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2-User Multiple Access Spatial Modulation

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Abstract—Spatial modulation (SM) is a recently proposed approach to multiple–input–multiple–output (MIMO) systems which entirely avoids inter–channel interference (ICI) and requires no synchronisation between the transmit antennas, while achieving a spatial multiplexing gain. SM allows the system designer to freely trade off the number of transmit antennas with the signal constellation. Additionally, the number of transmit antennas is independent from the number of receive antennas which is an advantage over other multiplexing MIMO schemes. Most contributions thus far, however, have only addressed SM aspects for a point–to–point communication systems, i.e. the single–user scenario. In this work we seek to characterise the behaviour of SM in the interference limited scenario. The proposed maximum–likelihood (ML) detector can successfully decode incoming data from multiple sources in an interference limited scenario and does not suffer from the near–far problem.

I. INTRODUCTION

Multiple–antenna systems are fast becoming a key technology for modern wireless systems. They offer improved error performance and higher data rates, at the expense of increased complexity and power consumption [1]. Spatial modulation (SM) is a recently proposed approach to multiple–input–multiple–output (MIMO) systems which entirely avoids inter–channel interference (ICI) and requires no synchronisation between the transmit antennas, while achieving a spatial multiplexing gain [2]. A spatial multiplexing gain is achieved by mapping a block of information bits into a constellation point in the signal and spatial domains [3]. In SM, the number of information bits, ℓ, encoded in the spatial domain can be related to the number of transmit antennas, Nt, as Nt = 2ℓ. This means that the number of transmit antennas must be a power of two unless fractional bit encoding is used [4]. Additionally, compared to other MIMO schemes, the spatial multiplexing gain i.e. the number of transmit antennas, is independent of the number of receive antennas. This offers the flexibility to trade off the number of transmit antennas with the modulation order in the signal domain to meet the desired data rate without regard for the number of receive antennas. It should also be noted that SM is shown to outperform other MIMO schemes in terms of bit–error–ratio (BER) [3].

A number of papers are available in the literature which are aimed at understanding and improving the performance of SM in various scenarios. Trellis coding on the transmit antenna is proposed in [5], a reduced complexity decoder is given in [6] and the performance of SM over a wide range of channels is presented in [7]. The optimal detector is known with and without channel state information at the receiver in [8–10]. The optimal power allocation problem for a 2 transmit with 1 receive antenna system is solved in closed form in [11] and the performance of SM in correlated fading channels is considered in [12]. Recent work has also shown that SM can be combined with space–time block codes to attain spectral efficiency gains [13]. SM has also been applied to relaying systems in [14] where it exhibits significant signal–to–noise–ratio (SNR) gains when compared to non–cooperative decode and forward.

Most contributions thus far, however, have only addressed SM aspects for a point–to–point communication systems, i.e. the single–user scenario. These scenarios include the application of SM in traditional orthogonal access systems such as frequency division multiple access (FDMA), time division multiple access (TDMA) or orthogonal frequency division multiple access (OFDMA) where co–channel interference is managed by ensuring orthogonal transmissions by all nodes in the system. A notable exception is given in [15], where the authors focus their analysis on a limited two user scenario employing only space–shift–keying (SSK). It should be noted, that SSK is similar to SM in that the antenna index is used for data transmission, but instead of a full signal–symbol only a reference signal is sent to enable channel estimation at the receiver.

In this work we seek to characterise the behaviour of SM in the interference limited scenario. In particular, we propose a maximum–likelihood (ML) detector which can successfully decode incoming data in the case of simultaneous transmission and does not suffer from the near–far problem, i.e. the detector can successfully decode data from a user with a lower signal–to–noise–ratio (SNR). The proposed jointly optimum multi–user detector minimises the BER for all users and does not suffer from the near–far problem.

