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Abstract— **In this paper, we study the sensitivity and robustness of Space Shift Keying (SSK) modulation to imperfect channel knowledge at the receiver. Unlike the common widespread belief, we show that SSK modulation is more robust to imperfect channel knowledge than other state–of–the–art transmission technologies, and only few training pilots are needed to get reliable enough channel estimates for data detection. More precisely, we focus our attention on the so–called Time–Orthogonal–Signal–Design (TOSD–) SSK modulation scheme, which is an improved version of SSK modulation offering transmit–diversity gains, and provide the following contributions: i) we develop a closed–form analytical framework to compute the Average Bit Error Probability (ABEP) of a mismatched detector for TOSD–SSK modulation, which can be used for arbitrary transmit–antenna, receive–antenna, channel fading, and training pilots; ii) we perform a comparative study of the performance of TOSD–SSK modulation and the Alamouti code under the same imperfect channel knowledge, and show that TOSD–SSK modulation is more robust to channel estimation errors; iii) we point out that only few pilot pulses are required to get performance very close to the perfect channel knowledge lower–bound; and iv) we verify that transmit– and receive–diversity gains of TOSD–SSK modulation are preserved even for a mismatched receiver.**

I. Introduction

Space modulation is a novel digital modulation concept for Multiple–Input–Multiple–Output (MIMO) wireless systems, which is receiving a growing attention due to the possibility of realizing low–complexity and spectrally–efficient MIMO implementations [1]–[4]. The space modulation principle is known in the literature in different forms, such as Information–Guided Channel Hopping (IGCH) [1], Spatial Modulation (SM) [2], and Space Shift Keying (SSK) modulation [3]. Although different from one another, all these transmission technologies share the same fundamental working principle, which makes them different from conventional modulation schemes: they encode part of the information bits into the spatial position of the antenna–array, which plays the role of a constellation diagram (the so–called "spatial–constellation diagram") for data modulation [4]. In particular, SSK modulation exploits only the spatial–constellation diagram for data modulation, which results in a very low–complexity modulation concept for MIMO systems [3]. Recently, we have introduced in [5], [6] and generalized in [7], respectively, an improved version of SSK modulation, which can achieve transmit–diversity gains without any spectral efficiency loss with respect to the original SSK modulation proposal.

In SSK modulation, blocks of information bits are mapped into the index of a single transmit–antenna, which is switched on for data transmission while all the other antennas radiate no power [3]. Regardless of the information message to be transmitted, and, thus, the active transmit–antenna, SSK modulation exploits the location–specific property of the wireless channel for data modulation [4]: the messages sent by the transmitter can be decoded at the destination since the receiver sees a different channel impulse response on any transmit–to–receive wireless link. In [3] and [4], it has been shown that the achievable performance of SSK modulation depends on how different the channel impulse responses are. In other words, the channel impulse responses are the points of the spatial–constellation diagram, and the receiver performance depends on the distance among these points. Recent results have shown that SSK modulation can provide better performance than conventional modulation schemes with similar complexity if the receiver has Perfect Channel State Information (P–CSI) [1]–[3]. However, due to its inherent working principle, the major criticism about the application of SSK modulation in a realistic propagation environment is the robustness of the space modulation principle to the imperfect knowledge, at the receiver, of the wireless channel. In particular, since the wireless channel is the actual modulation unit, it is often argued that the space modulation concept is more sensitive to channel estimation errors. The main contribution of this paper is to shed light on this matter.

Although some research works on the performance of the space modulation principle with imperfect channel knowledge are available in the literature, these results are insufficient and only based on numerical simulations. In [3], the authors study the Average Bit Error Probability (ABEP) of SSK modulation with non–ideal channel knowledge. However, there are four limitations in this paper: i) the ABEP is obtained only through Monte Carlo simulations, which is not very much insightful; ii) the arguments in [3] are applicable only to Gaussian fading channels and do not take into account the cross–product between channel estimation error and Additive White Gaussian Noise (AWGN) at the receiver; iii) it is unclear from [3] how the ABEP changes with respect to the pilot symbols used by the channel estimator; and iv) the robustness/weakness of SSK modulation with respect to conventional modulation schemes is not analyzed. In [8], we have studied the performance of SSK modulation when the receiver does not exploit the knowledge of the phase of the channel gains for data detection (semi–blind receiver). The main message of this paper is that semi–blind receivers are much worse than coherent detection schemes, and, thus, that the assessment of the performance of coherent detection with imperfect channel knowledge is a crucial aspect for SSK modulation. A very interesting study has been recently conducted in [9], where the authors have compared the performance of SM and V–BLAST (Vertical Bell Laboratories Layered Space–Time) [10] schemes with practical channel estimates. It is shown that the claimed sensitivity of space modulation to channel estimation errors is simply a misconception and that, on the contrary, SM is more robust than V–BLAST to imperfections on the channel estimates, and that less training is, in general, required by SM. However, the system in [9] is studied only through Monte Carlo simulations, which does not give too much insights for performance analysis and system optimization.

