage probability value.

Figure 7 shows the outage probability maps of the simulation scenario for the case $\xi = 1$. The base stations are positioned in $(x_1, y_1) = (750, 750)$m and in $(x_2, y_2) = (2250, 750)$m and transmit power of each base station is 10 dB. The blue squares indicate the areas where

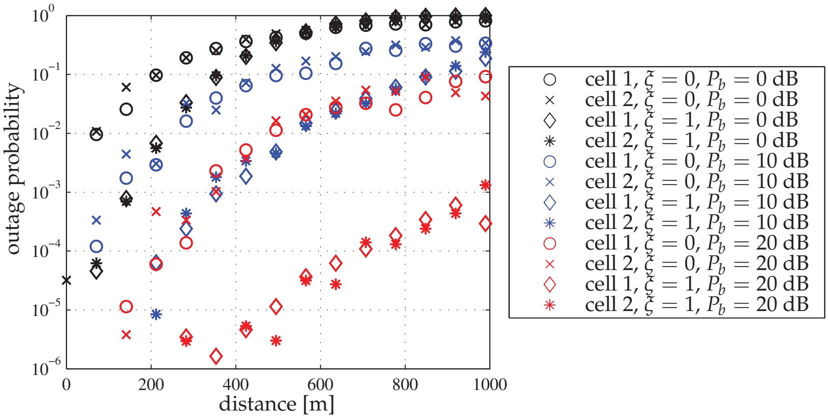

Fig. 6. Outage probability versus distance for given values of transmit power.

user terminals achieve the lowest outage probability values and the red squares indicate where user terminals have higher outage values. Figure 7a shows that the cells have similar performance when user terminals have the same target rate. On the other hand, Figure 7b shows the case when the user terminal in cell 2 (on the right side) requires three times the target rate of the one in cell 1. Cell 2 has worse performance the cell 1 because equal power allocation is performed.

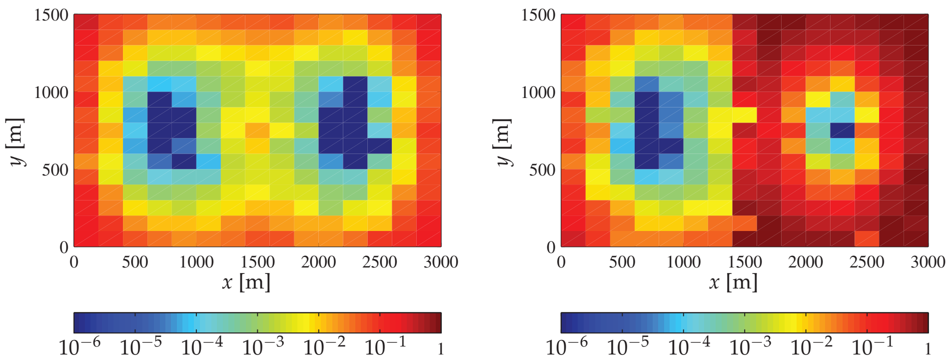

Fig. 7. Outage probability map for a) $\gamma = [1, 1]$ bits/s/Hz and b) $\gamma = [1, 1]$ bpcu

4. Resource allocation strategies for multiple carrier systems

This section is dedicated to the study of allocation strategies for multiple carrier systems. There are much more variables that impact the performance of these systems when compared to single carrier systems. That is why the challenge of allocating resources for such systems deserves special attention.

The difficulties encountered in this general case will be discussed in the next subsections, where we assume similar assumptions regarding channel state information (perfect, partial and no channel state information) similarly to the assumptions that were made for the case of single carrier systems.

