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# PREDICTIVE CONTROL FOR TRAJECTORY TRACKING AND DECENTRALIZED NAVIGATION OF MULTI-AGENT FORMATIONS

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Abstract: This paper addresses a predictive control strategy for multi-agent formations with a time-varying topology. The goal is to guarantee a trajectory tracking, where a reference trajectory is specified for an agent designed as the leader. Then, a predictive control strategy combined with the Potential Field method is used in order to derive a control action based only on local information within the group of agents. The main concern is that the interconnections between the agents are time-varying, affecting the neighborhood around each agent. The proposed method exhibits effective performance validated through some illustrative examples.

## 1 INTRODUCTION

Control and coordination of multi-agent systems, such as pedestrians in the crowd, vehicles, spacecraft and unmanned vehicles, are emerging as a challenging field of research. There exist several classes of multi-agent systems where the interconnections between the agents could be time-varying (e.g. traffic control, pedestrian behavior in the crowd etc.). Guaranteeing stability with the existing cooperative control techniques is still an open problem for multi-agent systems with time-varying (constrained) topologies. This paper addresses a new methodology based on predictive control in order to answer to some of these difficulties; an illustrative example proves the interest of the proposed methodology.

Collision avoidance can be difficult in the context of managing multiple agents, since certain (static or dynamic) constraints are non-convex. A common point of most publications in the collision avoidance problem is devoted to the case of *punctiform* agents, which is far from real world applications. In many of them the relative positioning between agents becomes important, such as the NASA's mission to construct a large interferometer from multiple telescopes (Schneider, 2009). Also, in air traffic management, two aircraft are not allowed to approach each other closer than a specific alert distance.

A class of methods for collision avoidance prob-

lems uses artificial potential fields to directly obtain feedback control actions steering the agents over the entire workspace. There is a large literature dedicated to the formation control for collections of vehicles using the potential field approach. The authors of (Jadbabaie et al., 2003) and (Tanner et al., 2007) investigate the motions of vehicles modeled as double integrators. Their objective is that the vehicles achieve a common velocity while avoiding collisions with obstacles and/or agents assumed to be points.

The aim of the present paper is twofold: first, to provide a framework for non point-like shapes which may define obstacles and/or safety regions around an agent; second, to offer a novel control strategy derived from a combination of constrained receding horizon and potential field techniques for the trajectory tracking problem, applied to multi-agent systems with time-varying topologies.

This paper is organized as follows. Section 2 presents two constructions that take into account the shape of a convex region defining an obstacle and/or a safety region around an agent. Section 3 presents the trajectory tracking problem for a leader/followers formation. A flat trajectory is generated for the leader and using predictive control the tracking error is minimized. For the followers, a potential function is embedded within MPC in order to achieve the group formation with a collision free behavior. Further on, Section 4 presents illustrative simulation results. And fi-

nally, several concluding remarks are drawn in Section 5.

The following notations will be used throughout the paper. Given a vector  $v \in \mathbb{R}^n$ ,  $\|v\|_\infty := \max_{i=1, \dots, n} |v_i|$  denotes the infinity norm of  $v$ . Minkowski's addition of two sets  $\mathcal{X}$  and  $\mathcal{Y}$  is defined as  $\mathcal{X} \oplus \mathcal{Y} = \{A + B : A \in \mathcal{X}, B \in \mathcal{Y}\}$ . The interior of a set  $S$ ,  $\text{Int}(S)$  is the set of all interior points of  $S$ . Denote as  $\mathbb{B}_p^n = \{x \in \mathbb{R}^n : \|x\|_p \leq 1\}$  the unit ball of norm  $p$ , where  $\|x\|_p$  is the  $p$ -norm of vector  $x$ . Let  $x_{k+1|k}$  denote the value of  $x$  at time instant  $k+1$ , predicted upon the information available at time  $k \in \mathbb{N}$ .

## 2 PREREQUISITES

For safety and obstacle avoidance problems the feasible region in the space of solutions is a non-convex set. Usually this region is considered as the complement of a (union of) convex region(s) which describes an obstacle and/or a safety region.

