Network Code Design from Unequal Error Protection Coding: Channel-Aware Receiver Design and Diversity Analysis

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Michela Iezzi(1), Marco Di Renzo(2), Fabio Graziosi(1)
(1) University of L'Aquila, College of Engineering
Department of Electrical and Information Engineering (DIEI), Center of Excellence for Research DEWS
Via G. Gronchi 18, Nucleo Industriale di Pile, 67100 L'Aquila, Italy
(2) L2S, UMR 8506 CNRS – SUPELEC – Univ Paris–Sud
Laboratory of Signals and Systems (L2S), French National Center for Scientific Research (CNRS)
École Supérieure d'Électricité (SUPÉLEC), University of Paris–Sud XI (UPS)
3 rue Joliot–Curie, 91192 Gif–sur–Yvette (Paris), France
E–Mail: marco.direnzo@lss.supelec.fr, {michela.iezzi, fabio.graziosi}@univaq.it

***Abstract*— In this paper, we propose Unequal Error Protection (UEP) coding theory as a viable and flexible method for the design of network codes for multi–source multi–relay cooperative networks. As opposed to state–of–the–art solutions available for improving the diversity gain of cooperative networks, it is shown that the proposed method allows us to assign each source node the desired diversity gain, according to, *e.g.*, the requested Quality–of–Service (QoS) or power constraints. The diversity advantage of the UEP–based network code design over conventional relay–only and XOR–only solutions is shown for the canonical two–source two–relay network. Furthermore, Maximum–Likelihood (ML–) optimum channel–aware receivers for multi–source multi–relay cooperative networks are developed, and their Average Bit Error Probability (ABEP) and achievable diversity over fading channels analytically studied. It is shown that only a cross–layer (joint) implementation of de–modulation and network–decoding allows the destination to fully exploit the diversity inherently provided by the distributed network code. Finally, analytical derivations and findings are substantiated via Monte Carlo simulations.**

I. Introduction

Network Coding (NC) is a recent field in engineering and computer science that generalizes conventional routing techniques: instead of simply forwarding data, intermediate network nodes may recombine several input packets into one or several output packets [1]. NC offers the promise of improved performance over state–of–the–art routing, and, in particular, it finds successful application in multi–hop/cooperative wireless networks [2], [3], as it offers an efficient way for boosting the achievable throughput (see [4], [5], and references therein).

In this context, the design of network codes for multi–hop/cooperative networks is receiving an upsurge of research interest. Originally, the design of network codes has mainly been concerned with methods to achieve the maximum information flow [1], [6]–[8]. However, in the recent period considerable effort has been devoted to the design of efficient network codes to attain the maximum diversity gain [9]–[11], which is known to determine the Bit Error Probability (BEP) for high Signal–to–Noise–Ratios (SNRs) [12]. More specifically, as far as a multi–source multi–relay cooperative scenario is concerned, in [9] it has been shown that binary NC is sub–optimal for achieving full–diversity, and in [11] it has been pointed out that *max–diversity–achieving* network codes can be obtained by resorting to the theory of non–binary linear block codes. For example, for the canonical two–source two–relay cooperative network the methods proposed in [9]–[11] can achieve full–diversity equal to three, when, instead, XOR–based binary NC can achieve only diversity equal to two. The solution proposed by all these papers to overcome the limitation in the achievable diversity is to use network codes in a non–binary Galois field. However, the price to be paid for this performance improvement is the additional complexity required at the relay nodes, which must network–code the received packets by using non–binary arithmetic. Also, longer decoding delays are, in general, required to design full–diversity–achieving network codes [11].

Motivated by these considerations, this paper aims at providing the following contributions:

- The network code design in [9]–[11] implicitly foresees that the sources involved in the cooperative protocol all require the same Quality–of–Service (QoS), and, thus, the same diversity gain. However, in a multi–source multi–relay scenario different sources are likely to have different QoS requirements. In this context, we believe that the development of a flexible network code design, which can accommodate a different diversity gain for each source, can be useful and valuable. In this paper, we show that this flexibility can be achieved by using Unequal Error Protection (UEP) coding theory [13], [14]. To the best of our knowledge, this is the first time that UEP coding theory is exploited for the design of distributed network codes for diversity purposes.
- In [9]–[11], network code design is performed under the assumption of the so–called *erasure* channel model. In other words, the system relies on powerful enough channel codes at the physical layer, which allow each relay to detect correct and wrong packets, and enable them to forward only the former ones. In this paper, we assume that the relays do not attempt to either recover from any errors or to apply error detection codes (*e.g.*, Cyclic Redundancy Check). We show that this approach leads to the design of network codes for the *error* channel. More specifically, we simply assume that the relay only Decode–and–Forward (DF) the received symbols, without attempting any guess on their reliability. Our main goal is to keep the complexity of the relay nodes at a very low level.
- We propose and study the performance of three network–wise detectors for optimal and sub–optimal de–modulation and network–decoding at the destination: i) Minimum Distance Decoder (MDD), ii) Hard–decision Maximum–Likelihood Decoder (H–MLD), and iii) Soft–decision Maximum–Likelihood Decoder (S–MLD). It is shown that they require different cross–layer interactions between physical and network layers (*i.e.*, Channel State Information – CSI), and provide different diversity, coding gain, and robustness to error propagation of wrong decoded bits. In particular, H–MLD and S–MLD receivers are best known as "channel–aware detectors" [15].
- We develop very accurate analytical frameworks to analyze the performance of the proposed channel–aware detectors

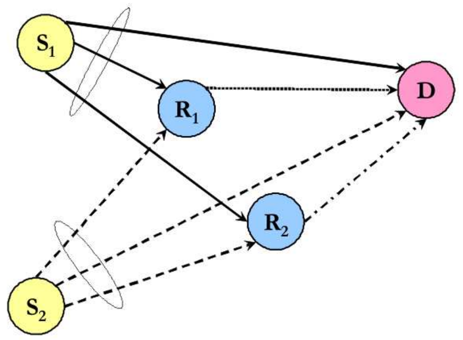

Fig. 1. Two–source two–relay network topology. Different line–styles denote transmission over orthogonal channels (*e.g.*, time–slots) to avoid mutual interference: S_1 transmits in time–slot 1 (solid lines), S_2 in time–slot 2 (dashed lines), R_1 in time–slot 3 (dotted lines), and R_2 in time–slot 4 (dashed–dotted lines).

using relay–only, XOR–only, and UEP–based network codes. It is shown that the proposed frameworks allows us to estimate the diversity gain of each receiver, and enable a simple comparison among the distributed network codes.

