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Marco Di Renzo, Harald Haas

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Abstract—In this paper, we study the performance of Space Shift Keying (SSK) modulation for Multiple-Input-Multiple-Output (MIMO) wireless systems in the presence of multiple-access interference. More specifically, a synchronous multi-user scenario is considered. The main technical contributions of this paper are as follows. Two receiver structures based on the Maximum-Likelihood (ML) criterion of optimality are developed and analytically studied, i.e., the single- and multi-user detectors. Accurate frameworks to compute the Average Bit Error Probability (ABEP) over independent and identically distributed (i.i.d.) Rayleigh fading channels are proposed. Furthermore, simple and easy-to-use lower- and upper-bounds for performance analysis and system design are introduced. The frameworks account for the near-far effect, which significantly affects the achievable performance in multiple-access environments. Also, we extend the analysis to Generalized SSK (GSSK) modulation, which foresees multiple-active antennas at the transmitter. With respect to SSK modulation, GSSK modulation achieves higher data rates at the cost of an increased complexity at the transmitter. The performance of SSK and GSSK modulations is compared to conventional Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) schemes, and it is shown that SSK and GSSK modulations can outperform conventional schemes for various system setups and channel conditions. In particular, the performance gain of SSK and GSSK modulations increases for increasing values of the target bit rate and of the number of antennas at the receiver. Finally, we put forth the concept of Coordinated Multi-Point (or network MIMO) SSK (CoMP-SSK) modulation, as a way of exploiting network cooperation of antennas at the receiver. The framework to Generalized SSK (GSSK) modulation, which is determined by conventional modulation schemes (e.g., Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM)), and the spatial-constellation diagram, which is determined by SSK modulation [6]. By exploiting signal- and spatial-constellation diagrams, SM introduces a multiplexing gain with respect to single-antenna systems that increases logarithmically with the number of antennas at the transmitter. Furthermore, with respect to spatial-multiplexing MIMO systems, the multiplexing gain is obtained with no inter-channel interference. This enables very simple single-stream and Maximum-Likelihood (ML-) optimum decoding at the receiver [2], [6], [7]. Recent analytical and simulation results have highlighted that SM and SSK modulation can provide better performance with reduced decoding complexity than state-of-the-art single- and multi-antenna wireless systems [2], [5]–[15]. The reader can find a comprehensive and detailed overview of the contribution of recent papers on SM and SSK modulation in [3], [13], and [16].

By carefully looking at recent research works on SM and SSK modulation, it is possible to notice that all the studies available so far consider the point-to-point reference scenario. For example, in [2] and [7], the Average Bit Error Probability (ABEP) of SSK modulation and SM, respectively, is studied over independent and identically distributed (i.i.d.) Rayleigh fading channels; in [8], the framework in [2] is generalized to the so-called Generalized SSK (GSSK) modulation, which is an improved version of SSK modulation where more than one antenna can be active at any time instance; in [10], the study in [2] is extended by taking into account channel (Trellis) coding to reduce the error probability of detecting the active transmit-antenna; in [11], the performance of SM over Nakagami-m fading channels is investigated; in [13], the ABEP of SSK modulation with and without transmit-diversity is analyzed over generically-correlated and distributed Rician fading channels; and, finally, in [3], [16], [17], the performance of SSK modulation is analyzed over correlated and non-identically distributed Nakagami-m fading with and without perfect Channel State Information (CSI). However, to the best of the authors knowledge, none of these papers address

I. INTRODUCTION

SPACE SHIFT KEYING (SSK) is a recently proposed modulation scheme for Multiple-Input-Multiple-Output (MIMO) wireless systems [1], [2]. It encodes the information bits onto the spatial position (i.e., the index) of the antennas at the transmitter, and enables data decoding by exploiting the differences in the Channel Impulse Responses (CIRs) of the transmit-to-receive wireless links [3]. It is receiving an increasing attention due to its simple transmitter and receiver design, and, more important, because it represents the fundamental building block of Spatial Modulation (SM) [4]–[7]. SM is a low-complexity hybrid modulation scheme for MIMO systems, which maps the information bits onto two information carrying units: the signal-constellation diagram, which is determined by conventional modulation schemes (e.g., Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM)), and the spatial-constellation diagram, which is determined by SSK modulation [6]. By exploiting signal- and spatial-constellation diagrams, SM introduces a multiplexing gain with respect to single-antenna systems that increases logarithmically with the number of antennas at the transmitter. Furthermore, with respect to spatial-multiplexing MIMO systems, the multiplexing gain is obtained with no inter-channel interference. This enables very simple single-stream and Maximum-Likelihood (ML-) optimum decoding at the receiver [2], [6], [7]. Recent analytical and simulation results have highlighted that SM and SSK modulation can provide better performance with reduced decoding complexity than state-of-the-art single- and multi-antenna wireless systems [2], [5]–[15]. The reader can find a comprehensive and detailed overview of the contribution of recent papers on SM and SSK modulation in [3], [13], and [16].
receiver design and performance analysis of SSK modulation in the presence of multiple-access interference. The present research work is motivated by the fact that, due to the simultaneous transmission of various users over the same physical wireless channel, the vast majority of wireless communication networks are interference limited [18]. Therefore, the potential merits of SSK modulation for its application to the next generation wireless communication systems highly depends on its robustness to multiple-access interference. The main aim of this paper is to understand advantages and disadvantages of SSK modulation in multiple-access environments (i.e., multi-user SSK modulation), and figure out if the claimed benefits of SSK modulation in a single-user environment are retained. Similar to [2], [3], and [16], we focus our attention only on SSK modulation as it enables a simple analytical derivation and insightful understanding of the role played by the spatial-constellation diagram, which is the main innovative enabler for performance improvement of SM/GSSK/SSK modulation [12]. In this context, we feel important to notice that the adoption of SM/GSSK/SSK modulation schemes inherently requires that each user is equipped with multiple antennas at the transmitter and at the receiver. Also, since in some cases, e.g., in SSK modulation, only a single antenna is active and the information is conveyed only by the spatial-constellation diagram, the number of antennas in each device might be quite large to achieve good data rates. However, this large number of antennas seems not to be a critical bottleneck for the development of the next generation multiple-access cellular systems, as current research is moving towards the utilization of the millimeter-wave frequency spectrum [19]. In fact, in this band compact horn antenna-arrays with 48 elements and compact patch antenna-arrays with more than 4 elements at the Base Station (BS) and at the mobile terminal, respectively, are currently being developed to support multi-gigabit transmission rates [20].

In detail, the main contributions and technical novelties of this paper can be summarized as follows: i) for the first time, the performance of modulation schemes exploiting the “space modulation” concept in a multi-user interference environment is investigated and compared to traditional modulation schemes; ii) two ML-optimum receiver structures are investigated, i.e., the single-user detector that is interference-unaware and the joint multi-user detector that is interference-aware; iii) for analytical tractability, we limit our performance study to the synchronous case, which is often considered as the initial reference scenario for performance analysis in the presence of interference [21]; iv) accurate frameworks for SSK and GSSK modulations are developed to compute the ABEP over i.i.d. frequency-flat Rayleigh fading channels. The frameworks can take into account the so-called near-far effect that significantly affects the performance of wireless systems in multiple-access environments [21]; v) simple bounds are derived to compare various modulation schemes (e.g., SSK, GSSK, PSK, QAM) and to understand advantages and disadvantages of each of them for various MIMO configurations; vi) the proposed frameworks and bounds are useful for an arbitrary number of antennas at the transmitter and at the receiver, as well as for any modulation order and bit-to-antenna-index mapping; and vii) we put forth the concept of Coordinated Multi-Point (or network MIMO) SSK (CoMP-SSK) modulation, as a way of exploiting network cooperation and the spatial-constellation diagram to achieve high bit rates.

Our comprehensive analytical study highlights the following general results: i) it is shown that SSK and GSSK modulations can outperform conventional modulation schemes in the presence of multiple-access interference; ii) if a single-user detector is used, SSK modulation provides better performance than PSK modulation for bit rates greater than 2 bits/s/Hz per user, while it outperforms QAM for bit rates greater than 2 bits/s/Hz per user if the receiver is equipped with at least two antennas; iii) GSSK modulation is always worse than SSK modulation, but it achieves higher bit rates for the same number of antennas at the transmitter; iv) the performance gain of SSK and GSSK modulations increases for increasing values of the requested bit rate per user; v) the robustness of SSK and GSSK modulations to multi-user interference increases by adding more antennas at the receiver; vi) when a multi-user detector is used, SSK and GSSK modulations seem to be more robust to multi-user interference than conventional modulation schemes. For example, for bit rates greater than 2 bits/s/Hz per user, SSK modulation always outperforms QAM regardless of the number of antennas at the receiver; and vii) it is shown that CoMP-SSK modulation can provide very high bit rates at the cost of network cooperation, which can be realized through a backhaul link. Also, it is shown that CSI at the transmitter is not needed to implement this scheme.

The reminder of this paper is organized as follows. In Section II, the system model is introduced. In Section III, the ABEP of SSK modulation with ML-optimum single-user detection is investigated. In Section IV, the performance of SSK modulation is compared to QAM and PSK modulation when a ML-optimum single-user detector is used. In Section V, the framework in Section III is extended to GSSK modulation, and SSK and GSSK modulations are compared. In Section VI, all the frameworks proposed in the previous sections are generalized to ML-optimum multi-user detection, and asymptotic analysis is used to reveal advantages and disadvantages of SSK and GSSK modulations. In Section VII, the concept of CoMP-SSK modulation is introduced. In Section VIII, numerical results are shown to substantiate our analytical derivations and findings. Finally, Section IX concludes this paper.