Let us define a bounded convex set in its polyhedral approximation, a polytope  $S \subset \mathbb{R}^n$  through the implicit half-space description:

$$S = \{x \in \mathbb{R}^n : h_a x \leq k_a, \quad a = 1, \dots, n_h\}, \quad (1)$$

with  $h_a \in \mathbb{R}^{1 \times n}$ ,  $k_a \in \mathbb{R}$  and  $n_h$  the number of half-spaces. We focus on the case where  $k_a > 0$ , meaning that the origin is contained in the strict interior of the polytopical region, i.e.  $0 \in \text{Int}(S)$ .

In the following, we are interested in measuring the relative position of an agent to such a region. In other words, we require a function which measures if and when a given state is inside or outside the polyhedral set (1). The forthcoming constructions will be used in a *repulsive potential function* to take into account the shape of the convex region in terms of (1).

### 2.1 Polyhedral function

Consider the class of (symmetrical) piecewise linear functionals defined using the specific shape of a polyhedral set. The following definitions will be instrumental for the rest of the paper.

**Definition 1** (Minkowski function – (Blanchini, 1995)). *Any bounded convex set  $S$  induces a Minkowski function defined as*

$$\mu(x) = \inf \{\alpha \in \mathbb{R}, \alpha \geq 0 : x \in \alpha S\} \quad (2)$$

**Definition 2** (Polyhedral function – (Blanchini, 1995)). *A polyhedral function is the Minkowski function of the polyhedral bounded convex set  $S$  defined in (1). This function has the following expression:*

$$\mu(x) = \|Fx\|_\infty, \quad (3)$$

where  $F \in \mathbb{R}^{n_h \times n}$  is a full column matrix with  $F_a = \frac{h_a}{k_a}$ ,  $a = 1, \dots, n_h$ .

In fact, any polytope can be defined in terms of the Minkowski function (2). Indeed there always exists a full column matrix  $F \in \mathbb{R}^{n_h \times n}$  such that the polytope  $S$  in (1) is equivalently defined as

$$S = \{x \in \mathbb{R}^n : \mu(x) \leq 1\}, \quad (4)$$

with  $\mu(x)$  defined by (3). From the avoidance point of view, the Minkowski function (2) denotes the inclusion of a value  $x$  to the given polytope (4) if  $\mu(x) \in [0, 1]$ . Conversely, if  $\mu(x) > 1$ , then  $x$  is outside the polytope (4).

**Remark 1.** *Note that if  $k_a < 0$  in (1), the origin is not contained in the strict interior of the polytopical region, i.e.  $0 \notin \text{Int}(S)$ ; then the polyhedral function can be brought to the form (3) by imposing*

$$F_a = \frac{h_a(x - x_s)}{k_a - h_a x_s}, \quad a = 1, \dots, n_h, \quad (5)$$

with  $x_s \in \mathbb{R}^n$  the analytic center of the polytope (1).

Note that, the polyhedral function (3) is piecewise affine and continuous. This means that each of the inequalities which compose its definition can provide the maximum, an explicit description of these regions being

$$X_a = \left\{ x \in \mathbb{R}^n : \frac{h_a}{k_a} x > \frac{h_b}{k_b} x, \forall a \neq b, \quad a, b = 1, \dots, n_h \right\}. \quad (6)$$

The entire space can thus be partitioned in a union of disjoint regions  $X_a$  which are representing in fact cones with a common point in the origin (respectively in  $x_s$  in the general case evoked in Remark 1).

Practically, the polyhedral function (2) can be represented in the form

$$\mu(x) = F_a x, \quad \forall x \in X_a, \quad a = 1, \dots, n_h \quad (7)$$

and the piecewise affine gradient is defined as:

$$\nabla \mu(x) = F_a, \quad \forall x \in X_a, \quad a = 1, \dots, n_h. \quad (8)$$

**Remark 2.** *Strictly speaking the gradient (8) is multivalued (the Minkowski function induced by a polytope is not differentiable in the classical sense, rather it is differentiable almost everywhere). However, an univocal candidate can be selected for the forthcoming computations, an alternative is to work with the multivalued expression of the gradient.*

### 2.2 Exemplification for the construction of repulsive potential function

In this subsection the previous theoretical tools will be integrated in order to describe a piecewise affine