The paper is organized as follows. In Section II, system model and network code design are introduced. In Section III, channel–aware receivers are developed. In Section IV, the frameworks to compute Average BEP (ABEP) and diversity gain are summarized. In Section V, some numerical results are shown. Finally, Section VI concludes the paper.

II. System Model

In this paper, we study the canonical two–source two–relay cooperative network shown in Fig. 1. Generalization to more general networks is possible, but it is not considered due to space constraints. The working principle is as follows. In time slot $t = 1, 2$, source node S_t broadcasts a modulated symbol, x_{S_t}, with average energy E_m. For analytical tractability, Binary Phase Shift Keying (BPSK) modulation is considered. Accordingly, $x_{S_t} = \sqrt{E_m}\left(1 - 2b_{S_t}\right)$, where $b_{S_t} \in \{0, 1\}$ is the bit emitted by S_t. Thus, the bits received at relays R_1, R_2, and destination D are:

$$\left\{ \begin{array}{l} y_{S_t R_1} = h_{S_t R_1} x_{S_t} + n_{S_t R_1} \\ y_{S_t R_2} = h_{S_t R_2} x_{S_t} + n_{S_t R_2} \\ y_{S_t D} = h_{S_t D} x_{S_t} + n_{S_t D} \end{array} \right. \tag{1}$$

where h_{XY} is the fading coefficient from node X to node Y, which is a circular symmetric complex Gaussian random variable with zero mean and variance σ_{XY}^2 per dimension (Rayleigh fading). For analytical tractability, independent and identically distributed (i.i.d.) fading over all the wireless links is considered, *i.e.*, $\sigma_0^2 = \sigma_{XY}^2$ for any X and Y. Furthermore, n_{XY} is the complex Additive White Gaussian Noise (AWGN) at the input of node Y and related to the transmission from node X to node Y. The AWGN in different time slots is i.i.d. with zero mean and variance $N_0/2$ per dimension.

Upon reception of $y_{S_1R_1}$, $y_{S_1R_2}$, $y_{S_2R_1}$, $y_{S_2R_2}$, the relays R_1 and R_2 attempt to decode the bits transmitted by S_1 and S_2 in a similar fashion as in a DF cooperative protocol. As described in Section I, the relays perform NC without checking if the packet/symbol is correct or wrong. Accordingly, the operation performed by each relay might be called decode–network–code–and–forward (D–NC–F). The relays perform coherent ML–optimum demodulation as follows ($t = 1, 2$):

$$\left\{ \begin{array}{l} \hat{b}_{S_t R_1} = \underset{\tilde{b}_{S_t} \in \{0,1\}}{\arg\min} \left\{ \left| y_{S_t R_1} - \sqrt{E_m} h_{S_t R_1} \left(1 - 2\tilde{b}_{S_t}\right) \right|^2 \right\} \\ \hat{b}_{S_t R_2} = \underset{\tilde{b}_{S_t} \in \{0,1\}}{\arg\min} \left\{ \left| y_{S_t R_2} - \sqrt{E_m} h_{S_t R_2} \left(1 - 2\tilde{b}_{S_t}\right) \right|^2 \right\} \end{array} \right. \tag{2}$$

where $\hat{}$ denotes the detected symbol and $\tilde{}$ denotes the trial symbol used in the hypothesis–detection problem. Note that (2) requires CSI about the source–to–relay channels.

After detecting $\hat{b}_{S_t R_q}$ ($t = 1, 2$, $q = 1, 2$) each relay R_q: i) performs NC on these two bits, ii) re–modulates the network–coded bit, and iii) transmits the modulated bit to the destination during the third ($q = 1$) and fourth ($q = 2$) time–slot. By denoting with $f_{R_q}(\cdot, \cdot)$ the NC operation performed by relay R_q, *i.e.*, $b_{R_q} = f_{R_q}\left(\hat{b}_{S_1 R_q}, \hat{b}_{S_2 R_q}\right)$, the bit received at the destination D is:

$$y_{R_q D} = h_{R_q D} x_{R_q} + n_{R_q D} \tag{3}$$

where $x_{R_q} = \sqrt{E_m}\left(1 - 2b_{R_q}\right)$.

After 4 time slots, the destination has 4 received signals, *i.e.*, y_{S_1D}, y_{S_2D}, y_{R_1D}, y_{R_2D}, from which it infers the pair of symbols b_{S_1} and b_{S_2} transmitted by S_1 and S_2, respectively. As mentioned in Section I, three receivers are studied:

- MDD: a two–step detector with no cross–layer interaction between physical and network layers. It works as follows. First, the receiver performs coherent de–modulation on y_{S_1D}, y_{S_2D}, y_{R_1D}, y_{R_2D} individually. Then, the detected bits are sent to the network layer for network–decoding, and jointly detecting b_{S_1} and b_{S_2}.
- H–MLD: a two–step detector with cross–layer interaction between physical and network layers. It works as follows. First, the receiver performs coherent de–modulation on y_{S_1D}, y_{S_1D}, y_{R_1D}, y_{R_2D} individually. Then, the detected bits are sent to the network layer along with some information about their reliability: network–decoding for b_{S_1} and b_{S_2} is performed by exploiting this information. Thus, even though the relays forward all the bits (correct and wrong), the destination exploits, at the network layer, CSI about the physical layer to cope with error propagation.
- S–MLD: a one–step receiver where the network layer is "pushed down" to the physical layer, and de–modulation and network–decoding are jointly performed.