Motivated by these considerations, in this paper we aim at developing a very general analytical framework to assess the performance of the space modulation concept with coherent detection and practical channel estimates. Our theoretical and numerical results corroborate the findings in [9], and highlight two important outcomes: 1) space modulation can be even more robust to channel estimation errors than conventional modulation schemes, and 2) the number of pilot symbols required to approach the lower–bound set by coherent detection with perfect channel knowledge is quite limited. More precisely, the contributions of this paper are as follows: i) we develop a general analytical framework to compute the ABEP of the TOSD–SSK modulation scheme with imperfect channel knowledge. The framework can handle arbitrary transmit–antenna, receive–antenna, fading channel statistics, and number of pilot symbols used by the channel estimation unit. It is shown that the mismatched detector of TOSD–SSK modulation [11] can be cast in terms of a quadratic–form in complex Gaussian Random Variables (RVs) when conditioning upon fading channel statistics, and that the ABEP can be computed by exploiting the Gil–Pelaez inversion theorem [12]; ii) we compare the performance of TOSD–SSK modulation with the Alamouti scheme [13], which similar to TOSD–SSK modulation can achieve transmit–diversity equal to two, and show that TOSD–SSK modulation is more robust to imperfect channel knowledge; and iii) we show that transmit– and receive–diversity of TOSD–SSK modulation with non–ideal channel estimates is always preserved.

The reminder of this paper is organized as follows. In Section II, the system model is introduced and the TOSD–SSK modulation scheme is briefly described. In Section III, the analytical framework to compute the ABEP with imperfect channel knowledge is developed. In Section IV, numerical results are shown to substantiate the main findings of the paper. Finally, Section V concludes the paper.

II. System Model

We consider a generic $N_t \times N_r$ MIMO system, with N_t and N_r being the number of transmit– and receive–antenna, respectively. TOSD–SSK modulation works as follows [3], [5]–[7]: i) the transmitter encodes blocks of $\log_2(N_t)$ data bits into the index of a single transmit–antenna, which is switched on for data transmission while all the other antennas are kept silent, and ii) the receiver solves a N_t–hypothesis detection problem to estimate the transmit–antenna that is not idle, which results in the estimation of the unique sequence of bits emitted by the encoder. With respect to conventional SSK modulation [3], in TOSD–SSK modulation the i–th transmit–antenna, when active, radiates a distinct pulse waveform $w_i(\cdot)$ for $i = 1, 2, \ldots, N_t$, and the waveforms across the antennas are time–orthogonal, *i.e.*[1], $\int_{-\infty}^{+\infty} w_i(t)\, w_j^*(t)\, dt = 0$ if $i \neq j$ and $\int_{-\infty}^{+\infty} w_i(t)\, w_j^*(t)\, dt = 1$ if $i = j$. We emphasize here that in TOSD–SSK modulation a single antenna is active for data transmission and that the transmitted message is still encoded into the index of the transmit–antenna and *not* into the impulse response of the shaping filter. In other words, the proposed concept is different from conventional Single–Input–Single–Output (SISO) schemes, which use Orthogonal Pulse Shape Modulation (O–PSM) [14] and are unable to achieve transmit–diversity, as only a single wireless link is exploited for communication [6], [7]. Also, the TOSD–SSK modulation principle is different from conventional transmit–diversity schemes [15]. Further details are available in [6] and [7], which are here omitted in order to avoid repetitions. In [6], [7], it is shown that the diversity gain of the TOSD–SSK modulation scheme is $2N_r$, which results in a transmit–diversity equal to two and a receive–diversity equal to N_r.

In this paper, the block of bits encoded into the index of the i–th transmit–antenna is called "message" and is denoted by m_i for $i = 1, 2, \ldots, N_t$. The N_t messages are equiprobable. Moreover, the related transmitted signal is denoted by $s_i(\cdot)$. It is implicitly assumed in this notation that, if m_i is transmitted, the analog signal $s_i(\cdot)$ is emitted by the i–th transmit–antenna while the other antennas radiate no power.

A. Notation

The main notation used in this paper is as follows. i) We adopt a complex–envelope signal representation. ii) $j = \sqrt{-1}$ is the imaginary unit. iii) $(x \otimes y)(t) = \int_{-\infty}^{+\infty} x(\xi)\, y(t-\xi)\, d\xi$ is the convolution of signals $x(\cdot)$ and $y(\cdot)$. iv) $|\cdot|^2$ is the square absolute value. v) $\mathrm{E}\{\cdot\}$ is the expectation operator. vi) $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ are the real and imaginary part operators, respectively. vii) $\Pr\{\cdot\}$ denotes probability. viii) $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^{+\infty} \exp\left(-t^2/2\right) dt$ is the Q–function. ix) $\hat{m}$ denotes the message estimated at the receiver. x) E_m is the average energy transmitted by each antenna that emits a non–zero signal. xi) T_m denotes the signaling interval for each information message m_i $(i = 1, 2, \ldots, N_t)$. xii) The noise η_l at the input of the l–th receive–antenna $(l = 1, 2, \ldots, N_r)$ is a complex AWGN process, with power spectral density N_0 per dimension. Across the receive–antenna, the noises η_l are statistically independent. xiii) E_p and N_p denote the energy transmitted for each pilot symbol and the number of pilot symbols used for channel estimation. xiv) $\delta(\cdot)$ and $\delta_{\cdot,\cdot}$ are the Dirac and Kronecker delta functions, respectively. xv) For ease of notation, we set $\bar{\gamma}{=}E_m/N_0$ and $r_{pm} = E_p/E_m$. xvi) $M_X(s) = \mathrm{E}\{\exp(sX)\}$ and $\Psi_X(\nu) = \mathrm{E}\{\exp(j\nu X)\}$ denote Moment Generating Function (MGF) and Characteristic Function (CF) of RV X, respectively.