The remainder of this work is organized as follows. In Section II, the system and channel models are introduced. In Section III, the performance of SM in the multiple access scenario is characterised and the analytical modelling for the multi–user detector is proposed. Section IV provides numerical and simulation results to substantiate the accuracy of the analytical framework developed. In Section V, we summarise and conclude the work.
II. SYSTEM MODEL

The basic idea of SM is to map blocks of information bits into two information carrying units [3]: i) a symbol, chosen from a complex signal–constellation diagram, and ii) a unique transmit–antenna, chosen from the set of transmit–antennas in the antenna–array, i.e. the spatial–constellation. The general SM constellation point is thus a combination of a signal–constellation point and a spatial–constellation point. The SM constellation diagram is presented in Fig. 1.

![Fig. 1. A transmission of four bits is assumed. The first two bits from right to left define the spatial–constellation point identifying the active antenna, while the remaining two bits determine the signal–constellation point that will be transmitted. This scenario means that a single SM constellation point carries four information bits.](image)

In the following work we assume a three node scenario as shown in Fig. 2 where we seek to characterise the behaviour of SM during simultaneous transmission i.e. in the presence of co-channel interference. We assume that the two transmit nodes, denoted as User1, node \((U_1)\), and User2, node \((U_2)\), in Fig. 2, transmit simultaneously to the receiver on the same time-frequency slot. Each node broadcasts a signal constellation symbol, \(x\), from one of its available antennas.

The received signal is given by:

\[
y_j = \sqrt{E_m}\sigma^2_{(U_1)}h_{i(v_1)}x(U_1) + \sqrt{E_m}\sigma^2_{(U_2)}h_{k(v_2)}x(U_2) + \eta \tag{1}
\]

where:

- \(E_m\) is the average energy per symbol for both nodes,
- \(i\) and \(k\) are the indices of the transmit antennas from nodes 1 and 2 respectively,
- \(j\) is the index of the receive antenna from a total of \(N_r\) available,

\(\sigma^2_{(U_1)}\) and \(\sigma^2_{(U_2)}\) are the channel attenuation coefficients on the \(U_1\) to receiver and \(U_2\) to receiver links in Fig. 2 respectively,

\(h_{i(v_1)}\) and \(h_{k(v_2)}\) are the fast fading channel coefficients of the link between the active antennas \((i, k)\) and the receiving antenna \(j\), and

\(\eta\) is a complex normal random variable with zero mean and variance \(N_0CN^2(0, N_0)\), and represents the additive white Gaussian noise (AWGN) at the receiver.

We note that all bold notations indicate vector notations. We now look at the analytical formulation of the system.

III. ANALYTICAL MODELLING

In this section, we develop a ML detector for use in the presence of co-channel interference. The detector computes the Euclidean distance between the received vector signal \(\hat{y}\) and the set of all possible received signals, selecting the closest one. The mathematical formulation of the ML detector used in the system is given in (2). We note that this formulation is valid for any channel vectors and any transmitted symbols. In particular, if the channels are correlated i.e. non-orthogonal, then it will be more difficult for the receiver to distinguish the individual antennas used in the transmission, which will result in an increase of the BER.

Starting from the system model presented in Section II, the decoded pair \((\hat{x}_{est}, n_{est})^{(U_1)}\), formed from the estimated symbol \(x_{est}\) emitted from antenna \(n_{est}\) on node \(\xi\), where \(\xi \in \{(U_1), (U_2)\}\), is given by:

\[
\{(\hat{x}_{est}, n_{est})^{(U_1)}, (\hat{x}_{est}, n_{est})^{(U_2)}\} = \operatorname{arg\, min}_{\hat{x}, n} \left\| \hat{y} - \sum_{u \in \{(U_1), (U_2)\}} \hat{x}(u)h_{n(u)}^{(u)} \right\|_F^2
\tag{2}
\]

\(\mathbf{X}^{(u)}\) is the set of all possible signal constellation points for node \(u\) with \(M^{(u)}\) number of elements, \(N_r^{(u)}\) is the number of available transmit antennas on node \(u\) and \(|| \cdot ||_F\) is the Frobenius norm.

