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Abstract—**This paper proposes an iterative method for a joint estimation of the signal-to-noise ratio (SNR) and the frequency selective transmission channel in OFDM systems. The noise variance is estimated using a pilot preamble scheme, with the minimum mean square error (MMSE) criterion which requires the channel estimation. The channel estimation uses the Linear MMSE (LMMSE) method which requires the noise variance estimation. As each estimatior feeds the other one, an iterative algorithm is proposed. We prove that this algorithm converges for any non-null initialization value. Simulations show the validity of the method with a very low number of iteration for both SNR and channel estimation. We show that the number of pilot symbols in the preamble with respect to the performance of SNR estimation is improved compared to existing SNR estimation methods. Our approach also requires only one pilot symbol. Furthermore, for a given BER value, the SNR gap between perfect estimation and our proposed method is less than 0.5 dB.**

I. Introduction

The performance quality of a communication system is strongly dependent on the design of the transmitter. The knowledge of the signal-to-noise ratio (SNR) is then very useful in order to adjust transmitter parameters as mapping constellation size or robustness of encoding. Moreover, in many algorithms such as linear minimum mean square error (LMMSE) channel estimation [1], or adaptive modulations [2], the SNR or the noise power is required, but most of the time, the SNR is only supposed to be known at receiver side. The context of this paper is orthogonal frequency division multiplexing (OFDM) broadcast systems (as digital television [3] or digital radio [4]) in frequency selective channels.

Reference [5] covers the usual maximum likelihood (ML), minimum mean square error (MMSE) and second- and forth-Order moment (M_2M_4) algorithms performance for SNR estimation in OFDM systems and makes a comparison with its proposed method, based on the use of the autocorrelation function given by the model of the channel (Jakes or Ricean models). ML, MMSE and M_2M_4 algorithms are depicted and their efficiencies are compared in the case of AWGN channel for single carrier systems in [6], [7] and [8]. Some iterative algorithms are presented in [8] and [9], but they are not suitable for frequency selective channels. ML SNR estimator, whose developments are given in [10], presents a prohibitive calculation complexity in the case of frequency selective channels. MMSE estimator, from which we derive our proposed method, requires the estimation of the transmission channel. Mainly for the same requirement, only theoretical expressions of the MMSE SNR estimator are proposed in the literature [5], [6], [11], and no practical algorithm. M_2M_4, firstly mentioned in [12], presents a low computational complexity and does not require any channel estimation. However, its efficiency is degraded in frequency selective channels. Simplifications of ML estimator have been brought by the recursive expectation maximisation EM algorithm [13] [14], wich also allows to estimate both noise variance and channel perturbations [15], [16]. However, each step of the EM algorithm necessitates a maximisation wich requires the estimation of all the parameters from the previous step. Furthermore, EM is used when the observed datas are not complete.

In order to avoid the need of the channel estimation, [11] develops a method for a 2×2 multi input multi output (MIMO) OFDM system assuming that the channel is invariant during a two-symbols preamble and for two consecutive carriers. Reference [17] presents a method using the estimation of the variance of the noise thanks to a two-pilots preamble and combines it with the estimation of the second moment (M_2) of the received signal. These two methods require a two-pilot preamble, what reduces the data rate of the transmission, especially if the preamble must be frequently repeated. In [18], an estimation of the SNR is presented using the properties of the channel correlation matrix, estimated thanks to a one-pilot preamble. This method is limited by an insufficient statistic on the channel, what degrades the performance, in particular for low SNR.

In this paper, we propose a method to estimate both SNR and frequency selective channel with MMSE criterion. The SNR is estimated using the noise variance estimation combined with the second moment of the received signal. The noise variance estimation method, based on the MMSE criterion, requires the channel estimate. In its turn, the channel estimation, performed with LMMSE method, needs the noise variance in addition to the channel correlation matrix. As a consequence, we propose an iterative approach in which both noise variance and channel estimations are computed for each iteration, using the results of the previous one. Here is the similarity with the EM, although in our algorithm, the estimation of each parameter only requires the estimation of the other one and not both. We show in the cases of perfect and approximated covariance matrix, that our method quickly converges for both noise variance and channel estimation. Furthermore, as our method only requires a one-pilot preamble by frame, the trade-off between the number of pilot and the performance is improved compared to existing methods.

The continuation of this paper is organized as follows : Section II presents the OFDM system model and the noise variance and channel estimation methods. In Section III, we present our iterative estimation method, and we prove in Section IV the convergence of the proposed algorithm. With the simulations in Section V, one can firstly verify the convergence of the method, secondly compare it with two existing methods of the literature [17], [18]. We draw the conclusions in Section VI.

II. Background

This section aims at describing the transmission system model and the techniques of estimation (noise variance, SNR, channel) required for the proposed method presented in Section III.