B. Channel Model

We consider a general frequency–flat slowly–varying channel model with generically correlated and non–identically distributed fading gains. In particular ($i = 1, 2, \ldots, N_t$, $l = 1, 2, \ldots, N_r$):

[1] $(\cdot)^*$ denotes complex–conjugate.

- $h_{i,l}(t) = \alpha_{i,l}\delta(t - \tau_{i,l})$ is the channel impulse response of the transmit–to–receive wireless link from the i–th transmit–antenna to the l–th receive–antenna. $\alpha_{i,l} = \beta_{i,l}\exp(j\varphi_{i,l})$ is the complex channel gain with $\beta_{i,l}$ and $\varphi_{i,l}$ denoting the channel envelope and phase, respectively, and $\tau_{i,l}$ is the propagation time–delay.
- The delays $\tau_{i,l}$ are assumed to be known at the receiver, *i.e.*, perfect time–synchronization is considered. Furthermore, we assume $\tau_{1,1} \cong \tau_{1,2} \cong \ldots \cong \tau_{N_t,N_r}$, which is a realistic assumption when the distance between the transmitter and the receiver is much larger than the spacing between transmit– and receive–antennas [4]. Due to the these assumptions, the propagation delays can be neglected in the reminder of this paper.

C. Channel Estimation

Similar to [16] and [17], we assume that channel estimation is performed by using a Maximum–Likelihood (ML) detector and by observing N_p pilot pulses that are transmitted before the modulated data. During the transmission of one block of pilot–plus–data symbols, the wireless channel is assumed to be constant, *i.e.*, a block–fading channel is considered. With these assumptions, the estimates of the channel gains $\alpha_{i,l}$ ($i = 1, 2, \ldots, N_t$, $l = 1, 2, \ldots, N_r$) can be written as follows:

$$\hat{\alpha}_{i,l} = \hat{\beta}_{i,l}\exp(j\hat{\varphi}_{i,l}) = \alpha_{i,l} + \varepsilon_{i,l} \tag{1}$$

where $\hat{\alpha}_{i,l}$, $\hat{\beta}_{i,l}$, and $\hat{\varphi}_{i,l}$ are the estimates of $\alpha_{i,l}$, $\beta_{i,l}$, and $\varphi_{i,l}$, respectively, at the output of the channel estimation unit, and $\varepsilon_{i,l}$ is the additive channel estimation error, which can be shown to be complex Gaussian distributed with zero mean and variance $\sigma_\varepsilon^2 = N_0/(E_p N_p)$ per dimension [16], [17]. The channel estimation errors $\varepsilon_{i,l}$ are statistically independent and identically distributed, as well as statistically independent of the channel gains and the AWGN at the receiver.

D. Mismatched ML–Optimum Detector

In this paper, for data detection we consider the so–called mismatched ML–optimum receiver according to the definition given in [11]. In particular, a detector with mismatched metric estimates the complex channel gains as in (1) and uses the result in the same metric that would be applied if the channel were perfectly known. This detector can be obtained as follows.

Let m_n with $n = 1, 2, \ldots, N_t$ be the transmitted message[2]. The signal received after propagation through the wireless fading channel and impinging upon the l–th receive–antenna can be written as follows:

$$r_l(t) = \tilde{s}_{n,l}(t) + \eta_l(t) \qquad \text{if } m_n \text{ is sent} \tag{2}$$

where $\tilde{s}_{n,l}(t) = (s_n \otimes h_{n,l})(t) = \alpha_{n,l}s_n(t) = \beta_{n,l}\exp(j\varphi_{n,l})s_n(t)$ for $n = 1, 2, \ldots, N_t$ and $l = 1, 2, \ldots, N_r$. Furthermore, in TOSD–SSK modulation we have $s_n(t) = \sqrt{E_m}w_n(t)$ for $n = 1, 2, \ldots, N_t$.

In particular, (2) is a general N_t–hypothesis detection problem [18, Sec. 7.1], [19, Sec. 4.2, pp. 257] in AWGN, when conditioning upon fading channel statistics. Accordingly, the mismatched ML–optimum detector with imperfect CSI at the receiver is as follows:

$$\hat{m} = \mathop{\arg\max}_{m_i \text{ for } i=1,2,\ldots,N_t} \left\{\hat{D}_i\right\} \tag{3}$$

where $\hat{D}_i$ is the mismatched decision metric:

$$\hat{D}_i = \sum_{l=1}^{N_r}\left[\mathrm{Re}\left\{\int_{T_m} r_l(t)\,\hat{s}_{i,l}^*(t)\,dt\right\} - \frac{1}{2}\int_{T_m}\hat{s}_{i,l}(t)\,\hat{s}_{i,l}^*(t)\,dt\right] \tag{4}$$

and $\hat{s}_{i,l}(t) = \hat{\alpha}_{i,l}s_i(t) = (\alpha_{i,l} + \varepsilon_{i,l})s_i(t)$ for $i = 1, 2, \ldots, N_t$ and $l = 1, 2, \ldots, N_r$. If the transmitted message is m_n, which results in switching on the n–th transmit–antenna for data transmission, the detector will be successful in detecting the transmitted message, *i.e.*, $\hat{m} = m_n$, if and only if $\max_{i=1,2,\ldots,N_t}\left\{\hat{D}_i\right\} = \hat{D}_n$.