4.1 Perfect channel state information

For the case where perfect channel state information is available at the central station, optimal power allocation is also found by a generalization of the classical waterfilling algorithm. The power allocation problem is modeled by a mathematical optimization problem that is solved using classical techniques. This is the case where the outage of the system is equal to the probability of γ being outside the capacity region $\mathcal{C}(\mathbf{a}, \mathbf{P})$:

$$P_{\text{out}} = 1 - \Pr\left(\gamma \in \mathcal{C}(\mathbf{a}, \mathbf{P})\right). \tag{26}$$

As well as for single carrier systems, the power allocation is performed by the algorithm that equalizes the individual outage probabilities. Hence, the following optimization problem has to be solved:

$$\min_{\sum w_k = 1} \max_{\mathbf{R} \in \mathcal{C}(\mathbf{a}, \mathbf{P})} \sum_{k=1}^{K} w_k \frac{R_k}{\alpha_k}, \tag{27}$$

where R_k is given by equation (3). Again, the inner optimization problem consists of maximizing the total system's capacity for a fixed $\mathbf{w} = (w_1, w_2, \ldots, w_K)$ and its solution can be found by applying the dual decomposition technique presented by Boyd & Vandenberghe (2004). The outer problem is identical to the one of the single carrier systems and it also consists of calculating subgradients and updating the weights. The overall algorithm is the same as Algorithm 1 and shall not be repeated in this subsection.

4.2 Partial channel state information

A feasible closed-form solution for the power allocation problem in multiple carrier systems with network MIMO has not been found yet. The proposal made by Souza et al. (2009b) consists of an iterative algorithm that finds the optimal number of allocated carriers as well as the optimal power allocation in multicell MIMO systems based on heuristics.

The solution to this problem was inspired by studies which demonstrated that, when $N \geq 2$ and considering the statistical channel knowledge, a closed-form for the outage probability can result in a complex and a numerical ill conditioned solution. The initial studies of Souza et al. (2009b) also included the analysis of Monte Carlo simulation results. of a very simple scenario with two base stations (equipped with two antennas each) and two user terminals. For this scenario the outage probability for different values of SNR and carriers, when the target rate tuple is $\gamma = [1, 1]$ bits per channel use and the both links have the same noise power, was evaluated. Results are presented in Figure 8. It is observed that the optimal strategy sometimes consists of allocating only a few carriers, even when more carriers are available. Hence, depending on SNR values, the distribution of power among carriers can result in rate reduction and increased outage of the system. Besides, frequency diversity gain only can be explored after a certain SNR value which is dependent of the number of carriers considered.

It was observed that the solution found in single-carrier case cannot be directed applied to the multicarrier case because the gains provided by frequency diversity were inferior to the loss due to the division of power between carriers. The proposed algorithm exploits this trade off and minimizes the outage probability of the system.

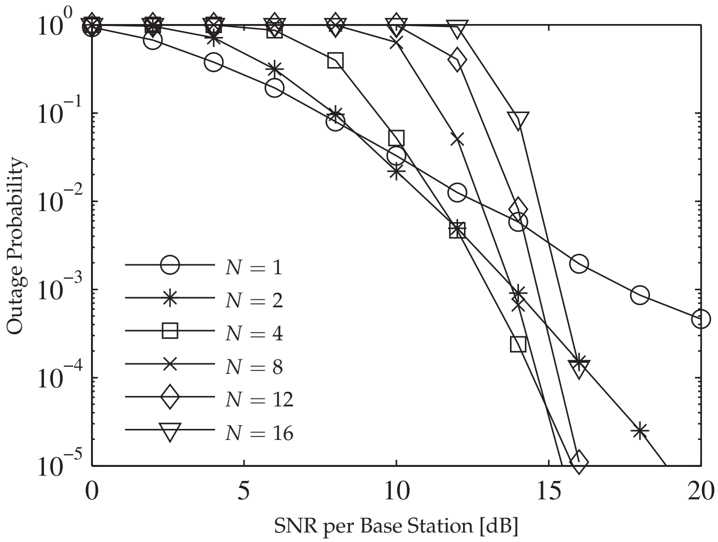

Fig. 8. Outage probability as a function of SNR

The heuristic solution is presented below. Let $\{p^*_{bk}[n]\}$ be the optimal power allocation for the multiple carrier case and $\{\theta^*_{bk}\}$ be the auxiliary variables that completely describe the power allocation so that the transmit power of each base station and each carrier can be defined as:

$$p^*_{bk}[n] = \theta^*_{bk}\frac{P_b}{N}, \tag{28}$$

with $\sum_k \theta^*_{bk} = 1$ for $b = 1, \ldots, B$. The solution is based on iterative calculations of the variables that represent the optimal power allocation and it is described by the Algorithm 3. Initially, equal power allocation is applied for each terminal and the optimal number number of carrier is defined as the total number of available carriers. In the next step, the optimal number of carriers is calculated based on the outage probability metric. Finally, for each carrier the optimal power allocation is obtained minimizing the Chernoff upperbound. The number of allocated carriers and the transmission powers are updated iteratively and minor optimization problems are solved until the convergence of the algorithm.

4.3 No channel state information

If there is no channel state information at the central station, the strategy is similar to the case of single-carrier networks and the total available power is divided among all carriers of the user terminals. Again, there is no optimization problem. The transmit power from base station b to user terminal k at carrier n is $p = P_b/KN$ and it means that $\Delta^e_{kn} = p\sum_b |a_{bk}[n]|^2$. Hence, the outage probability of the system is:

$$\begin{aligned} P_{\text{out}}(\gamma, \mathbf{p}) &= 1 - \prod_{k=1}^{K} \Pr\left(\frac{1}{N}\sum_{n=1}^{N}\log\left(1+\Delta^e_{kn}\right) > \gamma_k\right) \\ &= 1 - \prod_{k=1}^{K} \Pr\left(\prod_{n=1}^{N}\left(1+\Delta^e_{kn}\right) > 2^{N\gamma_k}\right), \end{aligned} \tag{29}$$

Algorithm 3 Resource allocation for multiple carrier systems with partial CSI

1: Initialize $\theta_{bk} = 1/K$ for $b = 1, \ldots, B$ and $k = 1, \ldots, K$
2: Initialize $N_{\text{opt}} = N$
3: **repeat**
4: Calculate $p_{bk}[n] = \theta_{bk} P_b / N_{\text{opt}}$
5: Find N_{opt} which minimizes the outage probability
6: Update $p_{bk}[n] = \theta_{bk} P_b / N_{\text{opt}}$
7: **for** each carrier **do**
8: Solve the single carrier optimization problem (18)
9: **end for**
10: Update θ_k for $k = 1, \ldots, K$
11: **until** convergence

where Δ^e_{kn} is a chi-squared random variable with $4(M - K + 1)$ degrees of freedom and its cumulative distribution function is given by:

$$F_{\Delta^e_{kn}}(y) = 1 - \exp\left[-\left(\frac{M-K+1}{p}y\right)\right]\sum_{k=0}^{2(M-K+1)-1}\frac{1}{k!}\left(\frac{M-K+1}{p}y\right)^k. \tag{30}$$

It is quite difficult to evaluate the analytical expression (29), but approximated values of the outage probability of the system may be easily found with Monte Carlo simulations.

4.4 Performance of resource allocation strategies

We considered a simulation scenario that consists of $B = 2$ base stations with $M = 2$ antennas each and $K = 2$ single-antenna terminals. Since $\theta_{b2} = 1 - \theta_{b1}$ in this case, it is sufficient to find the variables θ_{11} and θ_{21}. So, the results are presented in terms of the optimal values of θ_{11} and θ_{21} and the optimal number of allocated carriers N_{opt}.

The optimal values of θ_{bk} and N_{opt}, for the scenario where the target rate tuple is $\gamma = [1, 1]$ bits per channel use and when the both links have the same noise power, are presented in the Figure 9. As expected, θ_{11} and θ_{21} have the same values since the channel conditions and target rates are the same. Besides, as already observed, in order to minimize the outage probability, the optimal number of allocated carriers N_{opt} was found and it is greater than 1 only when SNR is above a certain value (around 9 dB in these simulations). Hence, in this scenario, both terminals are allocated with equal power and the system outage is minimized only for the optimal number of allocated carriers.