A. Transmitted Signal Model

We consider the transmission of OFDM symbols over a multipath channel. After the removal of the cyclic prefix (CP) and the Discrete Fourier Transform (DFT), (1) gives the formulation of the n^{th} received OFDM symbol in the frequency domain:

$$\mathbf{U}_n = \underline{\mathbf{C}}_n \mathbf{H}_n + \mathbf{W}_n. \tag{1}$$

$\mathbf{U}_n = [U_{0,n}, ..., U_{M-1,n}]^T$, $\mathbf{H}_n = [H_{0,n}, ..., H_{M-1,n}]^T$ and $\mathbf{W}_n = [W_{0,n}, ..., W_{M-1,n}]^T$ are the $M \times 1$ complex vectors of the received signal, the multipath channel and the gaussian white noise on the n^{th} time-slot respectively. M is the size of the DFT, which also tallies with the number of carriers per symbol in our model. $\underline{\mathbf{C}}_n$ is the $M \times M$ diagonal matrix of the transmitted signal containing the vector $[C_{0,n}, ..., C_{M-1,n}]$. $C_{m,n}$ are either data elements or pilots, whose gains, phases and positions are perfectly known at emission and reception sides. In this article, pilots are dedicated to channel estimation and noise variance estimation. We consider a pilot-preamble scheme with only one OFDM pilot symbol by frame and assume a constant channel between two consecutive preambles. In the rest of the paper, the pilot symbols are noted with the subscript p. $H_{m,n}$ are the components of the vector $\mathbf{H}_n$ given by

$$H_{m,n} = \sum_{l=0}^{L-1} h_{l,n} \exp\left(-2j\pi \frac{m}{M} \tau_l\right), \tag{2}$$

where m denotes the sub-carrier subscript, L the length of the impulse response and $h_{l,n}$ the zero-mean complex process of the l^{th} path of the channel. All L paths are considered independent.

B. Noise Variance Estimation

We note σ^2 the noise variance (or noise power), equal to $\sigma^2 = E\left\{|W_{m,n}|^2\right\}$, or in the same way in its vector form $\sigma^2 = \frac{1}{M} E\left\{||\mathbf{W}_n||^2\right\}$. We here consider the matrix Frobenius norm, defined for a matrix $\underline{\mathbf{A}}$ as $||\underline{\mathbf{A}}||^2 = tr(\underline{\mathbf{A}}\underline{\mathbf{A}}^H)$, where tr is the trace and $(.)^H$ is the Hermitian transpose. We use in this article the Minimum Mean Square Error (MMSE) criterion to estimate the noise variance (see [10]), noted $\hat{\sigma}^2$ and given by (3)

$$\hat{\sigma}^2 = \frac{1}{M} E\left\{||\mathbf{U}_p - \underline{\mathbf{C}}_p \hat{\mathbf{H}}_p||^2\right\}, \tag{3}$$

where $\hat{\mathbf{H}}_p$ is the channel estimation performed on the pilot symbol. In practice, the expectation can only be approximated by the mean on a sufficiently large number of sub-carriers, leading to

$$\hat{\sigma}^2 = \frac{1}{M} \sum_{m=0}^{M-1} |U_{m,p} - C_{m,p} \hat{H}_{m,p}|^2, \tag{4}$$

where $\hat{H}_{m,p}$ is the estimation of the m^{th} subcarrier. Equation (4) then shows that the efficiency of the MMSE noise variance estimation depends on the quality of the channel estimation.

C. SNR Estimation

The SNR noted ρ is basically obtained from the second moment M_2 of the received signal and the noise variance:

$$M_2 = \frac{1}{M} E\left\{||\mathbf{U}_n||^2\right\} = P_S + \sigma^2, \tag{5}$$

where P_S is the power of the useful transmitted signal. We then get the SNR:

$$\rho = \frac{M_2}{\sigma^2} - 1. \tag{6}$$

In practice, we estimate the SNR $\hat{\rho}$ in the same way:

$$\hat{\rho} = \frac{\hat{M}_2}{\hat{\sigma}^2} - 1, \tag{7}$$

where $\hat{\sigma}^2$ is defined in (4) and $\hat{M}_2$ by

$$\hat{M}_2 = \frac{1}{M} \sum_{m=0}^{M-1} |U_{m,n}|^2. \tag{8}$$

D. Channel Estimation

The two basic estimation methods are the Least Square (LS) and Linear Minimum Mean Square (LMMSE) presented in [19] and [20]. (9) gives the LS channel estimate:

$$\hat{\mathbf{H}}_p^{LS} = \underline{\mathbf{C}}_p^{-1} \mathbf{U}_p = \mathbf{H}_p + \underline{\mathbf{C}}_p^{-1} \mathbf{W}_p. \tag{9}$$