By using (2), the decision metric in (4), when conditioning upon the transmission of message m_n, *i.e.*, $\hat{D}_{i|m_n}$, can be written as follows ($n = 1, 2, \ldots, N_t$, $i = 1, 2, \ldots, N_t$):

$$\hat{D}_{i|m_n} = \sum_{l=1}^{N_r}\mathrm{Re}\left\{\alpha_{n,l}\hat{\alpha}_{i,l}^*E_m\delta_{i,n} + \hat{\alpha}_{i,l}^*\sqrt{E_m}\tilde{\eta}_{i,l}\right\} - \frac{E_m}{2}\sum_{l=1}^{N_r}\left|\hat{\alpha}_{i,l}\right|^2 \tag{5}$$

with $\tilde{\eta}_{i,l} = \int_{T_m}\eta_l(t)\,w_i^*(t)\,dt$.

III. Performance Analysis

In this section, we summarize the main steps to compute the ABEP of the mismatched detector in (3). To this end, we exploit the same methodology proposed in [4] for a receiver with P–CSI, but generalize the derivation to account for channel estimation errors. More specifically, the ABEP can be tightly upper–bounded as follows [4, Eq. (34)]:

$$\mathrm{ABEP} \le \frac{1}{2(N_t - 1)}\sum_{t_1=1}^{N_t}\sum_{t_2 \ne t_1 = 1}^{N_t}\mathrm{APEP}(\mathrm{TX}_{t_1} \to \mathrm{TX}_{t_2}) \tag{6}$$

where $\mathrm{APEP}(\mathrm{TX}_{t_1} \to \mathrm{TX}_{t_2})$ denotes the Average[3] Pairwise Error Probability (APEP) of the transmit–antenna TX_{t_1} and TX_{t_2} with $t_1, t_2 = 1, 2, \ldots, N_t$, *i.e.*, the probability of detecting TX_{t_2} when, instead, TX_{t_1} is actually transmitting. More specifically, $\mathrm{APEP}(\mathrm{TX}_{t_1} \to \mathrm{TX}_{t_2})$ is the ABEP of an equivalent $2 \times N_r$ MIMO system where only the transmit–antenna TX_{t_1} and TX_{t_2} can be switched on for transmission. In this section, exact closed–form expressions of the APEPs in (6) are given.

A. Computation of PEPs

Let us start by computing the PEPs, *i.e.*, the pairwise probabilities in (6) when conditioning upon fading channel statistics. From (3), the $\mathrm{PEP}(\mathrm{TX}_{t_1} \to \mathrm{TX}_{t_2})$ is as follows:

$$\mathrm{PEP}(\mathrm{TX}_{t_1} \to \mathrm{TX}_{t_2}) = \Pr\left\{\hat{D}_{t_1|m_{t_1}} < \hat{D}_{t_2|m_{t_1}}\right\} \tag{7}$$

where, from (5), $\hat{D}_{t_1|m_{t_1}}$ and $\hat{D}_{t_2|m_{t_1}}$ can be explicitly written as follows:

$$\hat{D}_{t_1|m_{t_1}} = \sum_{l=1}^{N_r}\mathrm{Re}\left\{\hat{\alpha}_{t_1,l}^*\left(\alpha_{t_1,l}E_m + \sqrt{E_m}\tilde{\eta}_{t_1,l}\right)\right\} - \frac{E_m}{2}\sum_{l=1}^{N_r}\left|\hat{\alpha}_{t_1,l}\right|^2 \tag{8}$$

$$\hat{D}_{t_2|m_{t_1}} = \sum_{l=1}^{N_r}\mathrm{Re}\left\{\hat{\alpha}_{t_2,l}^*\sqrt{E_m}\tilde{\eta}_{t_2,l}\right\} - \frac{E_m}{2}\sum_{l=1}^{N_r}\left|\hat{\alpha}_{t_2,l}\right|^2 \tag{9}$$

From (8) and (9), the PEP in (7) can be written as shown in (10) on top of the next page, where we have: i) used the identity $\mathrm{Re}\{ab^*\} = (1/2)ab^* + (1/2)a^*b$, which holds for

[2] In order to avoid any confusion with the adopted notation, let us emphasize that the subscript n denotes the actual message that is transmitted, while the subscript i denotes the (generic) i–th message that is tested by the detector to solve the N_t–hypothesis detection problem. More specifically, for each signaling interval, n is fixed, while i can take different values at the detector.

[3] The expectation is here computed over fading channel statistics.

$$\mathrm{PEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)=\Pr\left\{\hat{D}_{t_1|m_{t_1}}<\hat{D}_{t_2|m_{t_1}}\right\}=\Pr\left\{\frac{\hat{D}_{t_1|m_{t_1}}}{N_0}<\frac{\hat{D}_{t_2|m_{t_1}}}{N_0}\right\}$$
$$=\Pr\left\{\begin{array}{c}\sum_{l=1}^{N_r}\left[\frac{1}{2}\left(\alpha_{t_1,l}\sqrt{\bar\gamma}+\varepsilon_{t_1,l}\sqrt{\bar\gamma}\right)^*\left(\alpha_{t_1,l}\sqrt{\bar\gamma}+\frac{\tilde\eta_{t_1,l}}{\sqrt{N_0}}\right)+\frac{1}{2}\left(\alpha_{t_1,l}\sqrt{\bar\gamma}+\varepsilon_{t_1,l}\sqrt{\bar\gamma}\right)\left(\alpha_{t_1,l}\sqrt{\bar\gamma}+\frac{\tilde\eta_{t_1,l}}{\sqrt{N_0}}\right)^*-\frac{1}{2}\left|\alpha_{t_1,l}\sqrt{\bar\gamma}+\varepsilon_{t_1,l}\sqrt{\bar\gamma}\right|^2\right]\\ <\\ \sum_{l=1}^{N_r}\left[\frac{1}{2}\left(\alpha_{t_2,l}\sqrt{\bar\gamma}+\varepsilon_{t_2,l}\sqrt{\bar\gamma}\right)^*\frac{\tilde\eta_{t_2,l}}{\sqrt{N_0}}+\frac{1}{2}\left(\alpha_{t_2,l}\sqrt{\bar\gamma}+\varepsilon_{t_2,l}\sqrt{\bar\gamma}\right)\frac{\tilde\eta^*_{t_2,l}}{\sqrt{N_0}}-\frac{1}{2}\left|\alpha_{t_2,l}\sqrt{\bar\gamma}+\varepsilon_{t_2,l}\sqrt{\bar\gamma}\right|^2\right]\end{array}\right\} \tag{10}$$