II. SYSTEM MODEL

We consider a very general multi-user MIMO setup with N_u active users (i.e., transmitters) and N_r possible receivers. Each transmitter/receiver is equipped with an antenna-array of N_t/N_r antennas. This setup can accommodate various multiple-access situations, such as: i) the scenario in which a single receiver must decode the information of more than one transmitter, and ii) the so-called interference channel (e.g., the X channel) [18], where each receiver must decode the information from a single transmitter and can disregard the information transmitted by the other users. Without loss of generality, among the possible transmitter/receiver pairs,
we focus our attention on a particular link that is usually known as probe link or intended link (see, e.g., [22, Fig. 1]). More specifically, we are interested in studying the error probability of the data sent from a generic user $\xi$, with $\xi = 1, 2, \ldots, N_u$, to a generic receiver when the other $N_u - 1$ users are simultaneously transmitting on the same physical channel. So, there is no loss of generality in considering $N_u = 1$.

In our system model, the $N_u$ users exploit the differences (i.e., known as spatial signatures or channel fingerprints [3]) in the wireless channel of any transmit–to–receive link with a twofold objective: data modulation and multiple–access. Accordingly, the multi–user SSK modulation scheme analyzed in this paper can be seen as a generalization of the so–called Space–Division Multiple–Access (SDMA) [23]–[25] and Channel–Division Multiple–Access (ChDMA) [26] schemes, which exploit user–specific CIRs only for differentiating simultaneously transmitting users, i.e., for multiple–access only. The fundamental difference between multi–user SSK modulation and multi–user SDMA/ChDMA is that the former scheme uses the differences in the CIRs for data modulation in addition to multiple–access, while the latter schemes rely on conventional (e.g., PSK and QAM) modulation for transmitting the data of the users. In multi–user SSK modulation, the stochastic differences in the CIRs are exploited in a twofold way: i) at the microscopic level, i.e., differences among co–located transmit–antennas of the same user, for data modulation, and ii) at the macroscopic level, i.e., differences among spatially distributed antenna–arrays associated to different users, for multiple–access. To the best of the authors knowledge, the performance analysis of this multiple–access scheme has never been considered in literature.

### A. Assumptions and Notation

Throughout this paper, the following assumptions and notation are used. i) A synchronous multi–access channel with perfect time–synchronization at the receiver is considered [21]. Accordingly, for ease of notation, time delays can be neglected during the analytical derivation. ii) The receiver is assumed to have perfect CSI. More specifically, if a single–user detector is used, the receiver needs only the CSI of the probe link. While, if a joint multi–user detector is used, the receiver needs the CSI of all the active users. iii) In all wireless links, frequency–flat independent Rayleigh fading is assumed. In particular, identically distributed fading is considered for wireless links related to co–located antennas, while non–identically distributed fading among the users is considered. This allows us to include the near–far effect in the analytical derivation. In formulas, we denote by $h_u^{(t,r)}$ the complex CIR from the $t$–th ($t = 1, 2, \ldots, N_t$) transmit–antenna of the $u$–th ($u = 1, 2, \ldots, N_u$) user to the $r$–th ($r = 1, 2, \ldots, N_r$) receive–antenna of the destination. Moreover, we use the notation $h_u^{(t,r)} = \text{Re} \{ h_u^{(t,r)} \} + j \text{Im} \{ h_u^{(t,r)} \}$, where Re $\{ \cdot \}$ and Im $\{ \cdot \}$ are real and imaginary operators, and $j = \sqrt{\text{–}1}$ is the imaginary unit. $\text{Re} \{ h_u^{(t,r)} \}$ and $\text{Im} \{ h_u^{(t,r)} \}$ are independent real–valued Random Variables (RVs). iv) $X \sim \mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian RV with mean $\mu$ and standard deviation $\sigma$. v) Owing to the assumption of Rayleigh fading, we have $\text{Re} \{ h_u^{(t,r)} \} \sim \mathcal{N}(0, \sigma_u^2)$ and $\text{Im} \{ h_u^{(t,r)} \} \sim \mathcal{N}(0, \sigma_u^2)$. vi) $\overline{\{ \cdot \}}$, i.e., overbar, denotes complex–conjugate. vii) E $\{ \cdot \}$ denotes the expectation operator. viii) Pr $\{ \cdot \}$ denotes probability. ix) $\mathcal{Q}(x) = \left(1/\sqrt{2\pi}\right) \int_{\infty}^{\infty} \exp \left(–t^2/2\right) dt$ is the $Q$–function and $\mathcal{Q}^–(\cdot)$ is used to denote the inverse $Q$–function. x) $T_s$ denotes the duration of the time–slot where each information symbol is transmitted. xi) The noise $n_r^{(t)}$ at the input of the $r$–th receive–antenna ($r = 1, 2, \ldots, N_r$) is assumed to be an Additive White Gaussian Noise (AWGN) process, with both real and imaginary parts having a power spectral density equal to $N_0$. Across the receive–antennas, the noise is statistically independent. xii) $M$ denotes the modulation order of QAM and PSK modulation. The $M$ symbols of the signal–constellation diagram of user $u$ are denoted by the complex numbers $s_u^{(m)}$ for $m = 1, 2, \ldots, M$ and $u = 1, 2, \ldots, N_u$. For PSK modulation, we have $|s_u^{(m)}|^2 = 1$. xiii) If GSSK modulation is used, $N_{ta}$ denotes the number of simultaneously–active antennas at the transmitter, with $1 \leq N_{ta} \leq N_t$. The effective size of the spatial–constellation diagram is denoted by $N = 2^{\lceil \log_2(N_{ta}) \rceil}$ [8], where $\lceil \cdot \rceil$ is the binomial coefficient and $\lfloor \cdot \rfloor$ is the floor function. xiv) In SSK and GSSK modulations, $E_s$ and $E_{sw}/N_{ta}$ denote the average energy transmitted by each antenna of user $u$ ($u = 1, 2, \ldots, N_u$) that emits a non–zero signal, respectively. In particular, in GSSK modulation a uniform energy–allocation scheme is considered among the active transmit–antennas. In QAM and PSK modulation, $E_s$ is the average transmitted energy per information symbol of user $u$, $xv)$ $w(\cdot)$ is the real–valued unit–energy transmitted pulse shape in each time–slot $T_s$. xvi) $\Gamma(x) = \int_0^{\infty} t^{x–1} \exp (–t) dt$ is the Gamma function. xvii) $\delta_{x,y}$ is the Kronecker delta function, which is defined as $\delta_{x,y} = 1$ if $x = y$ and $\delta_{x,y} = 0$ elsewhere. xviii) $\approx$, $\approx$, and $\approx$ denote conventional equality ($\approx$), inequality ($\approx$), and approximation ($\approx$) operators, respectively, which have been labeled with $\approx$ as a short–hand to better identify them in the text, and provide comments on the analytical procedure that is used for their computation.

### III. SSK MODULATION WITH SINGLE–USER DETECTION

In SSK modulation, each user encodes blocks of $\log_2(N_t)$ data bits into the index of a single transmit–antenna, which is
Hamming distance between the bit–to–antenna–index map-
closed–form as shown in (3) on top of this page, where
the optimum estimate, \( \hat{x}_\xi \), for \( \xi = 1, 2, \ldots, N_r \), is given in (1) on top of the previous page.

As mentioned in Section II, (1) confirms that in multi–
user SSK modulation only the differences in the CIRs
are exploited for modulation and multiple–access. All
the users share the same time–slot, frequency–band, or spreading
code [26]. By assuming single–user detection, the interference
is not exploited for optimal detection, and the ML–
obtained estimate, \( \hat{x}_r \), for \( r \in N_r \) is shown in (2) on top of this page [27], where \( y_\xi \) denotes the trial instance of \( x_\xi \)
used in the \( N_r \)–hypothesis testing problem, and \( D_{\xi}(y_\xi) = 
\sum_{r=1}^{N_r} \int_{T_r} \left| z^{(r)}(t) - \sqrt{E} h^{(y_\xi,r)}(t) \right|^2 dt \).

The ABEP of the detector in (2) can be computed in
closed–form as shown in (3) on top of this page, where \( \subseteq \) comes from [28, Eq. (4) and Eq. (5)], and \( \subseteq \) is the well–
known asymptotically–tight–union–bound [29, Eq. (12.44)],
which has recently been used in [3, Eq. (35)] for point–
to–point SSK modulation. Furthermore, \( N_H (x_\xi, y_\xi) \) is the
Hamming distance between the bit–to–antenna–index map-
plings of \( x_\xi \) and \( y_\xi \); \( E_h \{ \cdot \} \) is the expectation computed over all
the fading channels (probe link and interference); and
\( \text{AEP}(x_\xi \rightarrow y_\xi) = E_h \{ \text{PEP}(x_\xi \rightarrow y_\xi) \} \) is the Average
Pairwise Error Probability (APEP), i.e., the probability of
estimating \( y_\xi \) when, instead, \( x_\xi \) is transmitted, under the
assumption that \( x_\xi \) and \( y_\xi \) are the only two antenna–indexes
possibly being transmitted. Accordingly, \( \text{APEP}(x_\xi \rightarrow y_\xi) = 
\text{Pr}\{\hat{x}_\xi = y_\xi, \hat{x}_\xi \neq y_\xi \} \) is conditioned on both antenna–indexes \( x_\xi \) and \( y_\xi \). Unlike [3], the main contribution of this paper consists in
taking into account multiple–access interference when com-
puting the APEPs.