LS estimation is very simple but sensitive to the noise. Furthermore, this estimation cannot be used for the noise variance estimation in (4) as we obtain $\hat{\sigma}^2 = 0$ for $\hat{\mathbf{H}}_p = \hat{\mathbf{H}}_p^{LS}$. (10) gives the efficient LMMSE channel estimation:

$$\hat{\mathbf{H}}_p^{LMMSE} = \underline{\mathbf{R}}_H(\underline{\mathbf{R}}_H + \sigma^2(\underline{\mathbf{C}}_p\underline{\mathbf{C}}_p^H)^{-1})^{-1}\hat{\mathbf{H}}_p^{LS}, \quad (10)$$

where $\underline{\mathbf{R}}_H$ is the channel covariance matrix. LMMSE channel estimation is more efficient than LS, but requires a matrix inversion. We assume in the rest of the paper that: $\forall m = 0, ..., M-1, C_{m,p} = 1$ on a given preamble position p. Consequently, the pilot matrix $\underline{\mathbf{C}}_p$ is equal to the identity matrix noted $\underline{\mathbf{I}}$. Thus, we get from (10): $\hat{\mathbf{H}}_p^{LMMSE} = \underline{\mathbf{R}}_H(\underline{\mathbf{R}}_H + \sigma^2\underline{\mathbf{I}})^{-1}\hat{\mathbf{H}}_p^{LS}$. $\underline{\mathbf{R}}_H$ is usually unknown at receiver, we then proposed in [21] a LMMSE-based estimation method which can replace (10). Furthermore, this estimation method requires the noise variance knowledge generally unknown and estimated by (4). This problem leads to our proposed algorithm for both noise variance and channel iterative estimation.

III. Proposed Algorithm for Noise Variance and channel Estimation

From (4), we notice that the accuracy of the noise variance estimation is directly linked to the channel estimation technique that is used. As, by combining (3) and (9), a LS channel estimation gives a noise variance estimation equal to zero, we then consider a solution that is based on the efficient LMMSE channel estimation. However, from its expression given in (10), the knowledge of the noise variance is necessary and also determines the quality of the channel estimation. Consequently, as the variance estimation feeds the channel estimation and vice versa, we here propose an iterative technique allowing a joint estimation of the noise variance and the channel coefficients, whose principle is described in Fig. 1. In this figure, we then observe that the iterative improvement of the noise variance estimation (respectively the channel coefficients estimation) allows a better estimation of the channel coefficients (respectively the noise variance).

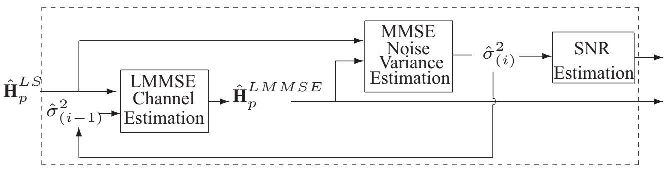

Fig. 1. Block diagram of the proposed iterative algorithm.

Let us then consider the i^{th} iteration of our algorithm, with $i \geq 1$. At this step, due to the iterative nature of our algorithm, the noise variance $\hat{\sigma}^2_{(i-1)}$, estimated at the iteration $(i-1)$, can be used for the LMMSE estimation of the channel by using (10), giving then:

$$\hat{\mathbf{H}}_{p(i)}^{LMMSE} = \underline{\mathbf{R}}_H(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i-1)}\underline{\mathbf{I}})^{-1}\hat{\mathbf{H}}_p^{LS}. \quad (11)$$

Consequently, by using (3) with $\hat{\mathbf{H}}_p = \hat{\mathbf{H}}_{p(i)}^{LMMSE}$, we can propose a new estimation of the noise variance. Recalling that $\underline{\mathbf{C}}_p = \underline{\mathbf{I}}$, this new estimation is expressed by

$$\begin{aligned}
\hat{\sigma}^2_{(i)} &= \frac{1}{M}E\left\{||\mathbf{U}_p - \underline{\mathbf{C}}_p\hat{\mathbf{H}}_{p(i)}^{LMMSE}||^2\right\} \\
&= \frac{1}{M}E\left\{||\mathbf{H}_p + \mathbf{W}_p - \hat{\mathbf{H}}_{p(i)}^{LMMSE}||^2\right\} \\
\hat{\sigma}^2_{(i)} &= \frac{1}{M}E\left\{||\hat{\mathbf{H}}_p^{LS} - \hat{\mathbf{H}}_{p(i)}^{LMMSE}||^2\right\}. \quad (12)
\end{aligned}$$

This algorithm finally necessitates a non-null value of $\hat{\sigma}^2_{(0)}$ as initialization step in order to be completely described. Indeed, if we choose $\hat{\sigma}^2_{(0)} = 0$, the channel estimation in (11) is equivalent to the LS one. Consequently, applying this result in (12), we then get a noise variance estimation equal to zero and the algorithm enters an endless loop. These both expressions (11) and (12) are obtained for pilots in the preamble equal to 1. In a different case, the pilot matrix $\underline{\mathbf{C}}_p$ (respectively the pilot total energy) has to be taken into account in (11) (respectively in (12)). Note also that for practical issues, (12) is replaced by

$$\hat{\sigma}^2_{(i)} = \frac{1}{M}\sum_{m=0}^{M-1}|\hat{H}_{m,p}^{LS} - \hat{H}_{m,p(i)}^{LMMSE}|^2. \quad (13)$$