$$\mathrm{PEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)=\frac{1}{2}-\frac{1}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}\left\{\Upsilon(\nu)\Upsilon(-\nu)\exp\left(\Delta_{t_1}(\nu)\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2+\Delta_{t_2}(-\nu)\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2\right)\right\}}{\nu}d\nu \tag{19}$$

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any pair of complex numbers a and b; ii) normalized to N_0 both decision metrics in order to explicitly show the Signal–to–Noise Ratio (SNR) $\bar\gamma=E_m/N_0$; and iii) used (1).

Let us now define:

$$\begin{cases}d_{t_1,l}=A\left|X_{t_1,l}\right|^2+B\left|Y_{t_1,l}\right|^2+CX_{t_1,l}Y^*_{t_1,l}+C^*X^*_{t_1,l}Y_{t_1,l}\\ d_{t_2,l}=A\left|X_{t_2,l}\right|^2+B\left|Y_{t_2,l}\right|^2+CX_{t_2,l}Y^*_{t_2,l}+C^*X^*_{t_2,l}Y_{t_2,l}\end{cases} \tag{11}$$

where $X_{t_1,l}=\alpha_{t_1,l}\sqrt{\bar\gamma}+\varepsilon_{t_1,l}\sqrt{\bar\gamma}$, $Y_{t_1,l}=\alpha_{t_1,l}\sqrt{\bar\gamma}+\tilde\eta_{t_1,l}/\sqrt{N_0}$, $X_{t_2,l}=\alpha_{t_2,l}\sqrt{\bar\gamma}+\varepsilon_{t_2,l}\sqrt{\bar\gamma}$, $Y_{t_2,l}=\tilde\eta_{t_2,l}/\sqrt{N_0}$, and $A=-1/2$, $B=0$, $C=1/2$.

With these definitions, the PEP in (10) can be simplified as follows:

$$\mathrm{PEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)=\Pr\left\{d_{t_1}-d_{t_2}<0\right\}=\Pr\left\{d_{t_1,t_2}<0\right\} \tag{12}$$

where $d_{t_1}=\sum_{l=1}^{N_r}d_{t_1,l}$, $d_{t_2}=\sum_{l=1}^{N_r}d_{t_2,l}$, $d_{t_1,t_2}=d_{t_1}-d_{t_2}$.

From [12, Sec. III], we note that the PEP in (12) can be studied by exploiting the theory of "quadratic–form" receivers in complex Gaussian RVs. More specifically, after a few algebraic manipulations, it can be shown that, when conditioning upon fading channel statistics, d_{t_1} and d_{t_2} are two quadratic forms with CF equal to:

$$\Psi_{d_t}\left(\nu\right|\boldsymbol{\alpha}_t)=\frac{\left(v_av_b\right)^{N_r}\exp\left\{\frac{v_av_b\left(-\nu^2\gamma_{t,a}+j\nu\gamma_{t,a}\right)}{(\nu+jv_a)(\nu-jv_b)}\right\}}{\left(\nu+jv_a\right)^{N_r}\left(\nu-jv_b\right)^{N_r}} \tag{13}$$

where we have emphasized the conditioning upon all the fading channels $\alpha_{i,l}$ for $i=1,2,\ldots,N_t$ and $l=1,2,\ldots,N_r$ in the channel vector $\boldsymbol{\alpha}_t$ with $t\in\{t_1,t_2\}$, and have defined:

$$\begin{cases}\gamma_{t_1,a}=\frac{\bar\gamma}{2}\left(1+\frac{1}{N_pr_{pm}}\right)\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2=g_{t_1,a}\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2\\ \gamma_{t_1,b}=\frac{\bar\gamma}{2}\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2=g_{t_1,b}\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2\\ \gamma_{t_2,a}=\frac{\bar\gamma}{2}\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2=g_{t_2,a}\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2\\ \gamma_{t_2,b}=-\frac{\bar\gamma}{2}\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2=g_{t_2,b}\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2\end{cases} \tag{14}$$

where $g_{t_1,a}=(1/2)\,\bar\gamma\left(1+1/(N_pr_{pm})\right)$, $g_{t_1,b}=g_{t_2,a}=-g_{t_2,b}=(1/2)\,\bar\gamma$, and $v_a=\sqrt{(1/4)+N_pr_{pm}}+(1/2)$, $v_b=\sqrt{(1/4)+N_pr_{pm}}-(1/2)$.