On the other hand, when the terminals have different rate requirements ($\gamma = [1, 3]$ bpcu), more power is allocated to the terminal with the highest target rate in order to minimize the outage probability (see Figure 10). However, this power difference only happens when SNR is greater than a certain value (9 dB in this case) because in the low SNR regime the single carrier optimization subproblem cannot be solved. In this scenario, the minimum system outage is only achieved with one allocated carrier, more carriers are allocated only when SNR values are greater than 19dB.

Figure 11 presents the results for the scenario where noise power of the links is different (asymmetric links). The noise power is modeled as follows: $\sigma_{ii} = \alpha\sigma_{ij}$ for $\alpha < 1$, $i, j = 1, 2$ and $i \neq j$. Considering $\alpha = 0.5$, it is possible to see that the algorithm allocates more power to the links which are in better conditions. This fact is observed specially for intermediate values of SNR; in the high SNR regime the allocation approximates to the equal power allocation

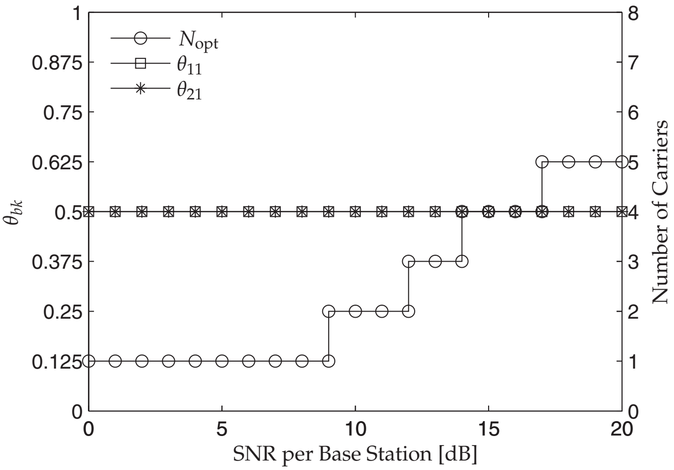

Fig. 9. Optimal values of θ_{11}, θ_{21} and N_{opt} for $\gamma = [1, 1]$ bpcu and symmetric links

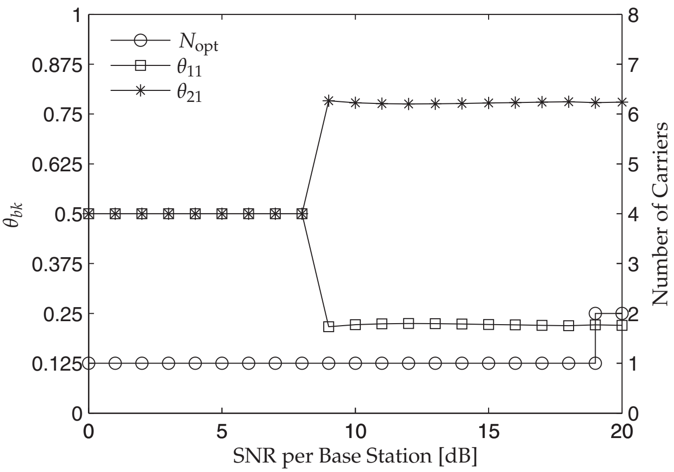

Fig. 10. Optimal values of θ_{11}, θ_{21} and N_{opt} for $\gamma = [1, 3]$ bpcu and symmetric links

because the difference of performance of the links decreases as the total available power increases.

Finally, Figure 12 shows the performance of the multicarrier system with perfect and partial channel state information. These curves represent the performance that may be achieved with the respective optimal allocation strategies together with the optimization of number of allocated carriers. Is has to be remarked that, for a given number of carriers, strategies for perfect and partial channel state information present similar trend as equal power allocation (see Figure 8).