Then if i_0 is the final algorithm iteration, from (7), the SNR is estimated from the noise variance $\hat{\sigma}^2_{(i_0)}$ by the expression:

$$\hat{\rho} = \frac{\hat{M}_2}{\hat{\sigma}^2_{(i_0)}} - 1. \quad (14)$$

In practice, the covariance matrix $\underline{\mathbf{R}}_H$ in (11) is unknown. However, if $\gamma(\tau)$ is the power intensity profile of the channel and if we use the notations of (2), the covariance matrix may be approximated by the matrix $\underline{\breve{\mathbf{R}}}_H$, whose elements $(\underline{\breve{\mathbf{R}}}_H)_{u,v}$ are derived by the expression given in [19]:

$$(\underline{\breve{\mathbf{R}}}_H)_{u,v} = \sum_{l=0}^{L-1}\int_0^{\tau_{max}}\gamma(\tau_l)e^{-2j\pi\frac{(u-v)}{M}\tau_l}d\tau_l. \quad (15)$$

Consequently, our proposed algorithm can be described as follows:

1) Calculate the matrix $\underline{\breve{\mathbf{R}}}_H$ from (15) or consider $\underline{\mathbf{R}}_H$ if the channel is perfectly known.
2) Initialize the noise variance so that $\hat{\sigma}^2_{(0)} > 0$.
3) For $i \geq 1$, performe a LMMSE estimation of the channel by using (11).
4) For $i \geq 1$, performe the noise variance estimation $\hat{\sigma}^2_{(i)}$ from (12).
5) Back to step 3 with $i \leftarrow i+1$ or go to step 5.
6) Estimate the SNR $\hat{\rho}$ by (14) from the final noise variance estimation $\hat{\sigma}^2_{(i_0)}$.
7) end of the algorithm.

In the case of an unknown channel, where neither $\underline{\breve{\mathbf{R}}}_H$ nor $\underline{\mathbf{R}}_H$ can be computed, the covariance matrix is estimated by $\underline{\tilde{\mathbf{R}}}_H = \hat{\mathbf{H}}_p^{LS}(\hat{\mathbf{H}}_p^{LS})^H$. This matrix must be then regularly updated according to the channel fluctuations. An adaptation of our algorithm in this context is currently under development.

The similarity of the proposed method with EM algorithm comes from its iterative character. However, our method is MMSE-based, and not ML-based. Furthermore, the iterations of our technic are only made up of two steps of estimation (11) and (13), without step of maximisation. Lastly, EM is used when the observed data vector dimension is lower than the wanted estimated vector dimension [13], [14]. This is not the case here, the estimation being performed on a complete one-symbol pilot preamble.

IV. Convergence of the Algorithm

In this section, it is shown that our proposed algorithm converges for both noise variance $\hat{\sigma}^2_{(i)}$ and channel $\hat{\mathbf{H}}^{LMMSE}_{n(i)}$ estimations. From (11), it is easy to prove that $\hat{\mathbf{H}}^{LMMSE}_{n(i)}$ converges to a given channel estimation if the sequence $(\hat{\sigma}^2_{(i)})$ admits a limit for an infinite number of iterations. Then, after the derivation of a recursive expression of $\hat{\sigma}^2_{(i)}$, the proof of the convergence of this sequence is given.

A. Scalar Expression of the Iterative Noise Variance Estimation

In the following, the different mathematical formulations are based on the covariance matrix $\underline{\mathbf{R}}_H$. However, as $\breve{\underline{\mathbf{R}}}_H$ and $\tilde{\underline{\mathbf{R}}}_H$ have the same properties than $\underline{\mathbf{R}}_H$, these formulations remain valid if one of these matrices is considered. Consequently, from (12), we get

$$\begin{aligned}
\hat{\sigma}^2_{(i+1)} &= \frac{1}{M}E\left\{||\hat{\mathbf{H}}^{LS}_p - \hat{\mathbf{H}}^{LMMSE}_{p(i+1)}||^2\right\} \\
&= \frac{1}{M}E\left\{||\hat{\mathbf{H}}^{LS}_p - \underline{\mathbf{R}}_H(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1}\hat{\mathbf{H}}^{LS}_p||^2\right\} \\
&= \frac{1}{M}E\left\{||(\hat{\sigma}^2_{(i)}I(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1})\hat{\mathbf{H}}^{LS}_p||^2\right\} \\
&= \frac{1}{M}tr\left(E\left\{(\hat{\sigma}^2_{(i)}(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1}\hat{\mathbf{H}}^{LS}_p)\right.\right. \\
&\quad \left.\left.\times(\hat{\sigma}^2_{(i)}I(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1}\hat{\mathbf{H}}^{LS}_p)^H\right\}\right). \qquad (16)
\end{aligned}$$