These detectors are explicitly derived in Section III, and their performance is accurately studied in Section IV.

A. *UEP Coding for the Design of Network Codes – Motivation*

In (3), we have implicitly described the network code used by relay R_q with $b_{R_q} = f_{R_q}(\cdot, \cdot)$. In this paper, four network codes (or NC scenarios) are analyzed:

- *Scenario 1*: $b_{R_1} = \hat{b}_{S_1R_1}$ and $b_{R_2} = \hat{b}_{S_1R_2}$. This network code corresponds to the working scenario in which relays R_1 and R_2 only decode–and–forward the signals received from sources S_1 and S_2, respectively (relay–only scenario) [2], [3].
- *Scenario 2*: $b_{R_1} = \hat{b}_{S_1R_1} \oplus \hat{b}_{S_2R_1}$ and $b_{R_2} = \hat{b}_{S_1R_2} \oplus \hat{b}_{S_2R_2}$, where $\oplus$ denotes bit–wise XOR operations. This network code corresponds to the working scenario in which relays R_1 and R_2 decode–network–code–and–forward the signals received from sources S_1 and S_2, respectively, by using conventional binary NC (XOR–only scenario) [9].
- *Scenario 3*: $b_{R_1} = \hat{b}_{S_1R_1} \oplus \hat{b}_{S_2R_1}$ and $b_{R_2} = \hat{b}_{S_2R_2}$. This scenario corresponds to using a distributed network code obtained from a (4,2,2) UEP code [16, Table I], where a higher diversity gain is assigned to source S_2.
- *Scenario 4*: $b_{R_1} = \hat{b}_{S_1R_1}$ and $b_{R_2} = \hat{b}_{S_1R_2} \oplus \hat{b}_{S_2R_2}$. This scenario corresponds to using a distributed network code obtained from a (4,2,2) UEP code [16, Table I], where a higher diversity gain is assigned to source S_1.

While *Scenario 1* and *Scenario 2* correspond to state–of–the–art distributed coding techniques [11], *Scenario 3* and *Scenario 4* are the flexible network codes we are interested in studying in this paper, as motivated in Section I. The reason why UEP coding theory can be a suitable tool to design distributed network codes for application scenarios in which each source requires a different diversity gain (see Section I) has its information–theoretic foundation in [17]. In fact, in [17] it is shown that the minimum distance of a network code plays the same role as it plays in classical coding theory. Furthermore, from classical coding theory we know that the minimum distance of a linear block code directly determines the diversity gain over fully–interleaved fading channels [18, Ch. 8]. In UEP linear codes, each systematic bit has its own minimum distance, and the set of these distances is known as *separation vector* [13], [14]. From [16, Table I], the network codes of *Scenario 3* and *Scenario 4* are (4,2,2) UEP distributed codes with separation vector $[2,3]$ and $[3,2]$, which means that the minimum distance for the bits sent by source S_1 is 2 and 3, while the minimum distance for the bits sent by source S_2 is 3 and 2, respectively. Thus, from [17] it follows that by using a network code constructed from UEP coding theory we can individually assign different diversity gains to different sources. Also, unlike [9]–[11], this is obtained by neither using a non–binary Galois field nor introducing extra delays. The complexity and decoding latency of all the network codes studied in this paper are, on the other hand, the same. The downside is that only one user can achieve full–diversity. Extension to multi–source multi–relay networks is possible from [16, Table I].

III. Channel–Aware Receiver Design

As mentioned in Section II, in this section we analyze three detectors (MDD, H–MLD, and S–MLD) that require different a priori CSI. Due to space constraints, we omit the details of the analytical derivation, and report only the final result. Furthermore, to understand the error propagation effect introduced by realistic source–to–relay channels, and, in particular, to analyze if there is any degradation in the diversity gain, we study a so–called *Benchmark* scenario, where no decoding errors at the relays are considered.

The detectors are based on the Maximum Likelihood Sequence Estimation (MLSE) criterion of optimality. In other words, given y_{S_1D}, y_{S_2D}, y_{R_1D}, y_{R_2D}, they estimate the distributed codeword that has most probably been transmitted [18]. From Section II-A, it follows that the codebook $\mathcal{C} = \left\{\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, \mathbf{c}^{(3)}, \mathbf{c}^{(4)}\right\}$ (*i.e.*, the set of distributed codewords that can be transmitted) for the four NC scenarios studied in this paper is[1]: i) $\mathcal{C} = \{0000, 0101, 1010, 1111\}$ for *Scenario 1*; ii) $\mathcal{C} = \{0000, 0111, 1011, 1100\}$ for *Scenario 2*; iii) $\mathcal{C} = \{0000, 0111, 1010, 1101\}$ for *Scenario 3*; and iv) $\mathcal{C} = \{0000, 0101, 1011, 1110\}$ for *Scenario 4*.

Notation. The following notation is used: i) $\bar{\gamma} = 2E_m/N_0$; ii) $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^{+\infty} \exp\left(-t^2/2\right) dt$ is the Q–function; and iii) $P_{XY} = Q\left(\sqrt{\bar{\gamma}\left|h_{XY}\right|^2}\right)$ is the BEP over the wireless link from node X to node Y.