The PEPs in (3) can explicitly be written as shown in (4) on
top of this page, where \( \subseteq \) comes directly from (2), and \( \subseteq \) can
be obtained after lengthy analytical manipulations. Further-
more, we have defined \( \eta^{(r)} = \sum_{u \neq \xi}^{N_r} \left( \sqrt{E} h^{(y_\xi,u)}(r) + n^{(r)}_w \right) \)
and \( n^{(r)}_w = \int_{T_r} n^{(r)}(t) w(t) dt \). By direct inspection, it is
easy to show that \( n^{(r)}_w \) is a complex–valued Gaussian RV
with distribution \( \text{Re}\{n^{(r)}_w\} \sim N(0, N_0) \) and \( \text{Im}\{n^{(r)}_w\} \sim 
N(0, N_0) \).

By conditioning upon all the fading channels (probe
link and interference), it can be proved that \( \Omega = 
2\text{Re}\left\{ \sum_{r=1}^{N_r} \sqrt{E} h^{(y_\xi,r)} - h^{(x_\xi,r)} \left( \eta^{(r)} \right) \right\} \)
in (4) is a real–
valued Gaussian RV with distribution shown in (5) on top of
this page. Thus, from the definition of Q–function in Section
II–A, the PEP in (4) has closed–form expression given in (6) on
top of the next page. It is worth mentioning that the PEP in (6)
is very general and can be used for channel models different
from Rayleigh fading studied in this paper. Furthermore, (6) is
obtained without making any assumptions about the statistical
distribution of the interference. This enables us to use, with
minor changes, this formula for modulation schemes different
from SSK, such as PSK/QAM and GSSK, which are studied in
Section IV and Section V, respectively. Thus, the assumption
of Rayleigh fading is here made only for analytical tractability,
and to get simple and insightful closed–form expressions and
bounds, which: i) enable a fairly simple comparison among
different state–of–the–art modulation schemes; ii) provide
guidelines for system design and optimization; and iii) shed

\[
\hat{x}_\xi = \arg\min_{y_\xi = 1, 2, \ldots, N_r} \left\{ D_{\xi}(y_\xi) = \arg\min_{y_\xi = 1, 2, \ldots, N_r} \left\{ \sum_{r=1}^{N_r} \int_{T_r} \left| z^{(r)}(t) - \sqrt{E} h^{(y_\xi,r)}(t) \right|^2 dt \right\} \right. 
\]

\[
\text{APEP}_\xi = E_h \left\{ \frac{1}{N_r} \sum_{\eta_\xi = 1}^{N_r} \left[ \frac{1}{\log_2(N_r)} \sum_{y_\xi = 1}^{N_r} N_H (x_\xi, y_\xi) \Pr\{ \hat{x}_\xi = y_\xi | x_\xi \} \right] \right\} 
\]

\[
\text{PEP}(x_\xi \rightarrow y_\xi) = \text{Pr}\{D_{\xi}(x_\xi) > D_{\xi}(y_\xi) \} = \text{Pr}\left\{ 2\text{Re}\left\{ \sum_{r=1}^{N_r} \left[ \sqrt{E} h^{(y_\xi,r)} - \overline{h^{(x_\xi,r)}} \right] \eta^{(r)} \right\} \right\} > E_{\xi} \sum_{r=1}^{N_r} \left| h^{(y_\xi,r)} - \overline{h^{(x_\xi,r)}} \right|^2 
\]

\[
\Omega \sim N(2\sqrt{E} \text{Re}\left\{ \sum_{r=1}^{N_r} \left[ \sqrt{E} h^{(y_\xi,r)} - \overline{h^{(x_\xi,r)}} \right] \right\} ) \cdot 4N_0 E_{\xi} \sum_{r=1}^{N_r} \left| h^{(y_\xi,r)} - \overline{h^{(x_\xi,r)}} \right|^2 
\]
lights on the robustness of SSK modulation to multiple-access interference.

To compute the AEP in Rayleigh fading, i.e., to remove the conditioning over channel statistics, we use a two-step procedure: i) first, we condition the PEP in (6) upon the channel gains of the probe link and remove the conditioning over the channel gains of the interference; and ii) then, we remove the conditioning over channel gains of the probe link. By conditioning upon the probe link, \( \Omega_I \) in (6) is a conditional Gaussian RV with distribution \( \Omega_I \sim \mathcal{N}(0, \sigma_I^2) \), while \( \Omega_P \) and \( \Omega_N \) are conditional constant terms. Accordingly, from (6) we have:

\[
\text{PEP}(x_\xi \rightarrow y_\xi) = Q \left( \frac{\Omega_P}{\sqrt{\Omega_N^2 + \sigma_I^2}} \right) - 2 \sqrt{\frac{E_r}{4}} \text{Re} \left( \sum_{r=1}^{N_r} \left[ \left( \sum_{\nu \neq \xi} h_{\xi}^{(y_\nu, r)} - h_{\xi}^{(x_\nu, r)} \right) \sum_{\nu \neq \xi} (E_{\nu} h_{\nu}^{(x_\nu, r)}) \right] \right)
\]

(6)

\[
\text{AEP}(x_\xi \rightarrow y_\xi) = E_{h_\xi} \left\{ \left[ 1 - \sqrt{\frac{\text{SINR}}{2 + \text{SINR}}} \right] R_{\alpha} \right\} = \frac{1}{2} \sum_{r=1}^{N_r} \left( N_r - 1 + r \right) \left[ \frac{1}{2} + \frac{\text{SINR}}{2 + \text{SINR}} \right]^r
\]

(8)

where \( E_{h_\xi} \{ \} \) denotes the expectation over all the fading gains except those of the probe link; \( \text{PEP} \) comes from (6); \( \text{AEP} \) is obtained by introducing the RV \( \tilde{\Omega}_I \), which is defined as \( \tilde{\Omega}_I = \Omega_I / \sigma_I \sim \mathcal{N}(0, 1) \); and \( \gamma_\xi \) is a notable integral that involves the Q-function and Gaussian RVs, and is tabulated in [21, Eq. (3.66)].

The last step consists in removing the conditioning over the channel gains of the probe link. From (7), we obtain (8) on top of this page, where we have defined: i) \( \gamma_\xi = \sum_{r=1}^{N_r} h_{\xi}^{(y_\nu, r)} - h_{\xi}^{(x_\nu, r)} \); ii) SINR = SNR_{\xi}/(1 + INR_{\xi}) is the Signal–to–Interference–Plus–Noise–Ratio (SINR); iii) SNR_{\xi} = (E_{\xi} \sigma_{\xi}^2)/N_0 is the Signal–to–Noise–Ratio (SNR) of the probe link; and iv) INR_{\xi} = \sum_{\nu \neq \xi} (E_{\nu} \sigma_{\nu}^2)/N_0 is the aggregate Interference–to–Noise–Ratio (INR) of all the interferers. The identity in \( \text{(a)} \) is obtained as follows: i) \( \gamma_\xi \) is the summation of the square absolute value of Gaussian RVs and, thus, is a Chi–Square RV [30, Eq. (2–1–136), Eq. (2–1–137)]; and ii) the Q–function is averaged over the resulting Chi–Square RV [30, Eq. (14–4–14), Eq. (14–4–15)].

Finally, the ABEP can be computed by substituting (8) in (3). By carefully looking at (8), we notice that the AEP is independent of \( x_\xi \) and \( y_\xi \), i.e., the actual and trial antenna–indexes, but it only depends on SNR and INR of probe link and interference, respectively. Thus, by defining in (8) AEP\((x_\xi \rightarrow y_\xi) = AEP_\xi \) for all \( x_\xi = 1, 2, \ldots, N_t \) and \( y_\xi = 1, 2, \ldots, N_r \), the ABEP in (3) simplifies as follows:

\[
\text{ABEP}_\xi \leq \frac{\text{AEP}_\xi}{N_0 \log_2 (N_t)} \sum_{x_\xi=1}^{N_t} \sum_{y_\xi=1}^{N_r} N_H(x_\xi, y_\xi)
\]

(9)

where \( \text{(a)} \) comes from the identity \( \sum_{x_\xi=1}^{N_t} \sum_{y_\xi=1}^{N_r} N_H(x_\xi, y_\xi) = N_t^2 / 2 \log_2 (N_t) \), which can be derived via direct inspection for all possible bit–to–antenna–index mappings.

In conclusion, (8) and (9) provide a very simple analytical framework of the ABEP of multi-user SSK modulation over Rayleigh fading channels. Very interestingly, from (9) we observe that the ABEP is independent of the bit–to–antenna–index mapping. This stems from the assumption of i.i.d. Rayleigh fading for the wireless links of co-located transmit–antennas. Also, we note that (8) and (9) reduce to known results if there is no multiple–access interference, i.e., INR_{\xi} = 0 (see, e.g., [2] and [3]).

A. Asymptotic Analysis

In this section, we study some asymptotic case studies to highlight some general behaviors when using the spatial–constellation diagram for modulation.