To extend that $(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i-1)}I)$ is an Hermitian invertible matrix, $(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i-1)}I)^{-1}$ is also an Hermitian matrix, we obtain

$$\begin{aligned}
\hat{\sigma}^2_{(i+1)} &= \frac{1}{M}tr\left((\hat{\sigma}^2_{(i)}(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1})\right. \\
&\quad \left.\times E\left\{(\hat{\mathbf{H}}^{LS}_p(\hat{\mathbf{H}}^{LS}_p)^H)\right\}(\hat{\sigma}^2_{(i)}I(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1})\right) \\
&= \frac{1}{M}tr\left(\hat{\sigma}^4_{(i)}(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1}(\underline{\mathbf{R}}_H + \sigma^2 I)\right. \\
&\quad \left.\times(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)^{-1}\right). \qquad (17)
\end{aligned}$$

We note $\underline{\mathbf{D}}_{H(i)}$ (respectively, $\underline{\mathbf{D}}_H$) the diagonal matrix obtained by the diagonalization of $(\underline{\mathbf{R}}_H + \hat{\sigma}^2_{(i)}I)$ (respectively, $(\underline{\mathbf{R}}_H + \sigma^2 I)$). The diagonal elements of $\underline{\mathbf{D}}_{H(i)}$ (respectively, $\underline{\mathbf{D}}_H$) are equal to $\lambda_m + \hat{\sigma}^2_{(i)}$ (respectively, $\lambda_m + \sigma^2$), where $\lambda_m, m = 0, 1, ..., M-1$ are the eigenvalues of $\underline{\mathbf{R}}_H$. We also note $\underline{\mathbf{Q}}$ as the orthonormal transformation matrix of $\underline{\mathbf{R}}_H$. Consequently, we can rewrite (17) as follows:

$$\hat{\sigma}^2_{(i+1)} = \frac{1}{M}tr\left(\hat{\sigma}^4_{(i)}\underline{\mathbf{Q}}(\underline{\mathbf{D}}_{H(i)})^{-1}(\underline{\mathbf{D}}_H)(\underline{\mathbf{D}}_{H(i)})^{-1}\underline{\mathbf{Q}}^{-1}\right). \qquad (18)$$

From (18), we obtain a recursive formulation of $\hat{\sigma}^2_{(i+1)}$:

$$\hat{\sigma}^2_{(i+1)} = \frac{\hat{\sigma}^4_{(i)}}{M}\sum_{m=0}^{M-1}\frac{\lambda_m + \sigma^2}{(\lambda_m + \hat{\sigma}^2_{(i)})^2}. \qquad (19)$$

B. Proof of Convergence

As the length of the channel impulse response is equal to L, the eigenvalues λ_m for $m = L, L+1, ..., M-1$ are equal to zero, and the eigenvalues λ_m for $m = 0, 1, ..., L-1$ are positive. If we note λ_{max} the largest eigenvalue and $\lambda_{min} = 0$ the lowest one, we express the following upper and lower bounds for $\hat{\sigma}^2_{(i+1)}$:

$$\frac{\hat{\sigma}^4_{(i)}}{M}\sum_{m=0}^{M-1}\frac{\lambda_m + \sigma^2}{(\lambda_{max} + \hat{\sigma}^2_{(i)})^2} \leq \hat{\sigma}^2_{(i+1)} \qquad (20)$$

$$\hat{\sigma}^2_{(i+1)} \leq \frac{\hat{\sigma}^4_{(i)}}{M}\sum_{m=0}^{M-1}\frac{\lambda_m + \sigma^2}{(\lambda_{min} + \hat{\sigma}^2_{(i)})^2}. \qquad (21)$$

As $M_2 = \frac{1}{M}\sum_{m=0}^{M-1}(\lambda_m + \sigma^2)$, we have :

$$\frac{\hat{\sigma}^4_{(i)}}{(\lambda_{max} + \hat{\sigma}^2_{(i)})^2}M_2 \leq \hat{\sigma}^2_{(i+1)} \leq M_2. \qquad (22)$$

We can write $\hat{\sigma}^2_{(i+1)} = f(\hat{\sigma}^2_{(i)})$, where the function f is defined by :

$$f(x) = \frac{x^2}{M}\sum_{m=0}^{M-1}\frac{\lambda_m + \sigma^2}{(\lambda_m + x)^2}. \qquad (23)$$

As $\hat{\sigma}^2_{(i)} > 0$, the variable x is necessarily positive. From (22), we obtain:

$$\forall x \geq 0, \frac{x^2}{(\lambda_{max} + x)^2}M_2 \leq f(x) \leq M_2. \qquad (24)$$

The proof of the convergence of the sequence $(\hat{\sigma}^2_{(i)})$ is based on the fixed-point theorem. Indeed, if f possesses at least one fixed point on an closed interval $[a, b]$ and if the sequence $(\hat{\sigma}^2_{(i)})$ is bounded and monotonous, $(\hat{\sigma}^2_{(i)})$ necessarily converges to one of the fixed points of f. In order to prove that f has at least one fixed point on a closed interval $[a, b]$ (i.e. that the equation $f(x) = x$ has at least one solution in $[a, b]$), we first prove that $f([a, b]) \subset [a, b]$, with a and b correctly chosen. Indeed, from (24), we show that

$$f(x) \xrightarrow[x \to +\infty]{} M_2. \qquad (25)$$

Due to the fact that $\lambda_{min} = 0$, f is not defined for $x = 0$. However:

$$f(\epsilon) \xrightarrow[\epsilon \to 0, \epsilon > 0]{} \frac{M-L}{M}\sigma^2 > 0. \qquad (26)$$