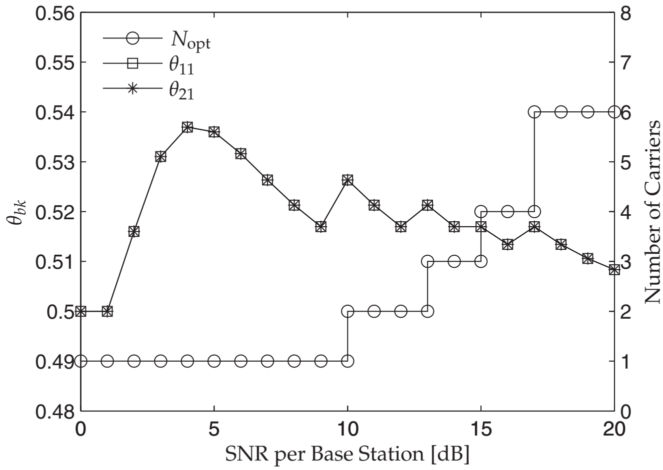

Fig. 11. Optimal values of θ_{11}, θ_{21} and N_{opt} for $\gamma = [1, 1]$ bpcu and asymmetric links

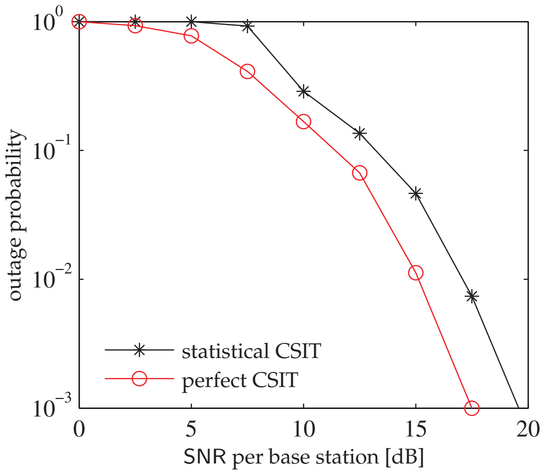

Fig. 12. Outage probability versus SNR with optimal number of allocated carriers.

5. Distributed diversity scheduling

In this section it is considered the importance of user scheduling when the number of user terminals is greater than the number of transmit antennas per base station. In order to apply the zero-forcing beamforming for each base station in a distributed manner, a set of $\tilde{K} < M$ user terminals shall be selected beforehand. It is assumed that the user scheduling is handled by the central station together with the power allocation for a system with B base stations with M antennas each. In this section, the Distributed Diversity Scheduling (DDS) scheme that was proposed by Kobayashi et al. (2010) is presented. This scheme achieves a diversity gain of $B\frac{K}{\tilde{K}}\left(M - \tilde{K} + 1\right)$ and scales optimally with the number of cooperative base stations as well as user terminals while limiting the amount of side information necessary at the central station and at the base stations.

Basically, the proposed scheduling scheme can be described as follows. Assuming local channel state information, each base station chooses its best set of user terminals over the predefined partition and reports the corresponding index and value to the central station. For its part, the central station decides and informs the selected set to all base stations. Finally, the selected user terminals are served exactly in the same manner as the previous case of $K < M$. Let $\mathcal{S},\mathcal{U}$ denote the set of all K users, the $\tilde{K}$ selected users, with $|\mathcal{S}| = K$, $|\mathcal{U}| = \tilde{K}$, respectively. In addition, let $\mathcal{Q}(\tilde{K})$ be the set of all possible user selections, i.e., $\mathcal{Q}(\tilde{K}) = \{\mathcal{U} \,|\mathcal{U} \subseteq \mathcal{S},\ |\mathcal{U}| = \tilde{K}\}$ for $\tilde{K} \leq M$. Then, the equivalent channel from the base stations to the selected users is:

$$y_k = \mathbf{a}^k \mathbf{u}^k + z_k, \quad k \in \mathcal{U}, \tag{31}$$

which is a MISO channel with $\mathbf{a}^k = [a_{1k} \cdots a_{Bk}]$ and $\mathbf{u}^k = \left[\sqrt{p_{1k}} s_{1k} \cdots \sqrt{p_{Bk}} s_{Bk}\right]^T$. For convenience, we only consider the diversity order of the worst user and refer it as the diversity of the system hereafter. Since the diversity order of a given channel depends solely on the Euclidean norm of the channel matrix, the following user selection scheme maximizes the diversity of the system:

$$\mathcal{U}^* = \arg_{\mathcal{U}} \max_{\mathcal{U} \in \mathcal{Q}} \min_{k \in \mathcal{U}} \|\mathbf{a}^k\|^2. \tag{32}$$

Unfortunately, this scheduling scheme has two major drawbacks: 1) it requires perfect knowledge at the central station on $\{\mathbf{a}_k\}$, which is crucial for the scheduling, is hardly implementable as aforementioned, and 2) the maximization over all $|\mathcal{Q}(\tilde{K})| = \binom{K}{\tilde{K}}$ possible sets $\mathcal{U}$ grows in polynomial time with K.