1) \( \text{SNR}_\xi \gg 1 \) and \( \text{INR}_\xi \ll 1 \) (noise–limited scenario): This corresponds to a scenario in which multi–access interference can be neglected and large–SNR analysis for the probe
Thus, if \( N \) is drastically reduced. However, even though the AWGN multiple-access interference is negligible, the ABEP goes to zero for increasing SNR and that the diversity order is equal to \( N_r \). These findings agree with [2] and [3]. This formula highlights a trend that was not shown either in [2] or [3]: the ABEP linearly increases with the number of transmit–antennas.

\[ \text{SNR}_\gamma \text{particular, if } \gamma \text{ is drastically reduced. However, even though the AWGN} \]

\[ \frac{\sqrt{N_r} \text{to } N_r \text{ becomes small we can expect worse performance than SSK}}{\}^2] \text{ does not hold as the APEPs depend on the actual pair of points} \]

\[ \begin{align*}
\mathbb{E}[\mathcal{E}_h^{(r)}(r)] & = \mathbb{E}[\mathcal{E}_h^{(r)}(r)] \\
\text{This is a very simple formula that can be used for a simple system design when the assumption } \gamma & \text{ is reasonable.}
\end{align*} \]

\[ \text{IV. SINGLE–USER DETECTION: COMPARISON OF PSK, QAM, AND SSK MODULATION} \]

In this section, we aim at studying the performance of conventional QAM and PSK modulation, and at comparing them with SSK modulation. We consider a detector similar to (2), but the search space is given by the signal–constellation diagram rather than by the spatial–constellation diagram. Also, the methodology we use for performance analysis is similar to Section III. For this reason, and due to space constraints, we omit the details of the analytical derivation and report only the final results. Instead, we focus our attention on trying to understand advantages and disadvantages of using the spatial–constellation diagram as a source of information. Finally, we note that for QAM and PSK modulation the transmitter is equipped with a single–antenna, i.e., \( N_t = 1 \).

Received signal and ML–optimum detector are shown in (10) on top of this page, where we have defined

\[ D_t^{(r)}(y) = \sum_{s=1}^{N_r} \int_{T_s} \left| z^{(r)}(t) - \sqrt{E_h^{(r)}(s)} s^{(r)}(t) \right|^2 dt \]

\[ \text{for } \gamma \text{ is drastically reduced. However, even though the AWGN} \]

\[ \text{which means that the ABEP does not go to zero as the AWGN goes to zero. This is because the single–user detector is interference–unaware; ii) the error–floor is lower (better performance) when either } N_t \text{ or the SIR increase; and iii) the error–floor is higher (worse performance) when } N_t \text{ increases. To overcome the error–floor, we need to increase either the transmit–energy (} E_t^{(r)} \text{) of the probe link or the number of antennas, } N_r, \text{ at the destination.} \]

\[ 0 \text{ we have just remarked that multiple–access interference can be, in part, mitigated by adding more antennas at the receiver. We are interested in analyzing the asymptotic ABEP when } N_r \text{ is very large. To derive this result, we start from the last equality in the first line of (8). From [30, Eq. (2–1–136) and Eq. (2–1–139)], it follows that the RV } \sqrt{N_r} \text{ has mean and variance equal to } E\left\{ \sqrt{N_r} \right\} = 2\sigma^2 \text{ and } E\left( \frac{N_r}{\sqrt{N_r}} \right) = N_r \text{ and } E\left( \frac{\left( \sqrt{N_r} - E \left( \sqrt{N_r} \right) \right)^2}{\sqrt{N_r}} \right) = \frac{4\sigma^2}{N_r / \sqrt{N_r}} \text{ respectively. Since } \sqrt{N_r} \text{ tends to a constant, i.e. } \sqrt{N_r} \text{ to } 2\sigma \sqrt{N_r} \text{ and, from (8), the ABEP reduces to } \text{ABEP}_{\gamma} \text{ to } (N_r / 2) Q(\sqrt{N_r}\text{SNR}). \text{ We notice that the ABEP has a typical “waterfall” behavior and the effect of fading is drastically reduced. However, even though the AWGN is negligible with respect to multiple–access interference, we still have an error–floor. But it is much reduced. In particular, if } \text{SNR}_{\gamma} \gg 1 \text{ and } \text{SNR}_\gamma / \text{SNR}_{\gamma} \gg 1, \text{ then the number of receive–antenna } N_r^* \text{ that provide the target ABEP}_{\gamma} \text{ is equal to } (1 / \text{SNR}) \left\{ Q^{-1} \left( 2\text{ABEP}_{\gamma} / N_t \right) \right\}^2. \text{ This is a very simple formula that can be used for a simple system design when the assumption } N_t \gg 1 \text{ is reasonable.} \]

\[ 0 \text{ we have just remarked that multiple–access interference can be, in part, mitigated by adding more antennas at the receiver. We are interested in analyzing the asymptotic ABEP when } N_r \text{ is very large. To derive this result, we start from the last equality in the first line of (8). From [30, Eq. (2–1–136) and Eq. (2–1–139)], it follows that the RV } \sqrt{N_r} \text{ has mean and variance equal to } E\left\{ \sqrt{N_r} \right\} = 2\sigma^2 \text{ and } E\left( \frac{N_r}{\sqrt{N_r}} \right) = N_r \text{ and } E\left( \frac{\left( \sqrt{N_r} - E \left( \sqrt{N_r} \right) \right)^2}{\sqrt{N_r}} \right) = \frac{4\sigma^2}{N_r / \sqrt{N_r}} \text{ respectively. Since } \sqrt{N_r} \text{ tends to a constant, i.e. } \sqrt{N_r} \text{ to } 2\sigma \sqrt{N_r} \text{ and, from (8), the ABEP reduces to } \text{ABEP}_{\gamma} \text{ to } (N_r / 2) Q(\sqrt{N_r}\text{SNR}). \text{ We notice that the ABEP has a typical “waterfall” behavior and the effect of fading is drastically reduced. However, even though the AWGN is negligible with respect to multiple–access interference, we still have an error–floor. But it is much reduced. In particular, if } \text{SNR}_{\gamma} \gg 1 \text{ and } \text{SNR}_\gamma / \text{SNR}_{\gamma} \gg 1, \text{ then the number of receive–antenna } N_r^* \text{ that provide the target ABEP}_{\gamma} \text{ is equal to } (1 / \text{SNR}) \left\{ Q^{-1} \left( 2\text{ABEP}_{\gamma} / N_t \right) \right\}^2. \text{ This is a very simple formula that can be used for a simple system design when the assumption } N_t \gg 1 \text{ is reasonable.} \]
similar conclusions can be drawn for QAM as well. For a fair comparison, i.e., to guarantee the same bit rate, we assume \( N_t = M \) and use the symbol \( N_t \) in what follows. By using the formulas valid in the asymptotic regime, the ratio in (11) on top of this page can be computed, which holds for both noise- and interference-limited scenarios.

By looking into (11), the following conclusions can be made: i) SSK modulation will never be superior to PSK modulation if \( s_{\xi}^{(y)} - s_{\xi}^{(x)} \geq 2 \). This happens, e.g., when \( N_t = M = 2 \) or \( N_t = M = 4 \). On the other hand, when \( N_t = M > 4 \) we can expect that a crossing point exists and that SSK modulation outperforms PSK modulation. In other words, the higher the target bit rate is, the more advantageous SSK modulation is. This conclusion holds for QAM as well. This result was argued by simulation in [2] for a noise–limited scenario, but no proof was given. Also, we have shown that the trend holds in the presence of multiple-access interference as well; and iii) by direct inspection of (11), we can compute the asymptotic SNR and SIR gains of a modulation scheme with respect to the other as \( \Delta_{\text{SNR}} = \Delta_{\text{SIR}} = (10/N_t) \log_{10} \left( \frac{\text{ABEP}_{\text{PSK}}}{\text{ABEP}_{\Xi}} \right) \).

V. GSSK MODULATION WITH SINGLE–USER DETECTION

The working principle of GSSK modulation is as follows [8]: i) each user encodes blocks of \( \left[ \log_2 \left( \frac{N_t}{N_{\text{ta}}} \right) \right] \) bits into a point of a spatial–constellation diagram of size \( N = 2^{\left[ \log_2 \left( \frac{N_t}{N_{\text{ta}}} \right) \right]} \), which enables \( N_{\text{ta}} \) antennas to be switched on for data transmission while all the other antennas are kept silent, and ii) similar to SSK modulation, the receiver solves a \( N \)--hypothosis testing problem to estimate the \( N_{\text{ta}} \) antennas that are not idle, which results in the estimation of the message emitted by the encoder of the probe link.