From here we can use techniques based on the union bound to describe the behaviour of the interference aware SM detector in the high SNR regions. The union bound for the interference aware SM detector, which estimates the average bit-error-ratio (ABER) for node \(\xi\), can be expressed as given in (3) where \(N_{\xi}(b, \tilde{b}) = N_{\xi}^t(n_{est}, \tilde{n}_{est}) + N_{\xi}^e(x, \tilde{x})\).\(N_{\xi}^t(n_{est}, \tilde{n}_{est})\) denotes the Hamming distance between the binary representations of the antenna indices \(n_{est}\) and \(\tilde{n}_{est}\) on node \(\xi\). Similarly, \(N_{\xi}^e(x, \tilde{x})\) denotes the Hamming distance between the binary representations of the symbols \(x\) and \(\tilde{x}\) on node \(\xi\).

We define PEP \(\xi^{(U_1), (U_2), n_{est}^{(U_1), (U_2)}, \tilde{n}_{est}^{(U_1), (U_2)}, \hat{x}^{(U_1), (U_2), \hat{n}_{est}^{(U_1), (U_2)}}\) to be the pairwise error probability between the symbol \(x^{(U_1), (U_2)}\) emitted from antennas \(n_{est}^{(U_1), (U_2)}\) being detected as symbol \(\hat{x}^{(U_1), (U_2)}\) emitted by antenna \(\hat{n}_{est}^{(U_1), (U_2)}\). It should be noted that the pairs, \((x^{(U_1), (U_2), n_{est}^{(U_1), (U_2)}})\) and \((\hat{x}^{(U_1), (U_2), \hat{n}_{est}^{(U_1), (U_2)}})\), come from the set of
ABERξ \leq \sum_{\hat{x}(U_1),\hat{x}(U_2)} \sum_{\hat{n}_1(U_1),\hat{n}_2(U_2)} \frac{N^2 \langle b, \hat{b} \rangle}{M} \log_2 \left( \frac{M \langle \xi^{(\xi)} \rangle}{M \langle \xi^{(\xi)} \rangle} \right) \left( E_{\text{H}} \left( \text{PEP} \left( \hat{x}(U_1),\hat{x}(U_2), n_1(U_1),n_2(U_2), \hat{x}(U_1),n_2(U_2) \right) \right) \right).

(3)

\text{PEP}(\cdot) = Q \left( \sqrt{\frac{E_m}{2N_0}} \left( \frac{h_{\hat{n}_1(U_1)}x(U_1) - h_{\hat{n}_1(U_1)}\hat{x}(U_1) + \sigma(U_2)}{h_{\hat{n}_2(U_2)}x(U_2) - h_{\hat{n}_2(U_2)}\hat{x}(U_2)} \right)^2 \right).

(4)

all possible symbol-antenna pairs for both nodes, i.e.,
\( \hat{x}(U_1),\hat{x}(U_2), n_1(U_1),n_2(U_2) \) and
\( \hat{x}(U_1),\hat{x}(U_2), n_1(U_1),n_2(U_2) \) represent the expectation of the system with respect to the channel and \( Q(\cdot) \) represents the expectation of the system with respect to the channel and \( Q(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( -\frac{x^2}{2} \right) dx \).

The ABER for node \( \xi \) is shown in (3), where the pairwise error probability is given in (4). Due to space constraints, we omit the derivation of (4). We note that thus far no assumptions have been made as to the distribution of the channel.

If we consider a Rayleigh fading channel, then we can derive the closed form solution for \( E_{\text{H}}[\text{PEP}(\cdot)] \) in (3) by employing the solution to [16, eq. 62]. We note that by assuming a Rayleigh fading channel, the argument within (4) can be represented as the summation of \( 2N_r \) squared Gaussian random variables, with zero mean and variance equal to 1, which means that they can be described by a central Chi-squared distribution with \( 2N_r \) degrees of freedom and a probability density function of:

\[ p_K(k) = \frac{1}{2^{N_r}} \left( \frac{N_r - 1}{k} \right)^{N_r - 1} \exp \left( -\frac{k}{2} \right) \]