By taking into account (14), the CF in (13) can be re–written in the very compact form as follows:

$$\Psi_{d_t}\left(\nu\right|\boldsymbol{\alpha}_t)=\Upsilon(\nu)\exp\left(\Delta_t(\nu)\sum_{l=1}^{N_r}\left|\alpha_{t,l}\right|^2\right) \tag{15}$$

where $\Upsilon(\cdot)$ and $\Delta_t(\cdot)$ are independent of the fading channel gains, and are defined as follows:

$$\begin{cases}\Upsilon(\nu)=\frac{\left(v_av_b\right)^{N_r}}{\left(\nu+jv_a\right)^{N_r}\left(\nu-jv_b\right)^{N_r}}\\ \Delta_t(\nu)=\frac{v_av_b\left(-\nu^2g_{t,a}+j\nu g_{t,b}\right)}{\left(\nu+jv_a\right)\left(\nu-jv_b\right)}\end{cases} \tag{16}$$

Let us emphasize that d_{t_1} and d_{t_2} are conditional quadratic forms in complex Gaussian RVs because both the receiver noise and the channel estimation errors are complex Gaussian RVs. Moreover, since noises and channel estimation errors are statistically independent if $t_1\neq t_2$, then it follows that d_{t_1} and d_{t_2} are two independent quadratic forms. As a consequence, the CF of d_{t_1,t_2} in (12) can be computed as:

$$\Psi_{d_{t_1,t_2}}\left(\nu\right|\boldsymbol{\alpha}_{t_1},\boldsymbol{\alpha}_{t_2})=\Psi_{d_{t_1}}\left(\nu\right|\boldsymbol{\alpha}_{t_1})\,\Psi_{d_{t_2}}\left(-\nu\right|\boldsymbol{\alpha}_{t_2}) \tag{17}$$

From (15) and (17), the PEP in (12) can be computed from [12, Eq. (11)], as follows:

$$\begin{aligned}\mathrm{PEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)&=\frac{1}{2}-\frac{1}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}\left\{\Psi_{d_{t_1,t_2}}\left(\nu\right|\boldsymbol{\alpha}_{t_1},\boldsymbol{\alpha}_{t_2})\right\}}{\nu}d\nu\\ &=\frac{1}{2}-\frac{1}{\pi}\int_0^{\pi/2}\frac{\mathrm{Im}\left\{\Psi_{d_{t_1,t_2}}\left(\tan(\xi)\right|\boldsymbol{\alpha}_{t_1},\boldsymbol{\alpha}_{t_2})\right\}}{\sin(\xi)\cos(\xi)}d\xi\end{aligned} \tag{18}$$

which yields an exact, single–integral, and closed–form expression for analysis and design of TOSD–SSK modulation with channel estimation errors.

B. Computation of APEPs

Let us now remove the conditioning upon the fading channel in (18). To this end, we first substitute (15) and (17) in (18), as shown in (19) on top of this page. Then, by averaging over the fading channels, we obtain:

$$\begin{aligned}&\mathrm{APEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)=\mathrm{E}\left\{\mathrm{PEP}\left(\mathrm{TX}_{t_1}\to\mathrm{TX}_{t_2}\right)\right\}\\ &\quad=\frac{1}{2}-\frac{1}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}\left\{\Upsilon(\nu)\Upsilon(-\nu)M_{\mathrm{A}_{t_1,t_2}(\nu)}(1)\right\}}{\nu}d\nu\end{aligned} \tag{20}$$

where we have introduced the RV $\mathrm{A}_{t_1,t_2}(\nu)$ as follows:

$$\mathrm{A}_{t_1,t_2}(\nu)=\Delta_{t_1}(\nu)\sum_{l=1}^{N_r}\left|\alpha_{t_1,l}\right|^2+\Delta_{t_2}(-\nu)\sum_{l=1}^{N_r}\left|\alpha_{t_2,l}\right|^2 \tag{21}$$

In summary, (20) provides an exact, single–integral, and closed–form expression of the APEP for a generic correlated and non–identically distributed MIMO wireless channel. To compute (20), only the MGF of RV $\mathrm{A}_{t_1,t_2}(\nu)$ in (21) has to be known in closed–form. This MGF might be computed for a large variety of fading channel models as shown in [18]. As an example, let us consider the scenario in which all the wireless links are independent. In this case, $M_{\mathrm{A}_{t_1,t_2}(\nu)}(\cdot)$ reduces to:

$$M_{\mathrm{A}_{t_1,t_2}(\nu)}(s) = \prod_{l=1}^{N_r} M_{|\alpha_{t_1,l}|^2}\left(s\Delta_{t_1}(\nu)\right) \times \prod_{l=1}^{N_r} M_{|\alpha_{t_2,l}|^2}\left(s\Delta_{t_2}(-\nu)\right) \tag{22}$$

where the MGFs $M_{|\alpha_{t,l}|^2}(\cdot)$ for $t \in \{t_1, t_2\}$ are available in [18] for many fading channel models.

From (22) and [20], we can observe that the diversity achieved by the TOSD–SSK modulation scheme with a mismatched receiver is the same as the diversity achieved with P–CSI, *i.e.*, $2N_r$. We will confirm this finding in Section IV with the help of some numerical examples.