Similar to Section III, received signal and ML–optimum detector are given in (12) on top of the next page, where \( D_{\Xi}^{(x^{(n)})} = \sum_{r=1}^{N_r} \int_{T_s} \left| z^{(r)}(t) - \sum_{n=1}^{N_{\text{ta}}} \sqrt{(E_{\Xi}/N_{\text{ta}})} s_{\xi}^{(n)(r)} w(t) \right|^2 dt; \)
**A. Asymptotic Analysis and Comparison with SSK Modulation**

Similar to SSK modulation, we can analyze the performance in noise- and interference-limited scenarios. In particular, the APEP in (13) reduces to $\dfrac{\text{APEP}_{\text{SSK}}(x_\xi \rightarrow y_\xi)}{\text{APEP}_{\text{GSSK}}(x_\xi \rightarrow y_\xi)} \rightarrow \left( \dfrac{2N_{\text{ta}}}{N_{\text{ta}}} \right)^{N_r} $, which lies in the interval $0 \leq \Delta_\gamma \leq 10 \log_{10}(N_{\text{ta}})$. This result is very important because it shows that the larger the number, $N_{\text{ta}}$, of active antennas is, the worse GSSK modulation with respect to SSK modulation is. The intuitive reason for this trend is as follows. If $N_{\text{ta}}^a(x_\xi, y_\xi) < N_{\text{ta}}$, it means that $x_\xi$ and $y_\xi$ have some antenna-indexes in common, which cancel out in the hypothesis testing problem. Since the transmit-energy is distributed among the active antennas, this results in a destructive interference cancellation effect:

$$
\begin{align*}
\text{APEP}_{\text{SSK}}(x_\xi \rightarrow y_\xi) & = \frac{N_{\text{ta}}}{N_{\text{ta}}} \sum_{(x_\xi, y_\xi) \in \Theta_{\text{SSK}}} \frac{\text{SNR}_{\xi}}{2 + \text{INR}_{\xi}} \frac{N_{\text{ta}}^a(x_\xi, y_\xi)}{N_{\text{ta}}} \\
\text{APEP}_{\text{GSSK}}(x_\xi \rightarrow y_\xi) & = \frac{N_{\text{ta}}}{N_{\text{ta}}} \sum_{(x_\xi, y_\xi) \in \Theta_{\text{GSSK}}} \frac{\text{SNR}_{\xi}}{2 + \text{INR}_{\xi}} \frac{N_{\text{ta}}^a(x_\xi, y_\xi)}{N_{\text{ta}}}
\end{align*}
$$

where $\Theta_{\text{SSK}}$ and $\Theta_{\text{GSSK}}$ are the spatial–constellation diagrams of SSK and GSSK modulations, respectively.
\[ \hat{x} = \arg \min_{y' \in \Delta} \left\{ D(y') \right\} = \arg \min_{y' \in \Delta} \left\{ \sum_{r=1}^{N_r} \int_{T_r} \left| z^{(r)}(t) - \sum_{u=1}^{N_u} \sqrt{E_u h_u^{(y_u,r)}} w(t) \right|^2 dt \right\} \] (15)

\[
\begin{align*}
\text{ABEP} & \lesssim \frac{1}{N_t} \sum_{x=1}^{N_x} \sum_{y=1}^{N_y} \sum_{x=1}^{N_x} \sum_{y=1}^{N_y} \left[ N_p \sum_{x=1}^{N_x} \sum_{y=1}^{N_y} \frac{1}{\log_2(N_t)} \text{ABEP}_{x \neq y} (x \rightarrow y) \right] \\
\text{ABEP}_{x \neq y} (x \rightarrow y) & \lesssim (1 - \delta_{x,y}) \text{E} [\{x \neq y\}] \approx (1 - \delta_{x,y}) \text{E} [\{x \neq y\}] \\
\text{E} [\text{PEP} (x \rightarrow y)] & = \left\{ \frac{1}{2} \left( 1 - \sqrt{\frac{\text{AggrSNR}}{2 + \text{AggrSNR}}} \right) \right\}^{N_r} \sum_{r=1}^{N_r} \left\{ N_r - 1 + r \right\} \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{\text{AggrSNR}}{2 + \text{AggrSNR}}} \right) \right]\right\} (18)
\end{align*}
\]

we transmit power on the common indexes, but it does not contribute to the ML–optimum decision process. Thus, for better performance we should keep the number of active antennas as small as possible. Finally, we emphasize that the conclusions in i) and ii) hold for the ABEP too, as it can be derived from (9) and (13).

VI. Multi–User Detection: Analysis and Comparison

In the sections above, we have studied the ABEP when the receiver is interference–unaware and exploits, for data detection, only the CSI of the probe link. The main advantage of this receiver is the low computational complexity, while its main disadvantage is the error–floor when multiple–access interference is the dominant effect. In this section, we study the performance of the ML–optimum joint multi–user detector [21], which exploits CSI of all the active users, and, thus, is interference–aware. We compute accurate union–bound estimates of the ABEP for all the modulation schemes (SSK, GSSK, PSK, and QAM) of interest, and, through asymptotic analysis, we provide some weaker bounds to better understand the behavior of the detector. Due to space constraints, we provide a detailed derivation of the ABEP of SSK modulation, and give only the final result for the other modulation schemes. Finally, we emphasize that with respect to, e.g., [21] our study does not exploit any signature code for multiple–access capabilities, but exploits only the differences/randomness of the CIRs among the active users. This agrees with the definition of SDMA/ChDMA multiple–access schemes described in Section II.

Let us consider SSK modulation. The received signal is always given by (1), but the detector is different. In particular, the ML–optimum joint multi–user detector is given in (15) on top of this page [21], [27], where the following notation is used: i) \( x = [x_1, x_2, \ldots, x_{N_t}] \) is the vector of antenna–indexes that is actually active in the considered time–slot; ii) \( x = [x_1, x_2, \ldots, x_{N_t}] \) is its ML–optimum estimate; iii) \( y = [y_1, y_2, \ldots, y_{N_t}] \) is the trial instance of \( x \) used in the hypothesis–testing problem; and iv) \( D(y) = \sum_{r=1}^{N_r} \int_{T_r} \left| z^{(r)}(t) - \sum_{u=1}^{N_u} \sqrt{E_u h_u^{(y_u,r)}} w(t) \right|^2 dt \).

Using arguments similar to (3), the ABEP of a generic user, e.g., \( \xi \), can be upper–bounded as shown in (16) on top of this page, where: i) in \(^{(a)}\), the scaling factor \( \frac{1}{N_t} \) takes into account that the \( N_t \) possible vectors \( x \) are equiprobable; ii) \(^{(b)}\) in \( \lesssim \), \( N_H (x_\xi, y_\xi) / \log_2 (N_t) \) accounts for the percentage of wrong bits between \( x_\xi \) and \( y_\xi \), which is related to bit–to–antenna–index mapping. It is worth mentioning that, as far as user \( \xi \) is concerned, an error occurs if and only if \( x_\xi \neq y_\xi \). In other words, even though \( x \neq y \), this does not imply that we have an error for all the \( N_u \) users. In the best case, and error occurs for one user only; iii) in \(^{(a)}\), the ABEP is conditioned upon the event \( x_\xi \neq y_\xi \) to take into account that we are interested in computing the ABEP of user \( \xi \); iv) in \(^{(b)}\), the factor \( (1 - \delta_{x_\xi,y_\xi}) \) takes into account that, as mentioned above, there is no contribution to the ABEP if \( x_\xi = y_\xi \), even though \( x \neq y \); and v) \(^{(c)}\) tells us that the ABEP is uniquely determined by the PEPs of the pair \( (x,y) \), i.e., PEP \( (x \rightarrow y) \).

The PEP, PEP \( (x \rightarrow y) \), conditioned upon all the fading channel gains, can be computed by using analytical steps similar to Section III. The final result is as follows:

\[
\text{PEP} (x \rightarrow y) = Q \left( \frac{1}{\sqrt{\text{AggrSNR}}} \sum_{r=1}^{N_r} \sum_{u=1}^{N_u} \sqrt{E_u} \left( h_u^{(y_u,r)} - h_u^{(x_u,r)} \right) \right)^2 = \left( \frac{1}{\sqrt{\text{AggrSNR}}} \sum_{r=1}^{N_r} \sum_{u=1}^{N_u} \sqrt{E_u} \left( h_u^{(y_u,r)} - h_u^{(x_u,r)} \right) \right)^2 (17)
\]

Finally, by exploiting, similar to (8), the properties of Chi–Square RVs, we can remove the conditioning over all fading channel statistics. After some algebra, and using [30, Eq. (2–1–136), Eq. (2–1–137)] and [30, Eq. (14–4–14), Eq. (14–4–15)], we can obtain (18) on top of this page, where \( \text{AggrSNR} = \sum_{u=1}^{N_u} \left\{ \left[ E_u \sigma_u^2 (1 - \delta_{x_u,y_u}) / N_0 \right] \right\} \) is the Aggregate SNR. We note that the delta function, \( \delta_{x_u,y_u} \), in \( \text{AggrSNR} \) takes into account that if \( x_u = y_u \), then \( h_u^{(y_u,r)} - h_u^{(x_u,r)} = 0 \) in (17) and, thus, it does not contribute to the SNR seen by the detector. In other words, the antenna–indexes shared by \( x \) and \( y \) cancel out in the hypothesis–testing problem.

By comparing (8) and (18), we notice, as expected, that the main difference between single– and multi–user detector is the absence of error–floor for high SNRs in (18), i.e., \( \text{E} [\{\text{PEP} (x \rightarrow y)\}] \to 0 \) if \( N_0 \to 0 \). The price to be paid
The only difference is that in spatial-multiplexing MIMO we use GSSK modulations. The performance comparison is postponed to Section VIII for various system setups. As far as QAM and PSK modulation are concerned, it is worth mentioning that: i) formally, the ABEP of both modulation schemes is the same, but the signal-constellation diagram is different; and ii) thanks to the assumptions of synchronous multiple-access interference and SDMA/ChDMA multiple-access scheme, the framework in Table II is useful to model spatial-multiplexing MIMO systems with ML-optimum detection as well. More specifically, in this case $N_u$ streams are simultaneously transmitted by a single user equipped with $N_c = N_u$ antennas [32]. The only difference is that in spatial-multiplexing MIMO we are interested in the average ABEP among all the streams, i.e., $\text{ABEP} = (1/N_u) \sum_{u=1}^{N_u} \text{ABEP}_u$, which can be derived from Table II. Some simulation results about spatial-multiplexing MIMO schemes are given in Section VIII.