A. Minimum Distance Decoder (MDD)

The MDD receiver works as follows:

[1] $\mathbf{c}^{(j)}$ is the j–th codeword of the $\mathcal{C}$, and $\mathbf{c}_i^{(j)}$ is the i–th element of $\mathbf{c}^{(j)}$.

a) Step 1 (Physical Layer): Hard–decision estimates of $[b_{S_1}, b_{S_2}, b_{R_1}, b_{R_2}]$ are provided by using a ML–optimum receiver with full–CSI about the source–to–destination and relay–to–destination channels ($t = 1, 2$ and $q = 1, 2$):

$$\begin{cases} \hat{b}_{S_tD} = \underset{\tilde{b}_{S_t} \in \{0,1\}}{\arg\min} \left\{ \left| y_{S_tD} - \sqrt{E_m} h_{S_tD} \left(1 - 2\tilde{b}_{S_t}\right)\right|^2 \right\} \\ \hat{b}_{R_qD} = \underset{\tilde{b}_{R_q} \in \{0,1\}}{\arg\min} \left\{ \left| y_{R_qD} - \sqrt{E_m} h_{R_qD} \left(1 - 2\tilde{b}_{R_q}\right)\right|^2 \right\} \end{cases} \tag{4}$$

b) Step 2 (Network Layer): The hard–decision estimates $\hat{\mathbf{c}} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4] = \left[\hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D}\right]$ are input to the network layer, which uses a minimum distance Hamming decoder [18] to retrieve the bits emitted by the sources:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\mathbf{c}^{(\tilde{j})} \text{ with } \tilde{j}=1,2,3,4}{\arg\min} \left\{ \sum_{i=1}^{4} \left|\hat{c}_i - c_i^{(\tilde{j})}\right| \right\} \tag{5}$$

We note that $\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right]$ because the distributed network codes studied in this paper can be regarded as *systematic* linear block codes [18].

1) Benchmark MDD: Since the MDD receiver does not exploit CSI about the source–to–relay channels, the Benchmark MDD (B–MDD) receiver is still given by (4) and (5). However, it is implicitly assumed that decoding at the relays is without errors, *i.e.*, $\hat{b}_{S_tR_q} = b_{S_t}$ for $t = 1, 2$ and $q = 1, 2$ in (2).

B. Hard–decision Maximum–Likelihood Decoder (H–MLD)

The H–MLD receiver works as follows:

a) Step 1 (Physical Layer): Similar to MDD, (4) is used.

b) Step 2 (Network Layer): The hard–decision estimates $\hat{\mathbf{c}} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4] = \left[\hat{b}_{S_1D}, \hat{b}_{S_2D}, \hat{b}_{R_1D}, \hat{b}_{R_2D}\right]$ are input to the network layer, which uses a MLSE–optimum decoder [18] with full–CSI to retrieve the bits emitted by the sources:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\mathbf{c}^{(\tilde{j})} \text{ with } \tilde{j}=1,2,3,4}{\arg\min} \left\{ \sum_{i=1}^{4} w_i \left|\hat{c}_i - c_i^{(\tilde{j})}\right| \right\} \tag{6}$$

where: i) $w_i = \ln\left[(1-\Psi_i)/\Psi_i\right]$; ii) $\Psi_1 = P_{S_1D}$ and $\Psi_2 = P_{S_2D}$ for *Scenario 1, 2, 3, 4*; iii) $\Psi_3 = P_{S_1R_1} + P_{R_1D} - 2P_{S_1R_1}P_{R_1D}$ for *Scenario 1, 4* and $\Psi_3 = P_{R_1D} + P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1} - 2P_{R_1D}\left(P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}\right)$ for *Scenario 2, 3*; and iv) $\Psi_4 = P_{S_2R_2} + P_{R_2D} - 2P_{S_2R_2}P_{R_2D}$ for *Scenario 1, 3* and $\Psi_4 = P_{R_2D} + P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2} - 2P_{R_2D}\left(P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}\right)$ for *Scenario 2, 4*.

We note that (6) exploits full–CSI for ML–optimum decoding: the network layer takes advantage of physical layer information (cross–layer), *i.e.*, the instantaneous BEP, P_{XY}, over all the wireless links. Furthermore, we remark that the weights w_i take into account the error propagation effects due to both DF relaying and NC operations.

1) Benchmark H–MLD: If the source–to–relay channels are reliable, there are no decoding errors at the relays. Thus, we still have $\hat{b}_{S_tR_q} = b_{S_t}$ for $t = 1, 2$ and $q = 1, 2$ in (2), and the Benchmark H–MLD (B–H–MLD) receiver is still given by (6), but with $\Psi_1 = P_{S_1D}$, $\Psi_2 = P_{S_2D}$, $\Psi_3 = P_{R_1D}$, and $\Psi_4 = P_{R_2D}$ for all NC scenarios.

C. Soft–decision Maximum–Likelihood Decoder (S–MLD)

Unlike MDD and H–MLD, S–MLD is a single–step detector in which there is no distinction between physical and network layers. In particular, the network layer is “pushed down” to the physical layer, and MLSE–optimum network decoding is performed directly on the signals $y_{S_1D}, y_{S_2D}, y_{R_1D}, y_{R_2D}$. Of course, decoding errors at the relays are still taken in account. The decoder is as follows:

$$\left[\hat{b}_{S_1}, \hat{b}_{S_2}\right] = \left[c_1^{(\hat{j})}, c_2^{(\hat{j})}\right] = \underset{\mathbf{c}^{(\tilde{j})} \text{ with } \tilde{j}=1,2,3,4}{\arg\min} \left\{ \begin{array}{l} f\left(y_{S_1D}, h_{S_1D}, c_1^{(\tilde{j})}\right) \times f\left(y_{S_2D}, h_{S_2D}, c_2^{(\tilde{j})}\right) \\ \times \left[(1-\lambda_1) f\left(y_{R_1D}, h_{R_1D}, c_3^{(\tilde{j})}\right) + \lambda_1 f\left(y_{R_1D}, h_{R_1D}, 1-c_3^{(\tilde{j})}\right)\right] \\ \times \left[(1-\lambda_2) f\left(y_{R_2D}, h_{R_2D}, c_4^{(\tilde{j})}\right) + \lambda_2 f\left(y_{R_2D}, h_{R_2D}, 1-c_4^{(\tilde{j})}\right)\right] \end{array} \right\} \tag{7}$$

where: i) $\lambda_1 = P_{S_1R_1}$ for *Scenario 1, 4* and $\lambda_1 = P_{S_1R_1} + P_{S_2R_1} - 2P_{S_1R_1}P_{S_2R_1}$ for *Scenario 2, 3*; ii) $\lambda_2 = P_{S_2R_2}$ for *Scenario 1, 3* and $\lambda_2 = P_{S_1R_2} + P_{S_2R_2} - 2P_{S_1R_2}P_{S_2R_2}$ for *Scenario 2, 4*; and iii) $f(\xi_1, \xi_2, \xi_3) = \exp\left[-(1/N_0)\left|\xi_1 - \sqrt{E_m}\xi_2(1-2\xi_3)\right|^2\right]$.