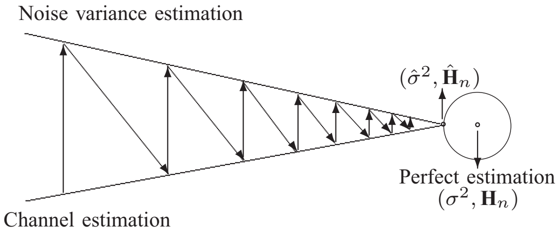

Fig. 2. Illustration of the convergence of the algorithm.

As a consequence, (26) ensures the existence of an $\epsilon \in]\,0, \frac{M-L}{M}\sigma^2\,]$ verifying $f(\epsilon) \geq \epsilon$. In addition, as

$$\forall x > 0, f'(x) = \frac{2}{M}\sum_{m=0}^{M-1}\frac{\lambda_m(\lambda_m+\sigma^2)x}{(\lambda_m+x)^3} > 0, \qquad (27)$$

we prove that f is a strictly growing function.

From (25) and (26), we easily obtain that $f([\,\epsilon, +\infty\,[) \subset [\epsilon, M_2]$. In addition, as f is a strictly growing function and upper-bounder by M_2, we get the following inclusion:

$$f([\epsilon, M_2]) \subset [\epsilon, M_2], \qquad (28)$$

proving then that f has at least one fixed point on $[\epsilon, M_2]$.

As it has been previously shown that f is stricty growing on the interval $[\,\epsilon, +\infty\,[$, equivalently on $[\epsilon, M_2]$, the sequence $(\hat{\sigma}^2_{(i)})$ is consequently monotonous. From (25) and (26), the sequence $(\hat{\sigma}^2_{(i)})$ is also lower bounded by ϵ and upper bounded by M_2. Finally, from the fixed-point theorem, $(\hat{\sigma}^2_{(i)})$ converges to one of the fixed point of f.

These mathematical formulations theoretically prove that our proposed algorithm converges, what is confirmed by simulations in the following section. Fig. 2 depicts the way our algorithm works. The noise variance and channel estimations alternatively feed each other until the algorithm reaches its limit, characterized by the couple $(\hat{\sigma}^2, \hat{\mathbf{H}}_n)$. This couple is normally different from the couple $(\sigma^2, \mathbf{H}_n)$ leading to the perfect estimation. Section V-A then depicts the performance of our algorithm and finally shows that our estimated couple $(\hat{\sigma}^2, \hat{\mathbf{H}}_n)$ is close to the perfect estimation couple $(\sigma^2, \mathbf{H}_n)$.

V. Simulation results

Simulations are based on the Digital Radio Mondiale (DRM) standard [4] designed as the digital audio broadcasting over the currently AM frequency bands. The OFDM modulation considered uses 201 sub-carriers for a sampling frequency equal to 12 kHz. The added cyclic prefix (CP) of time duration $T_{CP} = 2.66$ ms is supposed to be longer than the maximum channel delay. Although DRM standard recommends a scattered pilot repartition, we consider for the need of our method a pilot-preamble scheme. Each preamble is then composed of only one pilot symbol. The channel model considered is the $US\ Consortium$, also taken from the DRM standard. This four-paths channel has a maximum delay $\tau_{max} = 2.2$ ms and a maximum Doppler frequency $f_D = 2$ Hz. All paths are mutually independent and follow a Gaussian power density spectrum.

A. Convergence of noise variance estimation

Fig. 3 and 4 show the evolutions of two estimated noise variances compared to the real noise variance as a function of the number of iterations. Case 1 tallies with the channel estimation made with the real covariance matrix $\underline{\mathbf{R}}_H$ and Case 2 tallies with the channel estimation made with the approximated covariance matrix $\underline{\breve{\mathbf{R}}}_H$. Fig. 3 shows the evolution of the algorithm for the initialization $\hat{\sigma}^2_{(0)} = 0.1$ and Fig. 4 for $\hat{\sigma}^2_{(0)} = 2$. Furthermore, for both Fig. 3 and 4, two fixed values of the SNR ρ are considered : $\rho = 0$ dB (for high values of σ^2), and $\rho = 10$ dB (for the lower ones). All the curves are obtained with 4000 simulation runs.