A. SSK Modulation: Asymptotic Analysis and Bounds

In this section, we study the asymptotic ABEP and compute some bounds to shed lights on the performance of multi-user detectors for SSK modulation. For noise-limited scenario, in what follows we use the short-hand called Single-User-Lower-Bound (SULB), as it provides the performance without multiple-access interference reduces to $\text{ABEP}^{\text{SULB}} = 2 \sum_{u=1}^{N_u} \left[ E_u \sigma_u^2 \left( 1 - \delta_{x_u,y_u} \right) \right]$. As expected, $\text{ABEP}^{\text{SULB}}$ is equivalent to the ABEP of the single-user detector computed in Section III-A for the noise-limited scenario. It is interesting to understand the relation between the ABEP of the multi-user detector and the SULB. By direct inspection, the extra SNR needed in a multi-user scenario to achieve the same ABEP as in a noise-limited scenario is given in (20) on top of this page. The formula in (20) provides a quite accurate estimate of the extra SNR to get the same ABEP as in a noise-limited environment. However, it is not much insightful because it explicitly depends on the bit-to-antenna-index mapping. To deeper understand, we analyze some special cases and provide some weaker bounds, which better reveal the behavior of multi-user detection for SSK modulation.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>ABEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSK/QAM</td>
<td>$\text{ABEP} \leq \frac{1}{M \log N} \sum_{u=1}^{M} \sum_{v=1}^{N_u} \sum_{y_N=1}^{N_u} \sum_{y_{N_u}=1}^{N_u} \left( \frac{N_H (x_v, y_{N_u})}{\log_2 (M)} \right) \text{APEP}<em>{\xi \not\in y</em>{N_u}} (s(x) \rightarrow y(y))$</td>
</tr>
<tr>
<td>GSSK</td>
<td>$\text{ABEP} \leq \frac{1}{M \log N} \sum_{u=1}^{M} \sum_{v=1}^{N_u} \sum_{y_N=1}^{N_u} \sum_{y_{N_u}=1}^{N_u} \left( \frac{N_H (x_v, y_{N_u})}{\log_2 (N_u)} \right) \text{APEP}<em>{\xi \not\in y</em>{N_u}} (s(x_1, y_{N_u}) \rightarrow y_{1:N_u})$</td>
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</table>

$$\text{AggrSNR} = \frac{1}{M \log N} \sum_{u=1}^{M} \sum_{v=1}^{N_u} \sum_{y_N=1}^{N_u} \sum_{y_{N_u}=1}^{N_u} \left( \frac{1}{1 - \delta_{x_u,y_u}} \right) (19)$$

$$\Delta_{\text{SNR}_{\xi}} \rightarrow 10 \log_{10} \left( \frac{\text{ABEP}\_\xi}{\text{ABEP}\_\xi^{\text{SULB}}} \right) = 10 \log_{10} \left( \frac{1}{N_{t}^{\text{SNR}^{\xi}}} \sum_{x} \sum_{y} \left( \frac{1}{1 - \delta_{x,y}} \right) \frac{N_H (x, y) E\sigma_{\xi}^2}{2 \sum_{u=1}^{N_u} \left[ E_u \sigma_u^2 \left( 1 - \delta_{x_u,y_u} \right) \right]} \right) (20)$$

The asymptotic ABEP for GSSK modulation, we have explicitly used the identity $N = 2 \log_2 \left( \frac{N_u}{N_t} \right)$, and $x_{1:N_u} = [x_1, x_2, \ldots, x_{N_u}]$ is used as a short-hand for "vectors of vectors". Furthermore, $s(x) = \left[ s_1(x_1), s_2(x_2), \ldots, s_{N_u}(x_{N_u}) \right]$. 

TABLE II

Abbe of PSK, QAM, and GSSK Modulations with Multi-User Detection. For GSSK modulation, we have explicitly used the identity $N = 2 \log_2 \left( \frac{N_u}{N_t} \right)$, and $x_{1:N_u} = [x_1, x_2, \ldots, x_{N_u}]$ is used as a short-hand for "vectors of vectors". Furthermore, $s(x) = \left[ s_1(x_1), s_2(x_2), \ldots, s_{N_u}(x_{N_u}) \right]$. 

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$$\text{AggrSNR} = \frac{1}{M \log N} \sum_{u=1}^{M} \sum_{v=1}^{N_u} \sum_{y_N=1}^{N_u} \sum_{y_{N_u}=1}^{N_u} \left( \frac{1}{1 - \delta_{x_u,y_u}} \right) (19)$$

$$\Delta_{\text{SNR}_{\xi}} \rightarrow 10 \log_{10} \left( \frac{\text{ABEP}\_\xi}{\text{ABEP}\_\xi^{\text{SULB}}} \right) = 10 \log_{10} \left( \frac{1}{N_{t}^{\text{SNR}^{\xi}}} \sum_{x} \sum_{y} \left( \frac{1}{1 - \delta_{x,y}} \right) \frac{N_H (x, y) E\sigma_{\xi}^2}{2 \sum_{u=1}^{N_u} \left[ E_u \sigma_u^2 \left( 1 - \delta_{x_u,y_u} \right) \right]} \right) (20)$$

for this performance improvement is a higher computational complexity, as the detector in (15) has a complexity that increases exponentially with $N_u$. By using a similar analytical derivation, Table II summarizes the ABEP of PSK, QAM, and GSSK modulations. The performance comparison is postponed to Section VIII for various system setups. As far as QAM and PSK modulation are concerned, it is worth mentioning that: i) formally, the ABEP of both modulation schemes is the same, but the signal constellation diagram is different; and ii) thanks to the assumptions of synchronous multiple access interference and SDMA/ChDMA multiple access scheme, the framework in Table II is useful to model spatial multiplexing MIMO systems with ML optimum detection as well. More specifically, in this case $N_u$ streams are simultaneously transmitted by a single user equipped with $N_c = N_u$ antennas [32]. The only difference is that in spatial multiplexing MIMO we are interested in the average ABEP among all the streams, i.e., $\text{ABEP} = (1/N_u) \sum_{u=1}^{N_u} \text{ABEP}_u$, which can be derived from Table II. Some simulation results about spatial multiplexing MIMO schemes are given in Section VIII.
\[
\begin{align*}
\text{ABEP}_u^L \leq \text{ABEP}_u^L \leq \text{ABEP}_u^U \leq 2^{-\left(N_r+1\right)N_t \left(\frac{2N_r - 1}{N_r}\right) \left(\frac{E_u \sigma_u^2}{N_0}\right) - N_r} \leq \text{ABEP}_u^L \leq 2^{-\left(N_r+1\right)N_t \left(\frac{2N_r - 1}{N_r}\right) \left(\frac{E_u \sigma_u^2}{N_0}\right) - N_r}
\end{align*}
\]

2) \(E_u \sigma_u^2 \ll E_u \sigma_u^2 \forall u = 1, 2, \ldots N_u\) (strong interference scenario): Let us assume that among the \(N_u\) users there is a user, henceforth called “worst” (\(w\)) user, with the worst propagation channel. The ABEP of this user can be readily estimated with arguments similar to [21], as the user experiences very strong aggregate interference from the remaining \(N_u - 1\) users. In this case, it is known that the multi–user detector can perfectly estimate and reduce to the interference generated by the other users. In this case, its ABEP tends to the SULB, i.e., \(\text{ABEP}_w^L \rightarrow 2^{-\left(N_r+1\right)N_t \left(\frac{2N_r - 1}{N_r}\right) \left(\frac{E_u \sigma_u^2}{N_0}\right) - N_r}\).

3) \(E_u \sigma_u^2 \gg E_u \sigma_u^2 \forall u = 1, 2, \ldots N_u\) (weak interference scenario): Let us assume that among the \(N_u\) users there is a user, henceforth called “best” (\(b\)) user, with the best propagation channel. This scenario is more complicated to study than the strong interference case. However, we provide a tight bound on the ABEP. If \(E_u \sigma_u^2 \gg E_u \sigma_u^2 \forall u = 1, 2, \ldots N_u\), then from (18) we have \(\text{AggrSNR} \rightarrow (E_u \sigma_u^2)/N_0\). Accordingly, for high SNR we get:

\[
\text{ABEP}_b \rightarrow \frac{1}{N_t^- \log_2 (N_t)} \left(\frac{2N_r - 1}{N_r}\right) \left[\frac{2E_u \sigma_u^2}{N_0}\right]^{-N_r}
\]

By direct inspection, it is easy to show that \(\sum_x \sum_y N_H (x_\xi, y_\xi) = N_t^{-1} \left[\frac{2N_r - 1}{N_r}\right] \left(\frac{2E_u \sigma_u^2}{N_0}\right) - N_r\), which is a very simple and easy-to-compute formula. Thus, the SNR gap with respect to the SULB can be computed as \(\Delta_{\text{SNR}} = (10/N_r) \log_{10} \left(\text{ABEP}_b/\text{ABEP}_{\text{SULB}}^U\right) = 10 \left[\left(N_u - 1\right)/N_r\right] \log_{10} (N_t)\). This formula is very insightful, as it provides a simple relation among all the parameters of interests, and, so, can be used for a quick system design. For example, for a given \(\Delta_{\text{SNR}}\), \(N_t\) and \(N_r\), we can compute the maximum number of users that can share the wireless medium to guarantee the desired ABEP. Also, we notice that the larger \(N_t\) is, the smaller \(\Delta_{\text{SNR}}\) is, and the ABEP turns out to be very close to the SULB.