1) Benchmark S–MLD: If there are no decoding errors at the relays, we have $\lambda_1 = \lambda_2 = 0$ for all NC scenarios. Thus, (7) reduces to the well–known MLSE–optimum receiver based on the computation of the minimum Euclidean distance [18].

IV. Performance Analysis

In this section, we compute the ABEP of the detectors developed in Section III by considering the four NC scenarios described in Section II. The following assumptions are retained: i) we use union bound arguments to estimate the ABEP, which require the computation of the Pairwise Error Probability (PEP) for each pair of codewords in the codebook [18], ii) the codewords of the distributed network code are assumed to be equiprobable, and iii) we separately compute the ABEP of sources S_1 and S_2, since, as mentioned in Section I, we are interested in showing that UEP–based distributed network codes provide different performance for different sources, according to the specified separation vector. Due to space constrains, the formulas are given without proof. However, the accuracy of the frameworks is substantiated via Monte Carlo simulations in Section V.

The ABEP of source S_t $(t = 1, 2)$ ca be computed as:

$$\mathrm{ABEP}^{(S_t)} \le \frac{1}{4}\sum_{j_1=1}^{4}\sum_{j_2 \ne j_1 = 1}^{4} \mathrm{APEP}_{j_1,j_2}^{(S_t)} \tag{8}$$

where $\mathrm{APEP}_{j_1,j_2}^{(S_t)}$ is the Average (over fading channel statistics) PEP, defined as follows:

$$\mathrm{APEP}^{(S_t)} = \Pr\left\{\mathbf{c}^{(j_1)} \to \mathbf{c}^{(j_2)}\right\} \Delta\left(c_t^{(j_1)}, c_t^{(j_2)}\right) \tag{9}$$

with $\Pr\{\cdot\}$ denoting probability, and $\Delta(\cdot,\cdot)$ being the Kronecker delta function. We emphasize that the Kronecker delta function in (9) is needed because we are interested in computing the ABEP of each source individually. In other words, MDD, H–MLD, and S–MLD receivers might be wrong in estimating the transmitted codeword, but this does not necessarily lead to a decoding error in the bits transmitted by *both* sources. As an example, let us consider *Scenario 3* and the transmission of $\mathbf{c}^{(2)} = 0111$. If the receiver (wrongly) decodes $\hat{\mathbf{c}}^{(2)} = \mathbf{c}^{(4)} = 1101$, then this results in an error only for the bit emitted by S_1, while there is no error for S_2.

Notation. The following notation is used: i) $\mu = \sqrt{\bar{\gamma}\sigma_0^2/(1+\bar{\gamma}\sigma_0^2)}$; ii) $\bar{P} = (1/2)(1-\mu)$; iii) $\binom{\cdot}{\cdot}$ is the binomial coefficient; iv) $\Upsilon_L = [(1-\mu)/2]^L \sum_{l=0}^{L-1}\left\{\binom{L-1+l}{l}[(1-\mu)/2]^l\right\}$; v) $\Gamma(a) = \int_0^{+\infty} t^{a-1}\exp(-t)\,dt$ is the Gamma function; vi) $\gamma(a,x) = \int_0^x t^{a-1}\exp(-t)\,dt$ is the incomplete Gamma function; and vii) $F_L(\xi) = \gamma\left(L, \xi/\left(2\sigma_0^2\right)\right)/\Gamma(L)$.

A. Minimum Distance Decoder (MDD)

a) Scenario 1: $\mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} = \frac{1}{2}\bar{P}_1 + \frac{1}{2}\bar{P}_3 + \frac{1}{2}\bar{P}_1\bar{P}_2 + \frac{1}{2}\bar{P}_1\bar{P}_3 + \frac{1}{2}\bar{P}_1\bar{P}_4 + \frac{1}{2}\bar{P}_2\bar{P}_3 + \frac{1}{2}\bar{P}_3\bar{P}_4 - \frac{1}{2}\bar{P}_1\bar{P}_2\bar{P}_3 - \frac{1}{2}\bar{P}_1\bar{P}_2\bar{P}_4 - \frac{1}{2}\bar{P}_1\bar{P}_3\bar{P}_4 - \frac{1}{2}\bar{P}_2\bar{P}_3\bar{P}_4 - \frac{1}{2}\bar{P}_1\bar{P}_2\bar{P}_3\bar{P}_4$, with $\bar{P}_1 = \bar{P}_2 = \bar{P}$ and $\bar{P}_3 = \bar{P}_4 = 2\bar{P} - 2\bar{P}^2$.

b) Scenario 2: $\mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} = \frac{1}{2}\bar{P}_1 + \frac{1}{2}\bar{P}_2 + \bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + \bar{P}_3\bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_4 - \bar{P}_1\bar{P}_3\bar{P}_4$, with $\bar{P}_1 = \bar{P}_2 = \bar{P}$ and $\bar{P}_3 = \bar{P}_4 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$.

c) Scenario 3: $\mathrm{ABEP}^{(S_1)} = \frac{1}{2}\bar{P}_1 + \frac{1}{2}\bar{P}_3 + \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_4 + \bar{P}_2\bar{P}_4 - \bar{P}_1\bar{P}_2\bar{P}_4$ and $\mathrm{ABEP}^{(S_2)} = \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + 2\bar{P}_2\bar{P}_4 + \bar{P}_3\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_4 - 2\bar{P}_2\bar{P}_3\bar{P}_4$, with $\bar{P}_1 = \bar{P}_2 = \bar{P}$, $\bar{P}_3 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$, and $\bar{P}_4 = 2\bar{P} - 2\bar{P}^2$.

d) Scenario 4: $\mathrm{ABEP}^{(S_1)} = \bar{P}_1\bar{P}_2 + 2\bar{P}_1\bar{P}_3 + \bar{P}_1\bar{P}_4 + \bar{P}_2\bar{P}_3 + \bar{P}_3\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_3 - 2\bar{P}_1\bar{P}_3\bar{P}_4$ and $\mathrm{ABEP}^{(S_2)} = \frac{1}{2}\bar{P}_2 + \frac{1}{2}\bar{P}_4 + \bar{P}_1\bar{P}_2 + \bar{P}_1\bar{P}_3 + 2\bar{P}_2\bar{P}_3 - \bar{P}_2\bar{P}_4 - 2\bar{P}_1\bar{P}_2\bar{P}_3$, with $\bar{P}_1 = \bar{P}_2 = \bar{P}$, $\bar{P}_3 = 2\bar{P} - 2\bar{P}^2$, and $\bar{P}_4 = 3\bar{P} - 6\bar{P}^2 + 4\bar{P}^3$.