The result for \( E_{\text{H}}[\text{PEP}(\cdot)] \) is given as:

\[ E_{\text{H}}[\text{PEP}(\cdot)] = \int_{-\infty}^{\infty} f(c) \sum_{r=0}^{N_r-1} \binom{N_r}{r} \left( \frac{N_r - 1}{r} \right) \left( 1 - f(c) \right)^r \]  

(5)

such that

\[ f(c) = \frac{1}{2} \left( 1 - \sqrt{1 + c} \right) \]

where

\[ c = \frac{E_m}{4N_0} \sum_{\xi(U_1),\xi(U_2)} \sigma_{\xi(U)}^2 \lambda_{\xi(U)} \]

(6)

which is a quarter of the received SNR at the receiver, and

\[ \lambda_{\xi(U)} = \begin{cases} (|x_1|)^2 + |\hat{x}_1|)^2 & r_1 \neq \hat{r}_1, \\ (|x_2|)^2 + |\hat{x}_2|)^2 & r_2 = \hat{r}_2, \\ 0 & r_1 = \hat{r}_1 \text{ and } x_1 = \hat{x}_1. \end{cases} \]

IV. SIMULATION RESULTS AND DISCUSSION

In this section we aim to show that the interference aware detector proposed in (2) can successfully decode the incoming streams for the two users. Numerical results are shown which demonstrate that (3) provides a tight upper bound for the BER of the interference aware detector at high SNR. The aim of this work is to develop and test a viable multi-user detector for SM.

A. Simulation Setup

A frequency-flat Rayleigh fading channel with no correlation between the transmitting antennas and AWGN is assumed. Perfect channel state information (CSI) is assumed at the receiving node, with no CSI at the transmitter. Only one of the available transmit antennas for each node is active at any transmitting instance. In theory each user independently decides the number of transmit antennas and the symbol modulation it uses. For use in the simulation we assume each node has the same number of transmit antennas as well as the same spectral efficiency target. In each figure, for each user, there are three presented results: i) the simulation results for the interference aware detector, denoted by \( \text{Sim}(\text{User}\xi) \), ii) the theoretical results from (3) using (5), denoted by \( \text{Analytical}(\text{User}\xi) \), and iii) the single-user-lower-bound (SULB), denoted by \( \text{SULB}(\text{User}\xi) \). We define SULB as the system performance in a single-user-single-receiver scenario where the system performance is determined purely by its SNR, defined as \( \frac{E_m}{2N_0} \). The theory behind SULB is well developed in [7].

B. Results

Fig. 3 and Fig. 4 clearly demonstrate that the analytical model presented in (3) represents a tight upper bound for the system in the high SNR region. Additionally, we can see that the system with the lowest SNR has similar performance to that predicted by its SULB. It should be noted that this is not the case for the node with the better SNR. This difference in performance of the two systems can be explained by looking at the error contribution of each element from each node in the analytical prediction.

We define two sets, one for each pairwise possibility within a particular user, given by \( \Omega(\hat{U}_1) \) in (7) for User1. We can similarly define the set \( \Omega(\hat{U}_2) \) for User2. If we now consider (3) and (4) we see that the overall error for each user is inevitably influenced by the errors from the other user. However, since each element from \( \Omega(\hat{U}_1) \) is associated with the full set of possible errors from \( \Omega(\hat{U}_2) \), then all erroneous terms from \( \Omega(\hat{U}_1) \) will ‘carry’ the full error from the terms in \( \Omega(\hat{U}_2) \) and vice versa. This means that besides the pairwise error associated with the mis-detection of the antenna-symbol combination of User1 alone, the error term for User2 and vice versa, i.e. the overall error for node 1 has \( \left( \left\lfloor \text{card} \left( \Omega(\hat{U}_1) \right) \right\rfloor - M(U_1) \right) \left\lfloor \text{card} \left( \Omega(\hat{U}_2) \right) \right\rfloor \) number of error terms where \( \text{card} \{ \cdot \} \) denotes the cardinality of a given set.