4) Generic user (arbitrary interference scenario): The analysis of the ABEP for a generic user can accurately be performed by using (16) and (18), or by using the asymptotic result for \(\text{AggrSNR} \gg 1\). However, its performance can be easily lower– and upper–bounded as \(\text{ABEP}_u^L \leq \text{ABEP}_u \leq \text{ABEP}_u^U\).

\[
\begin{align*}
\text{ABEP}_u^L &= 2^{-\left(N_r+1\right)N_t \left(\frac{2N_r - 1}{N_r}\right) \left(\frac{E_u \sigma_u^2}{N_0}\right) - N_r} \\
\text{ABEP}_u^U &= 2^{-\left(N_r+1\right)N_t \left(\frac{2N_r - 1}{N_r}\right) \left(\frac{E_u \sigma_u^2}{N_0}\right) - N_r}
\end{align*}
\]

B. GSSK Modulation: Asymptotic Analysis and Bounds

As far as GSSK modulation is concerned, we can perform a similar asymptotic analysis. In particular, insightful bounds can be obtained by combining the study in Section V-A and in Section VI-A. More specifically, for a generic user, the ABEP is lower– and upper–bounded as shown in (23) on top of this page. In particular, \(\text{ABEP}_u^U\) corresponds to the SULB of GSSK modulation. It actually depends on the number, \(N_t^\phi (x_\xi, y_\xi)\), of non–shared antenna–indexes of \(x_\xi\) and \(y_\xi\). However, in Section V-A we have proved that \(N_t^\phi (x_\xi, y_\xi)\) can be lower–bounded by the SULB of SSK modulation, i.e., \(\text{ABEP}_u^L\) in (23), as well as upper–bounded by considering the worst–case scenario with \(N_t^\phi (x_\xi, y_\xi) = 2\) for every \(x_\xi\) and \(y_\xi\), as given in the right hand–side of (23). On the other hand, \(\text{ABEP}_u^L\) corresponds, similar to Section VI-A, to the worst–case scenario with weak interference. Its lower– and upper–bound shown in (23) can be obtained by setting \(N_t^\phi (x_\xi, y_\xi) = N_t^\phi (x_\xi, y_\xi)\). This result is very insightful, as it provides a simple relation among all the parameters of interests, and, so, can be used for a quick system design. For example, for a given \(\Delta_{\text{SNR}}\), \(N_t\) and \(N_r\), we can compute the maximum number of users that can share the wireless medium to guarantee the desired ABEP. Also, we notice that the larger \(N_t\) is, the smaller \(\Delta_{\text{SNR}}\) is, and the ABEP turns out to be very close to the SULB.
transmit diagram. With this approach, CoMP–SSK modulation can form a virtual (distributed and very large) spatial–constellation in each BS. Accordingly, the virtual MIMO system of \( N_t \) rates by exploiting just the spatial–constellation diagram. Also, modulation as a practical way of achieving very large bit rates need to be encoded into the signal–constellation diagram. In this paper, we are only interested in putting forth the concept of CoMP–SSK or CoMP–GSM are possible with their own advantages and disadvantages. A comprehensive study of all these solutions is out of the scope of this paper. In this paper, we are only interested in putting forth the concept of CoMP–SSK modulation as a practical way of achieving very large bit rates by exploiting just the spatial–constellation diagram. Also, we wish to understand the asymptotic performance gain of SSK modulation as \( N_t \) increases without bound. In Section VIII, we will show various results for, e.g., \( N_t > 64 \), which might not be achievable, in practice, with a single BS, but can be obtained by resorting to a CoMP approach (e.g., with \((N_t^{BS}, N_t^{AR}) = (4, 16)\) or \((N_t^{BS}, N_t^{AR}) = (8, 8)\)).

With respect to conventional BS cooperation methods [36], in CoMP–SSK modulation the backhaul has less stringent requirements as the coordinated BSs do not have to exchange data for cooperative beamforming, but the backhaul is used only for disseminating the information from the core network to the BSs. Furthermore, we emphasize that since the cooperative BSs do not perform distributed beamforming, no transmit–CSI is required, even though it might be beneficial [31].

VIII. NUMERICAL AND SIMULATION RESULTS

In this section, we provide some numerical results to compare the performance of various modulation schemes in the presence of multiple–access interference, and to substantiate our analytical findings. In particular, the system model introduced and described in Section II is accurately reproduced in our simulation environment, and various MIMO setups and interference scenarios are analyzed. The specific simulation parameters can be found in the caption of each figure. The results are obtained by assuming \( E_{tu} = E_{m} \forall u = 1, 2, \ldots, N_u \). So, the near–far effect is modeled through the fading parameters \( \sigma_u^2 \).

In Fig. 1, we observe the near–far effect for two MIMO setups with a different number of receive–antennas. As computed analytically in Section III, the ABEP gets worse as the interference increases. Also, the system is more robust to multiple–access interference as \( N_t \) increases. We notice that our analytical model (union–bound) is very accurate. Only when \( \text{ABEP} \geq 10^{-3} \), it starts being less accurate. This is a reasonable outcome as the union–bound is tight only for low values of ABEP, while the interference introduces an error–floor. However, the model can, in general, well track the error–floor, as predicted in Section III. In Fig. 2, we study the robustness of single–user detection to the number of active users \( N_u \). As predicted by the union–bound in Section III, the ABEP gets worse when increasing \( N_u \), but the receiver can work quite well when \( N_t \geq 3 \). We notice that the error–floor increases with \( N_u \). In Fig. 3 and Fig. 4, we compare the performance of SSK, GSSK, QAM, and PSK modulations for various target bit rates. We remind the reader that for large values of \( N_t \), SSK modulation is implemented through the CoMP approach. As predicted in Section IV and Section V, QAM and PSK modulation outperform SSK modulation only if the bit rate if less than 2 bits/s/Hz, and SSK modulation always outperforms GSSK modulation. Also, the higher the target bit rate is, the larger the gap is. This confirms the findings in Section IV, and highlights that using the spatial–constellation diagram is beneficial with and without multiple–access interference. Overall the bounds can very well capture the behavior of all the modulation schemes if the error floor is not too high. This means that they are useful for all scenarios of practical interest. In Fig. 5 and in Fig. 6, we

\[(10/N_t) \log_{10} \left( \frac{2^{2N_t[\log_2(N_t,N_u)]}}{N_t} \right) \]

\( \Delta_{SNR} \) provides reasonable and insightful outcomes about the behavior of GSSK modulation. Finally, we emphasize that ABEP \( u \) is the SULB of SSK modulation, and, so, we can readily estimate the relation among the two modulation schemes for generic MIMO systems.
analyze the performance of SSK modulation with respect to the number of receive–antennas. If $N_r = 3$ (Fig. 5), we observe a non–negligible performance gain, if the bit rate is greater than 2 bits/Hz, provided by SSK modulation with respect to QAM. The price to be paid is, of course, the need to exploit the CoMP principle to achieve very high bit rates, e.g., when $N_t = 64$. However, the SNR gain is so significant to motivate the CoMP approach. On the other hand, if $N_r = 1$ (Fig. 6), we notice that QAM is always superior to SSK modulation, while SSK modulation is better than PSK and GSSK modulations. This result is not available in the literature even for $N_u = 1$, as the vast majority of papers typically consider MIMO setups with $N_r > 1$. As a consequence, if a single–user scenario is considered and the receiver can be equipped with only one receive–antenna, then SSK modulation is not the best choice and we should use QAM. In all the other cases, SSK modulation is superior to QAM. Furthermore, in a multi–user scenario we can very unlikley design and use a MIMO system with $N_r > 1$, since, as shown in Fig. 6 for $N_u = 2$, the ABEP rapidly gets worse with the target bit rate. In scenarios with multi–user interference and single–user detection we are forced to increase $N_r$ to get adequate performance. In these situations, SSK modulation is always better than QAM. Finally, in Fig. 7 we study the ABEP for very high bit rates (CoMP–SSK has a large number of cooperative BSs). We can observe a significant performance gain of SSK modulation with respect to all the other modulation schemes. Also, GSSK modulation

Fig. 1. ABEP of SSK modulation with single–user detection. Setup: $N_t = 8$; $\sigma_i^2 = 1$; $\sigma_i^2 = 10^{-2}$ for $i = 2, 3, \ldots, N_u$. Markers show Monte Carlo simulations and solid lines the analytical model (i.e., (8) and (9)). The ABEP of user 1 (probe/intended link) is shown. SULB stands for Single–User Lower Bound, i.e., it represents the scenario with no multiple-access interference.

Fig. 2. ABEP of SSK modulation with single–user detection. Setup: $N_t = 8$; $\sigma_i^2 = 1$; $\sigma_i^2 = 10^{-2}$ for $i = 2, 3, \ldots, N_u$. Markers show Monte Carlo simulations and solid lines the analytical model (i.e., (8) and (9)). The ABEP of user 1 (probe/intended link) is shown.