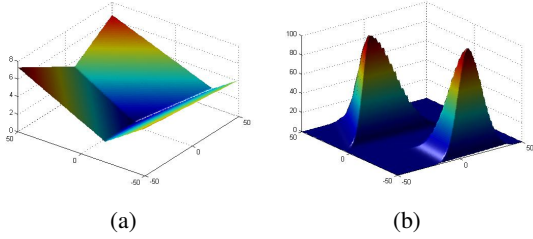


Figure 1: (a) The polyhedral function (7). (b) The repulsive potential using the polyhedral function (7).

function which measure the position of a state with respect to the frontier of a polyhedral set defined in (1). The derived potential function takes into account the shape of the convex region which will define a safety region for an agent and/or an obstacle. For a given convex region, Figure 1.a illustrates the polyhedral function according to (7).

Further, for the control design purpose, the construction based on the polyhedral function defined in (7) is proposed for the generation of a repulsive potential:

$$V_\mu(\mu(x)) = c_1 e^{-(\mu(x)-c_2)^2}, \quad (9)$$

where the parameters  $c_1$  and  $c_2$  are positive constants representing the strength and effect ranges of the repulsive potential.

Figure 1.b illustrates the proposed function (9) for two polyhedral obstacles. As it can be seen, the function has a high value inside the polytopes and a low value outside them. The repulsive potential will be further used in order to derive a control action such that the collision avoidance inside the formation is satisfied.

### 3 TRAJECTORY TRACKING FOR A LEADER/FOLLOWERS FORMATION

This section presents the formation trajectory tracking problem. The agents are required to follow a pre-specified trajectory while preserving a tight inter-agent formation in time. Each agent has an associated polyhedral safety region as defined in (1). Using a leader/followers approach, we generate a flat trajectory for the leader and formulate a receding horizon optimization problem in order to minimize the tracking error. For the followers, we propose a gradient method combined with a receding horizon approach which aims to follow the group leader and respects the collision avoidance formation specifications.

A set of  $N_a$  linear systems will be used to model the behavior of individual heterogeneous agents. The  $i^{th}$  system is described by the following continuous time dynamics:

$$\dot{x}^i(t) = A_{c,i}x^i(t) + B_{c,i}u^i(t), \quad i = 1, \dots, N_a, \quad (10)$$

where  $x^i(t) \in \mathbb{R}^n$  are the state variables and  $u^i(t) \in \mathbb{R}^m$  is the control input vector for the  $i^{th}$  agent. The components of the state are: the position  $p^i(t)$  and the velocity  $v^i(t)$  of the  $i^{th}$  agent such that  $x^i(t) = [p^i(t) \ v^i(t)]^T$ .

#### 3.1 Trajectory generation

The idea is to find a trajectory  $(x^l(t), u^l(t))$  that steers the model of the leader (10) with  $i = l$  from an initial state  $x_0$  to a final state  $x_f$ , over a fixed time interval  $[t_0, t_f]$ . Using the flatness theory (Fliess et al., 1995), the system is parameterized in terms of a finite set of variables  $z^l(t)$  and a finite number of their derivatives:

$$\begin{aligned} x^l(t) &= \xi(z^l(t), \dot{z}^l(t), \dots, z^{l,(q)}(t)), \\ u^l(t) &= \eta(z^l(t), \dot{z}^l(t), \dots, z^{l,(q)}(t)), \end{aligned} \quad (11)$$

where  $z^l(t) = \Upsilon(x^l(t), u^l(t), \dot{u}^l(t), \dots, u^{l,(q)}(t))$  is called the flat output<sup>1</sup>. The generation of a reference trajectory will be based on the class of polynomial functions. Using the parametrization (11) and imposing boundary constraints for the evolution of the differentially flat systems (De Doná et al., 2009) a reference trajectory  $z_{ref}^l(t)$  can be generated by solving a linear system of equalities. Therefore, the corresponding reference state and input for the system (10), with  $i = l$  are obtained by replacing the reference flat output  $z_{ref}^l(t)$ , with  $t \in [t_0, t_f]$  in (11):

$$\begin{aligned} x_{ref}^l(t) &= \xi(z_{ref}^l(t), \dot{z}_{ref}^l(t), \dots, z_{ref}^{l,(q)}(t)), \\ u_{ref}^l(t) &= \eta(\dot{z}_{ref}^l(t), \ddot{z}_{ref}^l(t), \dots, z_{ref}^{l,(q)}(t)), \end{aligned} \quad (12)$$

where  $t \in [t_0, t_f]$ .