1) Benchmark MDD: As far as the B–MDD detector is concerned, the formulas above can still be used. However, as there are no errors on the source–to–relay channels, we have $\bar{P}_1 = \bar{P}_2 = \bar{P}_3 = \bar{P}_4 = \bar{P}$ for all NC scenarios.

2) Diversity Analysis: To understand the diversity gain of the MDD receiver, let us study the performance (ABEP_∞) for high SNRs [12]. The following results can be proved. i) *Scenario 1*: $\mathrm{ABEP}_\infty^{(S_1)} = \mathrm{ABEP}_\infty^{(S_2)} = (3/2)\,\bar{P}$ for MDD, and $\mathrm{ABEP}_\infty^{(S_1)} = \mathrm{ABEP}_\infty^{(S_2)} = \bar{P}$ for B–MDD; ii) *Scenario 2*: $\mathrm{ABEP}_\infty^{(S_1)} = \mathrm{ABEP}_\infty^{(S_2)} = \bar{P}$ for MDD and B–MDD; iii) *Scenario 3*: $\mathrm{ABEP}_\infty^{(S_1)} = 2\bar{P}$ and $\mathrm{ABEP}_\infty^{(S_2)} = 16\bar{P}^2$ for MDD, and $\mathrm{ABEP}_\infty^{(S_1)} = \bar{P}$ and $\mathrm{ABEP}_\infty^{(S_2)} = 6\bar{P}^2$ for B–MDD; and iv) *Scenario 4*: $\mathrm{ABEP}_\infty^{(S_1)} = 16\bar{P}$ and $\mathrm{ABEP}_\infty^{(S_2)} = 2\bar{P}^2$ for MDD, and $\mathrm{ABEP}_\infty^{(S_1)} = 6\bar{P}$ and $\mathrm{ABEP}_\infty^{(S_2)} = \bar{P}^2$ for B–MDD. From these results, it follows that the diversity (Div) gain is [12]: i) $\mathrm{Div} = 1$ for both sources in *Scenario 1*; ii) $\mathrm{Div} = 1$ for both sources in *Scenario 2*; iii) $\mathrm{Div} = 1$ and $\mathrm{Div} = 2$ for source S_1 and S_2 in *Scenario 3*, respectively; and iv) $\mathrm{Div} = 2$ and $\mathrm{Div} = 1$ for source S_1 and S_2 in *Scenario 4*, respectively. There is no diversity difference between MDD and B–MDD receivers.

From these results, the following conclusions can be drawn: i) in our implementation, decoding errors at the relays introduce only a loss in the coding gain (the ABEP of MDD is slightly higher than the ABEP of B–MDD), but there is no loss in the diversity gain; ii) UEP–based design of network codes is the only method that allows at least one source to have a diversity greater than current implementations (relay–only and XOR–only); iii) the diversity achieved by the UEP–based network code design agrees with the theoretical prediction discussed in Section II-A. In particular, let SP denote the separation vector of a UEP–based network code, the diversity of MDD and B–MDD is equal to $\mathrm{Div} = \mathrm{SP} - 1$. In our case, in *Scenario 3* we obtain $\mathrm{Div} = 1$ and $\mathrm{Div} = 2$ for S_1 and S_2, respectively, while in *Scenario 4* we obtain $\mathrm{Div} = 2$ and $\mathrm{Div} = 1$ for S_1 and S_2, respectively. This is also the best achievable diversity for a (4,2,2) UEP code that uses minimum distance decoding at the destination [18]; and iv) interestingly, we note that the XOR–only network code (*Scenario 2*) is the most robust to error propagation caused by decoding errors at the relays. In fact, MDD and B–MDD yield, asymptotically, the same ABEP, but only $\mathrm{Div} = 1$ is obtained with this code.

B. Hard–decision Maximum–Likelihood Decoder (H–MLD)

The computation of the ABEP of the detector in (6) is analytically very complicated, and cannot be included in this paper. However, the analysis of the MDD receiver in Section IV-A has revealed that only a loss in the coding gain can be expected between the operating conditions with ideal (no decoding error at the relays) and realistic source–to–relay wireless links. Thus, in this paper we provide only a closed–form expression for the ABEP of the B–H–MLD receiver. In Section V, we prove by simulation that the diversity gain analytically predicted for the B–H–MLD receiver holds for the H–MLD receiver as well. The final formulas are summarized in (10)–(13) for *Scenario 1, 2, 3, 4*, respectively.