IV. Numerical and Simulation Results

In this section, we show some numerical examples to study the performance of TOSD–SSK modulation in the presence of channel estimation errors and compare it with the Alamouti scheme, which similar to TOSD–SSK modulation can offer a diversity gain equal to $2N_r$ [13]. The simulation setup used in our study is as follows: i) we consider independent Rayleigh fading with normalized unit power over all the wireless links. The MGF needed to compute (22) can be found in [18, Eq. (2.8)]; ii) in TOSD–SSK modulation the rate is qual to $R = \log_2(N_t)$; iii) as far the Alamouti scheme is concerned, we consider Multilevel Phase Shift Keying (M–PSK) modulation with constellation size M and rate $R = \log_2(M)$; and iv) the orthogonal shaping filters needed in TOSD–SSK modulation are obtained from Hermite polynomials [14]

From Figs. 1–4, we can observe, for various data rates, a very good agreement between Monte Carlo simulations and the analytical framework developed in Section III. By carefully analyzing these figures, the following conclusions can be drawn: i) TOSD–SSK modulation is quite robust to channel estimation errors and only a limited number of pilots N_p are needed to get performance very close to the P–CSI lower–bound. In particular, we notice that, in the analyzed scenario, with $N_p = 10$ there is almost no performance penalty; ii) the numerical examples confirm the diversity gain predicted in Section III-B, and we notice a steeper slope when increasing the number of antennas at the receiver. Furthermore, the diversity gain is preserved for any number of pilot pulses, and, so, the quality of the channel estimates; iii) the performance of both TOSD–SSK modulation and Alamouti scheme gets worse for increasing values of the data rate, as expected; iv) TOSD–SSK modulation is worse than the Alamouti scheme when the data rate is low (1 bits/s/Hz), it yields comparable performance for medium data rates (2 bits/s/Hz), while it noticeably outperforms the Alamouti scheme for high data rate (3 bits/s/Hz and 4 bits/s/Hz), while still keeping almost the same computational complexity. This confirms that TOSD–SSK modulation is a good candidate for high data rate applications, and where the data rate can be increased by adding more antennas at the transmitter but still keeping only one of them active at any time instance; and v) the TOSD–SSK modulation scheme is more robust to channel estimation errors than the Alamouti scheme, which agrees with the results obtained in [9] where SM is compared to the V–BLAST scheme. This result is very important and breaks with the misconception that the space modulation concept is inherently less robust to inaccuracies in the channel estimation because it maps data information on the impulse response of the wireless channel.

More specifically, the relative robustness of TOSD–SSK modulation with respect to the Alamouti scheme can be quantitatively analyzed in Table I, where we have (approximately) computed the SNRs needed to get $\mathrm{ABEP} = 10^{-4}$. For example, we notice that for TOSD–SSK modulation the SNR gap between the setups with P–CSI and $N_p = 1$ is approximately equal to 2dB, while in the same scenario the SNR penalty for the Alamouti scheme is approximately 3dB. Furthermore, the higher robustness of TOSD–SSK modulation to imperfect channel knowledge can be observed by carefully analyzing Table I for a rate equal to 2 bits/s/Hz. We observe that if channel estimation is perfect, the Alamouti scheme is slightly better than TOSD–SSK modulation. However, TOSD–SSK modulation gets slightly better when $N_p = 1$. This example clearly shows the potential benefits of TOSD–SSK modulation in practical scenarios with imperfect channel knowledge.

V. Conclusion

In this paper, we have analyzed the performance of the space modulation principle when CSI is not perfectly known at the receiver. A very accurate analytical framework has been proposed and it has been shown that, unlike common belief, transmission systems based on the space modulation concept can be more robust to channel estimation errors than conventional modulation schemes. Furthermore, it has been shown that only few pilot symbols are needed to achieve almost the same performance as the reference scenario with perfect channel knowledge. These results clearly point out the usefulness of the space modulation principle in practical operating conditions, as well as that it can be a promising low–complexity transmission technology for the next generation MIMO wireless systems.

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TABLE I

REQUIRED E_m/N_0 (dB) TO GET ABEP $= 10^{-4}$. THE FIRST AND SECOND LINES SHOW THE SETUP WITH $N_r = 1$ AND $N_r = 2$, RESPECTIVELY.

TOSD–SSK					Alamouti				
Rate	$N_p = 1$	$N_p = 3$	$N_p = 10$	P − CSI	Rate	$N_p = 1$	$N_p = 3$	$N_p = 10$	P − CSI
1 bits/s/Hz	27.1 18.2	26 16.9	25.5 16.4	25.3 16.2	1 bits/s/Hz	25.3 16.2	23.5 14.5	22.8 13.5	22.3 13.2
2 bits/s/Hz	28.7 19	27.5 17.8	27 17.3	26.8 17	2 bits/s/Hz	29.1 19.7	27.4 18	26.4 17.1	26.1 16.7
3 bits/s/Hz	30.2 19.8	29 18.6	28.5 18.2	28.4 17.8	3 bits/s/Hz	33.8 24.7	32.3 23	31.4 22.1	30.8 21.7
4 bits/s/Hz	31.7 20.7	30.5 19.4	30.1 18.9	29.9 18.7	4 bits/s/Hz	39.4 30.2	37.7 28.6	36.7 27.7	36.4 27.2