We further note that each pairwise error from the user with
the worse SNR makes a bigger contribution to the overall BER than the pairwise error from the node with the better SNR. This can be shown if we look at the Euclidean distance between the different pairwise errors. We classify a pairwise error if the Euclidean distance between the symbol-antenna pairs being tested is greater than zero. In particular, the greater the Euclidean distance becomes, the smaller the error from that term. From (4) it is clear that the pairwise error depends on the SNR as well as the Euclidean distance. It thus follows that given pairwise error terms with the same Euclidean distance, the worse the SNR is for each term, the greater the absolute pairwise error. Considering the above, it is clear that the node with the better channel gain never performs close to its SULB, while the node with the worse channel gain does perform near its SULB.

Fig. 3 and Fig. 4 demonstrate this behaviour. The gap in performance with respect to the SULB for the main contributor to the overall user error, i.e. the node with the lower SNR, effectively increases the BER of the node with the higher SNR. To further elaborate, we note that the difference between the simulation BER curves of the two nodes when $N_r = 2$ and $N_r = 3$ increases as more receive antennas are added. This can be explained if we consider that by increasing the number of receive antennas, the diversity of the system increases and the pairwise error terms for each node approach zero more rapidly. This mean that the absolute pairwise error contributed to the overall BER is less for each node. As a consequence, the node with the better channel gain i.e. the node with higher SNR, will perform closer to its SULB.

On the one hand, moving from Fig. 3 and Fig. 4 to Fig. 5, we notice that for a fixed spectral efficiency and a fixed number of transmit antennas, the addition of more receive antennas results in an increasing gap between the average analytical BER curves of the two nodes. In particular, a gap of around 4 dB between the performance of User1 and User2 with $N_r = 2$ is increased to around 7 dB when $N_r = 4$ and further increased to around 9 dB for $N_r = 8$. On the other hand, given that the two nodes experience a channel gain difference of 10 dB, we know that the interference aware detector cannot reach the performance of independent detection and the SULB for the node with the better SNR. Nonetheless, the gap between their respective BER curves tend toward the difference between the channel attenuations of the two users as $N_r$ grows to infinity but can never reach it i.e. the gap tends towards 10 dB.

The addition of more transmit antennas at each of the nodes results in SNR gains for each node as can be seen when we compare Fig. 4 and Fig. 6. Interestingly, however, increasing the number of transmit antennas does not change the relative behaviour of the system, i.e. the SNR difference between the BER curves of the two nodes remains constant. This behaviour is expected when we consider that (5) is independent of $N_t$ and heavily influenced by $N_r$. In particular, the BER of both nodes is dependent on the variance of the channel coefficients in (4) which follow a central chi-squared distribution with $2N_r$ degrees of freedom. This variance is defined in (6).

At this point it should be noted that while the proposed detector is jointly optimum for both nodes and does not suffer from the near-far problem, it needs full CSI from all possible transmitting antennas to each receiving antenna. Additionally, finding the optimal solution is an exponentially complex problem, i.e. if we assume each node has the same number of transmit antennas and uses the same signal constellation, then the multi user ML detector has $O((MN_s)^{N_u})$ computational complexity which is proven to be NP-complete [17]. Fortunately, recent work on sphere detection algorithms may be used to alleviate this computational cost [18].

V. CONCLUSION

In this work the performance of SM with simultaneous transmission was analysed. A ML detector for SM in the interference limited scenario was proposed. Its performance over uncorrelated Rayleigh fading channels was studied and a closed form solution for the upper bound of the system was provided. Numerical results verified that the proposed analysis was fairly accurate for the high SNR regions. On the one hand, increasing the number of transmit antennas at each of the nodes from 2 to 4 resulted in SNR gains of around 2 dB. This measure did not, however, have any effect on the
relative coding gain between the BER curves of the two nodes \(i.e\). the two nodes improved their performance by the same amount. On the other hand, increasing the number of receive antennas increased the diversity of the system and decreased the error contribution of each node, thus increasing the SNR gap between the BER curves of the two nodes.

The generalization of this work to a system with an arbitrary number of nodes, along with further investigation on the performance of SM in an interference limited scenario will be considered in the future.

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