To overcome the first drawback, the following selection scheme is used:

$$\mathcal{U}_{\mathrm{d}} = \arg_{\mathcal{U}} \max_{b=1\ldots B} \max_{\mathcal{U} \in \mathcal{Q}} \min_{k \in \mathcal{U}} |a_{bk}|^2. \tag{33}$$

This means that base station b selects the set $\mathcal{U}$ that maximizes $\min_{k \in \mathcal{U}} |a_{bk}|^2$ and sends both the index of the set and the corresponding maximum value to the central station. Upon the reception of B values and the corresponding sets from the B base stations, the central station makes a decision by selecting the largest one. Therefore, only a very small amount of information is sent through the links between the base station and the central station. To address the second drawback, the choices of $\mathcal{U}$ are narrowed down to $\kappa = K/\tilde{K}$ possibilities (It is assumed that κ is integer for simplicity of demonstration, but it can be shown that the same conclusion holds otherwise):

$$\mathcal{P}_{\mathcal{S}} = \{\mathcal{U}_1, \mathcal{U}_2, \ldots, \mathcal{U}_\kappa\}, \quad \bigcup_i \mathcal{U}_i = \mathcal{S},\ \mathcal{U}_i \cap \mathcal{U}_j = \emptyset,\ \forall\, i \neq j,\ |\mathcal{U}_i| = \tilde{K},\ \forall\, i. \tag{34}$$

In other words, $\mathcal{P}_{\mathcal{S}}$ is partition of the set of all users $\mathcal{S}$. Furthermore, it is assumed that the partition $\mathcal{P}_{\mathcal{S}}$ is fixed by the central station and known to all base stations. Hence, the proposed scheduling scheme selects the following set of users:

$$\mathcal{U}_{\mathrm{d}} = \arg_{\mathcal{U}} \max_{b=1\ldots B} \max_{\mathcal{U} \in \mathcal{P}_{\mathcal{S}}} \min_{k \in \mathcal{U}} |a_{bk}|^2. \tag{35}$$

To summarize, the scheduling scheme works as described in Algorithm 4.

An example of two base stations and six user terminals is shown in Figure 13. In this example, in order to serve two user terminals simultaneously, a partition of three sets is

Algorithm 4 Distributed Diversity Scheduling (DDS)

1: Central station fixes a partition $\mathcal{P}_\mathcal{S}$ and informs it to all base stations
2: Base station b finds $\max_{\mathcal{U}\in\mathcal{P}_\mathcal{S}} \min_{k\in\mathcal{U}} |a_{bk}|^2$ and sends this value and the index of the maximizing set $\mathcal{U}$ to the central station
3: Central station chooses the highest value and broadcasts the index of the selected set $\mathcal{U}_\mathrm{d}$ as defined in (35)
4: All base stations serve the user terminals in $\mathcal{U}_\mathrm{d}$ simultaneously

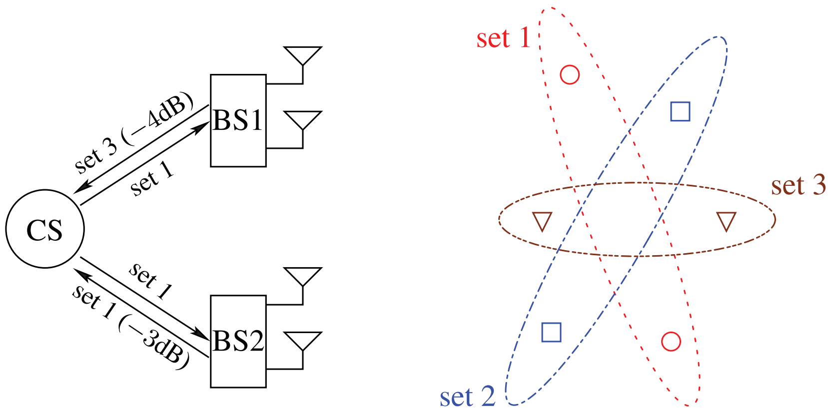

Fig. 13. An example scenario of user scheduling with two base stations and six user terminals.