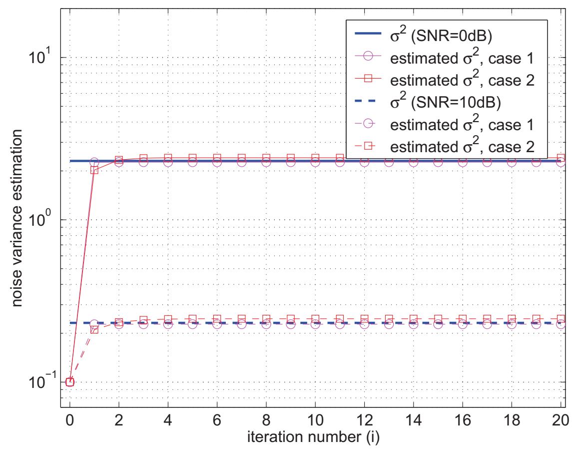

Fig. 3. Convergence of the algorithm for $\hat{\sigma}^2_{(0)} = 0.1$.

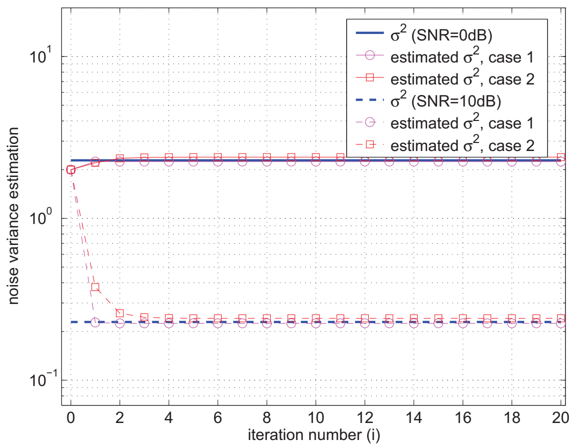

Fig. 4. Convergence of the algorithm for $\hat{\sigma}^2_{(0)} = 2$.

Figs. 3 and 4 display the convergence of the algorithm, for low and high values of the initializations $\hat{\sigma}^2_{(0)}$ and for two values of SNR (0 dB and 10 dB). Although the development of Section IV is not done with the matrix $\underline{\breve{\mathbf{R}}}_H$, we also observe the convergence of the algorithm in Case 2. Whatever the value of $\hat{\sigma}^2_{(0)}$, we verify that the sequence $(\hat{\sigma}^2_{(i)})$ is monotonous and converges to a given limit, as shown in Section IV. Furthermore, for both cases 1 and 2, the convergence is fast : the noise variance estimation $\hat{\sigma}^2_{(i)}$ is constant for $i \geq 1$ for Case 1 and for $i \geq 3$ for Case 2. Fig. 3 and 4 also allow to characterize the bias of the noise variance estimation method. For both cases 1 and 2, after a few iterations, we observe a very low bias. Indeed, if we note $\beta = |(\hat{\sigma}^2_{(i)}) - \sigma^2|/\sigma^2$ the normalized error, given in percentage, we have, in Case 1 (real covariance matrix) $\beta = 1.67$ % for $\rho = 0$ dB and $\beta = 4.57$ % for $\rho = 10$ dB. In Case 2 (approximated covariance matrix), we get $\beta = 1.96$ % for $\rho = 0$ dB and $\beta = 5.43$ % for $\rho = 10$ dB.

B. Comparison of SNR estimation with other methods

Fig. 5 shows the curves of the Normalized Mean Square Error (NMSE) of SNR estimations in the case of the frequency selective channel $US\ Consortium$. We remind that estimated SNR is $\hat{\rho} = \frac{\hat{M}_2}{\hat{\sigma}^2} - 1$ (see section II-C). The NMSE of the SNR estimation $\hat{\rho}$ is defined by $NMSE = E\left[|\hat{\rho} - \rho|^2/\rho^2\right]$. In our simulations, the expectation is performed on 200000 samples. The initialization is $\hat{\sigma}^2_{(0)} = 0.1$ and the number of iterations is $i = 3$. We compare the proposed algorithm to two existing methods using a preamble-based SNR estimation in a frequency selective channel supposed invariant between two consecutive preambles.

Ren's method, depicted in [17], uses a 2 pilot-symbols preamble which allows to remove the effect of the channel in order to estimate the noise variance only. The SNR is then estimated thanks to the second moment M_2, as done in (7). Xu's method, depicted in [18], uses a single pilot-symbol preamble to estimate the covariance matrix. A diagonalization of this matrix allows to estimate the noise variance in one hand and the second moment in the other hand. Unlike our technique, none of these two methods performs a joint channel and SNR estimation. It makes these techniques of SNR estimation less complex to apply than our algorithm, but our method makes the joint estimation of SNR and channel directly usable for the equalization. To perform equalization, Ren and Xu's methods require then a separated channel estimation.

Fig. 5 compares the efficiency of our algorithm in terms of NMSE to Ren and Xu's ones [17], [18]. The shape of the curves of Ren's and Xu's estimation methods matches with those of the simulations in [17]. Our theoretical case (Case 1) has a lower NMSE than Ren's method (which is better than Xu's method), while Ren's technique requires two pilot symbols (our technique requires only one pilot symbol). In Case 2, Ren's method has a lower NMSE than ours, due to the approximation of the covariance matrix. However, our algorithm has a globally lower NMSE than Xu's method, except for the SNR values between 3 and 7 dB. Xu's method and ours require only 1 pilot symbol, so the proposed algorithm is globally more efficient for the same useful bit rate. Furthermore, our method performs a channel estimation, whose efficiency is studied in next sub-section V-C.