Fig. 3. ABEP of SSK (left) and GSSK (right) modulations with single–user detection. Setup: $\sigma_i^2 = 1$; $\sigma_i^2 = 10^{-2}$ for $i = 2, 3, \ldots, N_u$. $N_r = 2$. Markers show Monte Carlo simulations and solid lines the analytical model (i.e., (8) and (9) for SSK modulation and (13) for GSSK modulation). The ABEP of user 1 (probe/intended link) is shown.

Fig. 4. ABEP of PSK (left) and QAM (right) modulations with single–user detection. Setup: $\sigma_i^2 = 1$; $\sigma_i^2 = 10^{-2}$ for $i = 2, 3, \ldots, N_u$. $N_r = 2$. Markers show Monte Carlo simulations and solid lines the analytical model (i.e., the union–bound in the first and second row of Table I). The ABEP of user 1 (probe/intended link) is shown.
significantly outperforms QAM. Overall, our analysis and simulations confirm the potential benefits of using the spatial–constellation diagram in both single– and multi–user scenarios.

In Figs. 3, 4, 7, we have compared the ABEP of SSK/GSSK modulations and single–antenna PSK/QAM with the main goal of understanding the performance gap among these transmission technologies when ML–optimum performance can be achieved with low–complexity single–stream decoding at the receiver. In other words, the receiver has almost the same complexity for all the modulation schemes. However, when considering the complexity of the transmitter, the comparison in Figs. 3, 4, 7 might appear a bit unfair for single–antenna PSK/QAM systems, as SSK/GSSK modulations need large antenna–arrays to achieve the same transmission rate. Motivated by this consideration, in Fig. 8 we study a complementary situation in which PSK/QAM systems using spatial–multiplexing MIMO [39, Sec. I–A] are compared to SSK/GSSK modulations. In this case, PSK/QAM systems need a multi–stream decoder to guarantee ML–optimum performance. More specifically, in a point–to–point link the detector is the same as in Section VI, with the only exception that all the streams are simul–

\[ \text{ABEP} = \frac{\text{number of bit errors}}{\text{number of bit attempts}} \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]

\[ \text{ABEP} = \frac{1}{N_u} \sum_{n=1}^{N_u} \frac{1}{N_t} \sum_{m=1}^{N_t} \frac{1}{N_M} \sum_{k=1}^{N_M} \frac{1}{N_M} \sum_{k=1}^{N_M} \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{2 \sqrt{R_b} \sigma}{N_0} \right) \right) \]
taneously transmitted from the same device under the total transmit–power constraint. Unlike Figs. 3, 4, 7, in this case the comparison is certainly unfair for SSK/GSSK modulations, which need much less decoding complexity for ML–optimum performance. However, we feel important to show this setup and to analyze all the possibilities. For simplicity, we limit the study to the setup with \( N_u = 1 \), since ML–optimum decoding for spatial–multiplexing MIMO in the presence of multiple–access interference would require a two–fold multi–stream decoder to cope with inner (i.e., due to multiplexing many streams at the same transmitter) and outer (i.e., due to the multi–user scenario) interferences. This scenario would require a more practical and sub–optimal decoder, e.g., based on sphere–decoding [40], to keep the complexity at a reasonable level. Accordingly, this study is postponed to future research. From Fig. 8, important considerations can be made. We notice that by increasing the number of antennas at the transmitter, spatial–multiplexing MIMO with QAM achieves, as expected, better performance than single–antenna QAM. However, the price to pay for this performance improvement is, among the others, multi–stream decoding at the receiver. Very interestingly, we see that SSK modulation is never worse than spatial–multiplexing MIMO, even though SSK modulation needs just low–complexity single–user decoding. In this case the price to pay is the need to have multiple radiating elements at the transmitter, even though just one of them is active and, thus, no multiple transmit–chains are needed. Similar to [6] and [7], these results clearly show the potential advantages of using the spatial–constellation diagram when comparing SSK modulation to more complicated MIMO schemes. However, it should be emphasized that, because of the very different hardware and computational complexity requirements, no conclusive statements can be made about the superiority of one transmission technology against the other. The only pragmatic conclusion that can be drawn is the clear potential gain that might come from using the spatial–constellation diagram, along with the inherent performance/complexity trade–off among different modulation schemes.

In Figs. 9–17, we show that ABEP with multi–user detection. The setup is the same as in Figs. 1–7. So, the interested reader can readily compare single– and multi–user detection for the same operating conditions. In Fig. 9 and Fig. 10, we study the accuracy of the lower– and upper–bound derived in Section VI–A. We observe that, for various strong/weak interference scenarios, the bounds well track the behavior of the system. In particular, Fig. 10 shows that the ABEP of a generic user that is subject to neither strong nor weak interference is well bounded by our analytical frameworks. For those users where the assumption of strong/weak interference can be made, the bounds are asymptotically–tight. Figure 11 clearly shows that no error floor is present with multi–user detection, and the ABEP goes to zero if the noise is very small. This is an important result and the confirmation that both modulation and multiple–access can be guaranteed by exploiting only the randomness of the wireless channels. Similar to Fig. 3 and Fig. 4, in Figs. 12–15 we compare the performance of various modulation schemes. Also in this case, we notice the non–negligible performance gain of SSK modulation. Overall, the predictions in Section VI are confirmed, and the bounds developed in Section VI–A and Section VI–B agree with Monte Carlo simulations. A very interesting result is shown in Fig. 16. Unlike Fig. 6, we observe that, for a bit rate greater than 2 bits/s/Hz, with multi–user detection SSK modulation is not worse than QAM even if \( N_r = 1 \). In particular, we observe a crossing point for high SNRs, where the ABEP of SSK modulation is at least the same as QAM. This result clearly highlights that SSK modulation with multi–user detection is inherently more robust than QAM to multiple–access interference. Finally, Fig. 17 shows a result
In this paper, we have proposed a comprehensive framework to study the ABEP of SSK/GSSK modulations over Rayleigh fading channels with multiple-access interference. The frameworks are useful for single- and multi-user detectors. Furthermore, simple upper- and lower-bounds have been developed, and have been used to get insightful information about advantages and disadvantages of using the spatial-constellation diagram as a source of information. Clear indications about the system behavior for various channel conditions, interference levels, and MIMO setups have been provided. Comprehensive performance comparisons with conventional modulation schemes for single- and multi-user detection have been given. Overall, our theoretical findings have been well substantiated by Monte Carlo simulations.

Ongoing research is concerned with: i) the extension of the framework to the asynchronous multiple-access scenario and to SM/GSM; ii) the extension of the analysis to frequency-
Fig. 15. ABEP of QAM with multi-user detection. Setup: \( N_0 = 3 \); \( \sigma_t^2 = 1 \) and \( \sigma_\sigma^2 = 10^{-2} \) for \( i = 2, 3, \ldots, N_r \); \( N_r = 2 \). Markers show Monte Carlo simulations and solid lines the analytical model (i.e., the formula in the first row of Table II). The ABEP of user 1 (probe/intended link) is shown. It is worth mentioning that some simulation results (markers) are not shown due to the long simulation time for medium/high values of \( E_m/N_0 \).

Fig. 16. ABEP of SSK (blue and green lines for \( N_r = 1 \) and \( N_r = 3 \), respectively) and QAM (red and magenta lines for \( N_r = 1 \) and \( N_r = 3 \), respectively) modulations with multi-user detection. Setup: \( N_0 = 2 \); \( \sigma_t^2 = 1 \) and \( \sigma_\sigma^2 = 5 \times 10^{-2} \). Markers show Monte Carlo simulations and solid lines the analytical model (i.e., (16) and (18) for SSK modulation and the formula in the first row of Table II for QAM). The ABEP of user 1 (probe/intended link) is shown.

Fig. 17. ABEP of, on the left, SSK (blue curves) and QAM (green curves), and, on the right, PSK (red curves) and GSSK (magenta curves) modulations with multi-user detection. Setup: \( N_0 = 2 \); \( \sigma_t^2 = 1 \) and \( \sigma_\sigma^2 = 10^{-2} \); \( N_r = 3 \). For GSSK modulation we have: \( (N_1, N_3) = (5, 2) \) if \( N = 8 \); \( (N_1, N_3) = (8, 4) \) if \( N = 64 \) and \( (N_1, N_3) = (11, 4) \) if \( N = 256 \). For SSK and GSSK modulations, markers show Monte Carlo simulations and solid lines the analytical model (i.e., (16) and (18) for SSK modulation and the formula in the second row of Table II for GSSK modulation). For QAM and PSK modulations, only Monte Carlo simulations (markers plus solid lines) are shown for ease of readability. The ABEP of user 1 (probe/intended link) is shown. Also, dashed lines and dashed lines plus markers show the performance estimated through the bounds (i.e., \( \text{ABEP}_U \) in (22) for SSK modulation and \( \text{ABEP}_{\text{U}} \) in (23) for GSSK modulation). More specifically: for SSK modulation, dashed lines show the estimated upper-bound if \( N = 8 \) and \( N = 64 \), and dashed lines with markers the upper-bound if \( N = 8 \); and, for GSSK modulation, dashed lines show the estimated upper-bound if \( N = 8 \) and dashed lines with markers the upper-bound if \( N = 64 \) and \( N = 256 \). This is done to improve the readability of the figure. Note that, for GSSK modulation the estimated lower-bound corresponds exactly to the upper-bound computed for SSK modulation and shown on the left-hand side of the figure. It is worth mentioning that some simulation results (markers) are not shown due to the long simulation time for medium/high \( E_m/N_0 \).

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