fixed by the central station. With local channel state information, each base station compares the coefficients $\min_{k\in\mathcal{U}} |a_{bk}|^2$ for all three sets $\mathcal{U}$, finds out the largest one and sends the corresponding "index(value)" pair to the central station. The central station compares the values and broadcasts the index of the selected set (set 1 in this example).

Figure 14 shows the outage probability versus signal-to-noise ratio when there are more users than the number of served users, i.e. $K \geq \tilde{K} = 2$. Assuming the same setting as Figure 2 for $M = 4$, the distributed diversity scheme is applied to select a set of two users among $K \in \{2, 4, 6\}$. Once the user selection is done, any power allocation policy presented in Sections 3 and 4 can be applied. However, it is non-trivial (if not impossible) to characterize the statistics of the overall channel gains in the presence of any user scheduling. Hence, it is illustrated here only the performance with equal power allocation in the absence of channel state information. As a matter of fact, any smarter allocation shall perform between the waterfilling allocation and the equal power allocation. It is observed in the figure that diversity gain increases significantly as the number K of users in the system increases.

6. Conclusions

In this chapter, we reviewed the litterature on the power allocation problems for coordinated multicell MIMO systems. It was seen that the optimal resource allocation is given by the waterfilling algorithm when the central station knows all channel realizations. Assuming a more realistic scenario, we also reviewed the solutions for the case of partial channel state information, i.e. local channel knowledge at each base station and statistical channel

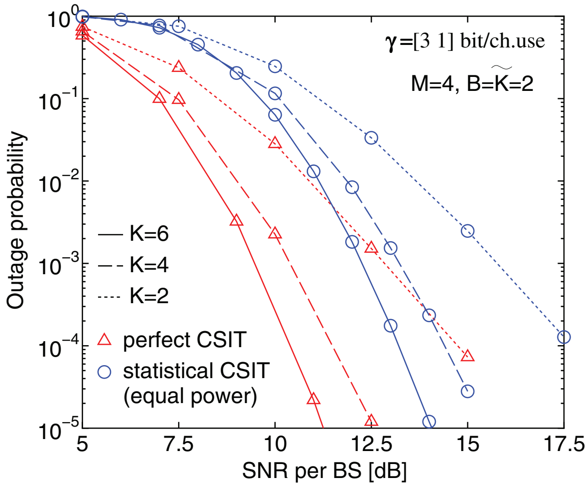

Fig. 14. Outage probability vs. SNR for many users with $B = \tilde{K} = 2$ and $M = 4$.

knowledge at the central station. Under this setting, it was presented an outage-efficient strategy which builds on distributed zero-forcing beamforming to be performed at each base station and efficient power allocation algorithms at the central station.
In addition, in the case of a small number of users $K \leq M$, it was proposed a scheme that enables each user terminal to achieve a diversity gain of $B(M - K + 1)$. On the other hand, when the number of users is larger than the number of antennas ($K \geq M$), the proposed distributed diversity scheduling (DDS) can be implemented in a distributed fashion at each base station and requires only limited amount of the backbone communications. The scheduling algorithm can offer a diversity gain of $B\frac{K}{\tilde{K}}(M - \tilde{K} + 1)$ and this gain scales optimally with the number of cooperative base stations as well as the number of user terminals. The main finding is that limited base station cooperation can still make network MIMO attractive in the sense that a well designed scheme can offer high data rates with sufficient reliability to individual user terminal. The proposed scheme can be suitably applied to any other interference networks where the transmitters can perfectly share the messages to all user terminals and a master transmitter can handle the resource allocation.

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