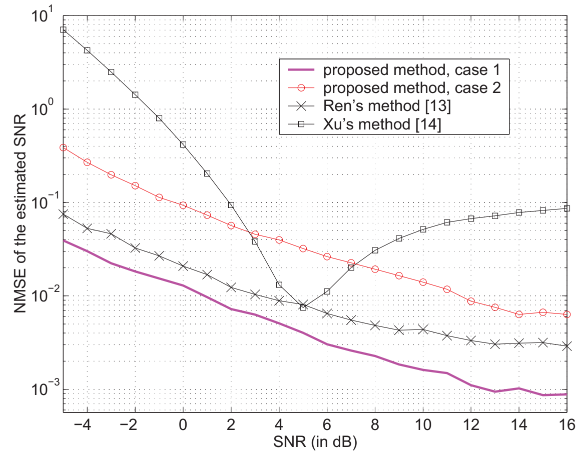

Fig. 5. NMSE of the SNR estimation of 3 methods.

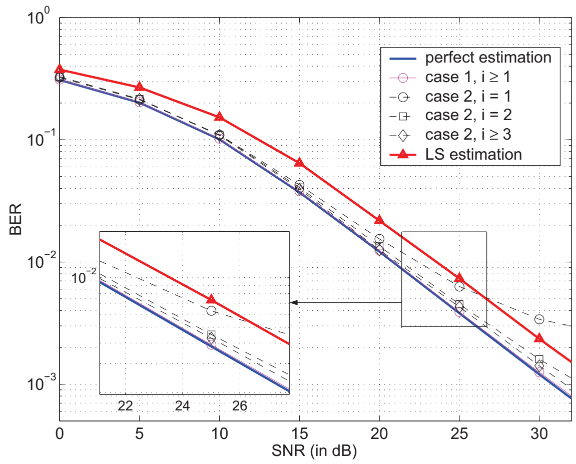

Fig. 6. BER of the iterative channel estimation.

C. Channel estimation

Fig. 6 shows the Bit Error Rate (BER) as a function of SNR of the proposed algorithm in both cases 1 and 2. The convergence of the channel estimation is proved by simulation. We use the frequency selective channel $US\ Consortium$. The initialization is $\hat{\sigma}^2_{(0)} = 0.1$. The BER is computed on 2.5×10^6 bits. We simulate frames of 22 OFDM symbols, including 1 pilot symbol. The channel is then considered as invariant over 22 consecutives symbols. The useful-data symbols are composed of samples mapped with a 16-QAM. For both cases 1 and 2, we study the efficiency of the channel estimation as a function of the number of iteration. The method is compared with the perfect estimation and the regular LS estimation.

The channel estimation made in Case 1 reaches its limit from the first iteration, what tallies with the convergence speed shown in Fig. 3 and 4. The error compared to the perfect estimation is less than 0.1 dB for $SNR = 25$ dB. This estimation matched with the theoretical LMMSE estimation using the real covariance matrix $\mathbf{\underline{R}}_H$ and the real noise variance σ^2. This theoretical LMMSE estimation is not displayed for a better visibility. In the same way for Case 2, the channel estimation reaches its limit from $i = 3$, what tallies with the number of iterations necessary for $(\hat{\sigma}^2_{(i)})$ reaches its limit (see Fig. 3 and 4). The error compared to the perfect estimation is less than 0.5 dB for $SNR = 25$ dB. Furthermore, the proposed iterative method is more efficient than the regular LS estimation. Indeed, for $SNR = 25$ dB, the error of the LS estimation compared to the proposed method in Case 2 is equal to 2.5 dB.

VI. Conclusion

This article presents an iterative and joint method for SNR and channel estimation. The SNR is estimated combining the second moment of the received signal and the noise variance estimation made thanks to a pilot-preamble scheme. We use the MSE criterion to estimate the noise variance. To do so, we require an efficient channel estimation, thus we use an LMMSE channel estimation, which requires itself the noise variance estimation. We then propose an iterative estimation of both noise variance and transmission channel. We prove that the algorithm converges whatever the non-null initialization, and the simulations show the low bias of the noise variance estimation. In comparison to two existing SNR estimation methods, our method improves the rate between the number of pilots in the preamble and the efficiency of the SNR estimation, as far as our method requires only one pilot. Furthermore, this pilot is used to perform in the same time the channel estimation. The simulations show that the channel estimation is close to the perfect estimation by less than 0.5 dB. The subjects of our future work is the theoretical proof of the unicity of the convergence of the algorithm and the extension of the algorithm for the case of the estimated channel covariance matrix $\tilde{\underline{\mathbf{R}}}_H$.

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