$$\begin{aligned} \mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} \cong \bar{P}^4 & \\ + \tfrac{1}{\sqrt{2\pi}} \int_0^{+\infty} \left[F_1\left(\tfrac{x^2}{\bar{\gamma}}\right)\right]^2 \exp\left(-\tfrac{x^2}{2}\right) dx & \\ + \tfrac{4}{\sqrt{2\pi}} \int_0^{+\infty} F_3\left(\tfrac{x^2}{\sqrt[3]{10}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx & \\ + \tfrac{3}{\sqrt{2\pi}} \int_0^{+\infty} \left[F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right)\right]^2 \exp\left(-\tfrac{x^2}{2}\right) dx & \end{aligned} \tag{10}$$

$$\begin{aligned} \mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} \cong \bar{P}^3 & \\ + \tfrac{1}{\sqrt{2\pi}} \int_0^{+\infty} \left[F_1\left(\tfrac{x^2}{\bar{\gamma}}\right)\right]^2 \exp\left(-\tfrac{x^2}{2}\right) dx & \\ + \tfrac{3}{\sqrt{2\pi}} \int_0^{+\infty} F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx & \end{aligned} \tag{11}$$

$$\begin{cases} \mathrm{ABEP}^{(S_1)} \cong \bar{P}^3 + \tfrac{1}{\sqrt{2\pi}} \int_0^{+\infty} \left[F_1\left(\tfrac{x^2}{\bar{\gamma}}\right)\right]^2 \exp\left(-\tfrac{x^2}{2}\right) dx \\ \qquad + \tfrac{3}{\sqrt{2\pi}} \int_0^{+\infty} F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx \\ \mathrm{ABEP}^{(S_2)} \cong 2\bar{P}^3 \\ \qquad + \tfrac{6}{\sqrt{2\pi}} \int_0^{+\infty} F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx \end{cases} \tag{12}$$

$$\begin{cases} \mathrm{ABEP}^{(S_1)} \cong 2\bar{P}^3 \\ \qquad + \tfrac{6}{\sqrt{2\pi}} \int_0^{+\infty} F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx \\ \mathrm{ABEP}^{(S_2)} \cong \bar{P}^3 + \tfrac{1}{\sqrt{2\pi}} \int_0^{+\infty} \left[F_1\left(\tfrac{x^2}{\bar{\gamma}}\right)\right]^2 \exp\left(-\tfrac{x^2}{2}\right) dx \\ \qquad + \tfrac{3}{\sqrt{2\pi}} \int_0^{+\infty} F_2\left(\tfrac{x^2}{\sqrt{3}\bar{\gamma}}\right) F_1\left(\tfrac{x^2}{\bar{\gamma}}\right) \exp\left(-\tfrac{x^2}{2}\right) dx \end{cases} \tag{13}$$

1) Diversity Analysis: From (10)–(13), the diversity gain can be computed by using [12]. It can be proved that the diversity of the B–H–MLD receiver is: i) $\mathrm{Div} = 2$ for both sources in *Scenario 1*; ii) $\mathrm{Div} = 2$ for both sources in *Scenario 2*; iii) $\mathrm{Div} = 2$ and $\mathrm{Div} = 3$ for source S_1 and S_2 in *Scenario 3*, respectively; and iv) $\mathrm{Div} = 3$ and $\mathrm{Div} = 2$ for source S_1 and S_2 in *Scenario 4*, respectively. In Section V, we show by simulation that a similar result holds for the H–MLD receiver. It can be noticed that: i) the diversity is higher than the MDD receiver, and this is due to the cross–layer interaction between physical and network layers; ii) UEP–based network codes provide a diversity that is equal to the separation vector; and iii) these results agree with the achievable diversity of conventional linear block codes over fully–interleaved fading channels and soft–decision decoding [18]. In fact, even though the H–MLD receiver is not a pure soft–decision detector, the weights in (6) mimic the behavior of a soft–decision decoder.

C. Soft–decision Maximum–Likelihood Decoder (S–MLD)

Similar to Section IV-B, we show only the ABEP of the Benchmark S–MLD (B–S–MLD) receiver. In Section V, it is shown by simulation that the ABEP of B–S–MLD is a lower bound for the ABEP of S–MLD. The ABEP is: i) $\mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} = \Upsilon_2 + \Upsilon_4$ for *Scenario 1*; ii) $\mathrm{ABEP}^{(S_1)} = \mathrm{ABEP}^{(S_2)} = \Upsilon_2 + \Upsilon_3$ for *Scenario 2*; iii) $\mathrm{ABEP}^{(S_1)} = \Upsilon_2 + \Upsilon_3$, $\mathrm{ABEP}^{(S_2)} = 2\Upsilon_3$ for *Scenario 3*; iv) $\mathrm{ABEP}^{(S_1)} = 2\Upsilon_3$, $\mathrm{ABEP}^{(S_2)} = \Upsilon_2 + \Upsilon_3$ for *Scenario 4*.

1) Diversity Analysis: From the ABEP above and [12], we conclude that B–S–MLD provides the same diversity as B–H–MLD. However, due to soft–decision decoding in the former case, a higher coding gain is shown in Section V.

V. Numerical Results

Here, some results to substantiate our claims are shown. A detailed description of the simulation setup is available in Section II. Due to space constraints, numerical results for the MDD receiver are not shown. The interested reader can find some numerical examples about this receiver in [19].

The results are shown in Figs. 2–5. The following conclusions can be drawn: i) our analytical model overlaps with Monte Carlo simulations, thus confirming our findings in terms of achievable performance and diversity analysis; ii) it can be noticed that the analytical frameworks for all the Benchmark receivers are lower bound for the performance in the presence of decoding errors at the relays; iii) it is confirmed that *Scenario 2* is very robust to error propagation, as the ABEP of MDD, H–MLD, and S–MLD is asymptotically the same as the ABEP of the related Benchmark detectors; iv) as expected, S–MLD is better than H–MLD, which is, in turn, better than MDD (see [19, Figs. 2–5]); and v) the achievable diversity does not depend only on the adopted network code, but also on the detector used at the destination. In particular, no cross–layer interaction between physical and network layers results in a receiver design (MDD) that cannot fully exploit the diversity provided by the network code. On the other hand, both H–MLD and S–MLD can exploit it by taking advantage of CSI from the physical layer. In other words, performing demodulation and network decoding separately is inherently sub–optimal regardless of decoding errors at the relays.