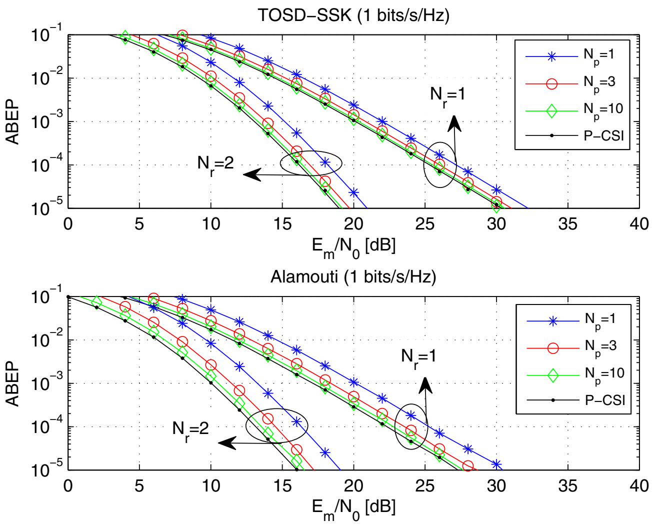

Fig. 1. ABEP against E_m/N_0 for various pilot pulses N_p and rate 1 bits/s/Hz. (top) TOSD–SSK modulation: solid lines show the analytical model and markers Monte Carlo simulations. (bottom) Alamouti scheme: only Monte Carlo simulations are shown.

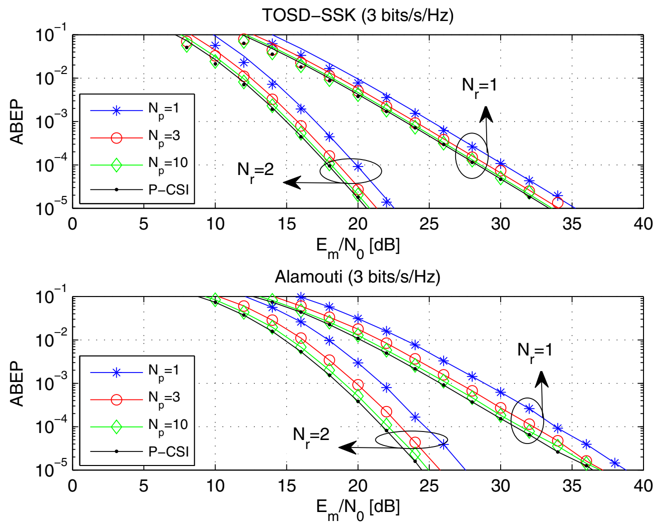

Fig. 3. ABEP against E_m/N_0 for various pilot pulses N_p and rate 3 bits/s/Hz. (top) TOSD–SSK modulation: solid lines show the analytical model and markers Monte Carlo simulations. (bottom) Alamouti scheme: only Monte Carlo simulations are shown.

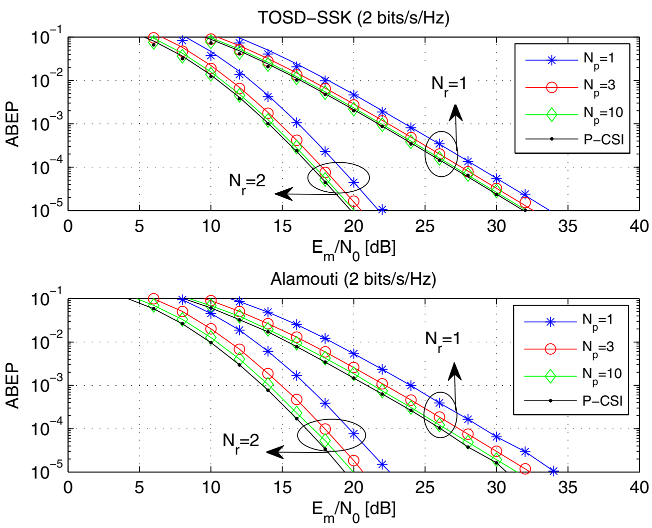

Fig. 2. ABEP against E_m/N_0 for various pilot pulses N_p and rate 2 bits/s/Hz. (top) TOSD–SSK modulation: solid lines show the analytical model and markers Monte Carlo simulations. (bottom) Alamouti scheme: only Monte Carlo simulations are shown.

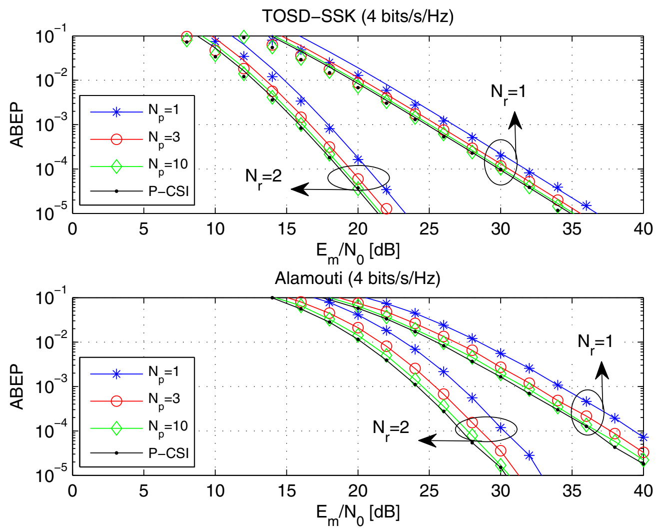

Fig. 4. ABEP against E_m/N_0 for various pilot pulses N_p and rate 4 bits/s/Hz. (top) TOSD–SSK modulation: solid lines show the analytical model and markers Monte Carlo simulations. (bottom) Alamouti scheme: only Monte Carlo simulations are shown.

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