Furthermore, as far as the achievable performance of our network code design based on UEP coding theory is concerned, it can be noticed that the proposed method is the only one providing $\mathrm{Div} = 3$ for at least one source, while conventional approaches can only achieve $\mathrm{Div} = 2$. Unlike [9]–[11], we note that this result is obtained without the need to use a network code in a non–binary Galois field and without requiring extra time slots. The complexity of our UEP–based network code design is the same as relay–only and XOR–only methods. For example, looking at the results in Fig. 5, we observe that the network code in *Scenario 3* is the best choice when the data sent by S_2 needs to be delivered (if compared to the data sent by S_1): i) either with a smaller ABEP and the same transmission power, or ii) with a reduced transmission power and the same ABEP. The working principle of the network code in *Scenario 3* has a simple interpretation: if S_2 is the "golden user", then we should dedicate one relay to only forward its data without performing NC. A similar comment can be made about *Scenario 4* if S_1 is the "golden user".

VI. Conclusion

In this paper, the performance of the canonical two–source two–relay cooperative network has been analyzed, and it has been highlighted that the achievable diversity depends on both the distributed network code and the detector at the destination. It has been shown that the inherent diversity provided by the network code can be achieved only with a cross–layer implementation of de–modulation and network–decoding, which foresees the physical and network layers to exchange some information about the quality of the wireless

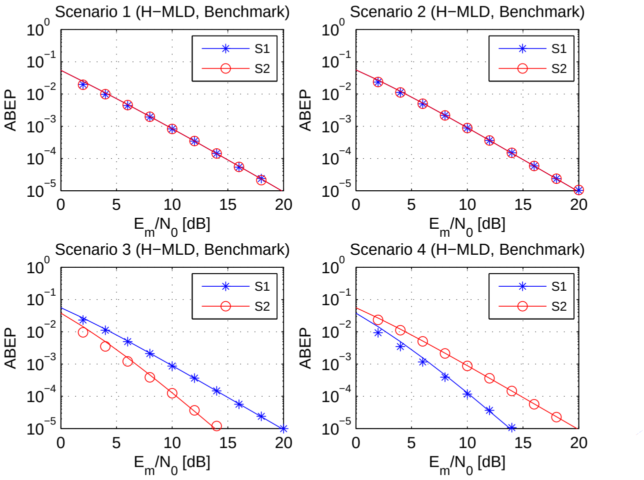

Fig. 2. B–H–MLD: ABEP against E_m/N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$).

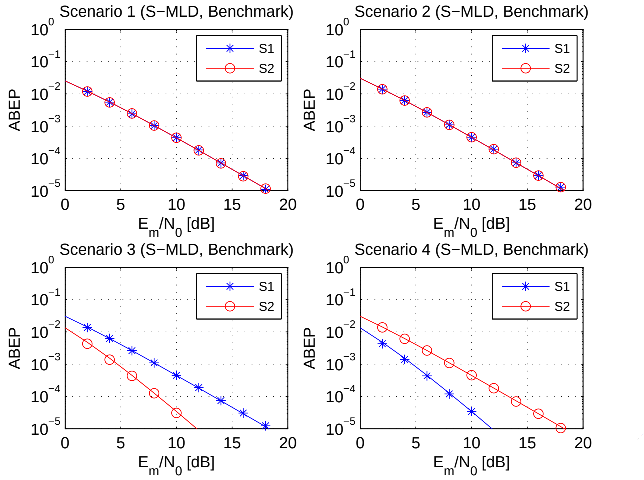

Fig. 4. B–S–MLD: ABEP against E_m/N_0. Solid lines show the analytical model and markers Monte Carlo simulations ($\sigma_0^2 = 1$).

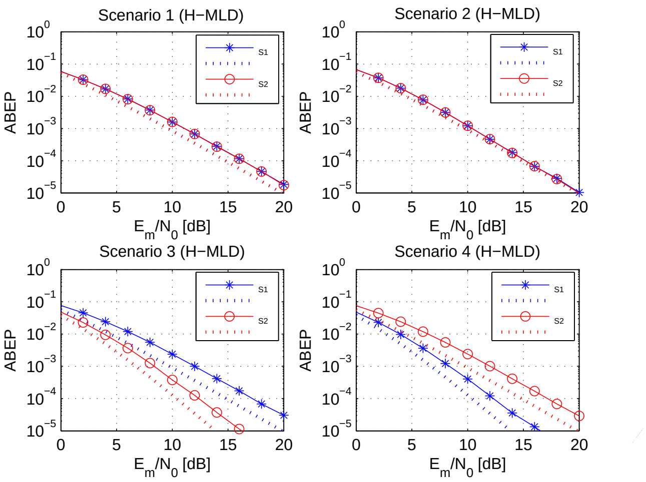

Fig. 3. H–MLD: ABEP against E_m/N_0. Solid lines with markers show Monte Carlo simulations and dotted lines the analytical model for B–H–MLD ($\sigma_0^2 = 1$).

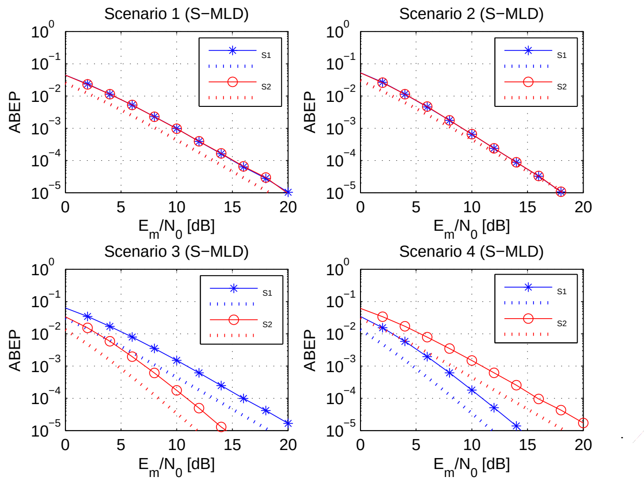

Fig. 5. S–MLD: ABEP against E_m/N_0. Solid lines with markers show Monte Carlo simulations and dotted lines the analytical model for B–S–MLD ($\sigma_0^2 = 1$).

links of the whole cooperative network. A new network code design based on the theory of UEP coding has been introduced, and it has been proved that it can improve the diversity of at least one source of the network, by only requiring binary operations at the relays and without additional time–slots.

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