In the rest of the paper we use the discrete correspondent of the reference signals in (12). Therefore, a corresponding discrete-time model for the equations (10) is constructed upon a chosen sampling period  $T_s$  by considering the time instants  $t_k = kT_s$ :

$$x^i(k+1) = A_i x^i(k) + B_i u^i(k), \quad k \in \mathbb{N}, \quad i = 1 : N_a, \quad (13)$$

where  $x^i(0)$  corresponds to the boundary condition in (12) and  $u^i(k) = u^i(t_k)$ . The pairs  $(A_i, B_i)$  are given by:

$$A_i = e^{A_{c,i}T_s}, \quad B_i = \int_0^{T_s} e^{A_{c,i}(T_s-\theta)} B_{c,i} d\theta.$$

<sup>1</sup>Hereafter we assume that the characteristics necessary for flat trajectory (controllability and existence of a flat output) are respected for the leader.

Considering the discrete-time model of the leader (13) with  $i = l$ , we compare the measured state and input variables with the reference trajectory  $(x_{ref}^l(k), u_{ref}^l(k))$  which satisfies the nominal dynamics:

$$x_{ref}^l(k+1) = A_l x_{ref}^l(k) + B_l u_{ref}^l(k). \quad (14)$$

Further on, the tracking error between the leader's state (13) and the state reference (14) becomes:

$$\tilde{x}^l(k+1) = A_l \tilde{x}^l(k) + B_l \tilde{u}^l(k), \quad (15)$$

with  $\tilde{u}^l(k) = u^l(k) - u_{ref}^l(k)$ ,  $\tilde{x}^l(k) = x^l(k) - x_{ref}^l(k)$ .

Since the reference trajectory is available beforehand, an optimization problem which minimizes the tracking error for the leader can be formulated in a predictive control framework (Maciejowski, 2002).

### 3.2 Predictive control for the leader

In what follows we present the predictive control problem, where an optimization is performed to compute the control law for the leader. The discrete model of the leader (i.e.  $i = l$  in (13)) is used in a predictive control context which permits the minimization of the tracking error.

A finite receding horizon implementation of the optimal control law is typically based on the real-time construction of a control sequence  $\tilde{u}^l = \{\tilde{u}^l(k|k), \tilde{u}^l(k+1|k), \dots, \tilde{u}^l(k+N_l-1|k)\}$  that minimizes the finite horizon quadratic objective function:

$$\begin{aligned} \tilde{u}^l* = \arg \min_{\tilde{u}^l} & (\|\tilde{x}^l(k+N_l|k)\|_P + \\ & + \sum_{s=1}^{N_l-1} \|\tilde{x}^l(k+s|k)\|_Q + \sum_{s=0}^{N_l-1} \|\tilde{u}^l(k+s|k)\|_R), \end{aligned} \quad (16)$$

subject to:

$$\begin{cases} \tilde{x}^l(k+s+1|k) = A_l \tilde{x}^l(k+s|k) + B_l \tilde{u}^l(k+s|k), \\ \tilde{x}^l(k+s|k) \in X_l, \quad s = 1, \dots, N_l, \\ \tilde{u}^l(k+s|k) \in U_l, \quad s = 1, \dots, N_l, \end{cases} \quad (17)$$

Here  $Q = Q^T \succeq 0$ ,  $R \succ 0$  are positive definite weighting matrices,  $P = P^T \succeq 0$  defines the terminal cost and  $N_l$  denotes the prediction horizon for the leader. The optimization problem (16) has to be solved subject to the dynamic constraints (17). In the same time, other security or performance specifications can be added to the system trajectory. These physical limitations (velocity, energy or forces) are stated in terms of hard constraints on the internal state variables and input control action as in (17). Note that the sets  $X_l$ ,  $U_l$  have to take into account the reference tracking type of problem delineated in (16). Thus, the absolute limitations have to be adjusted according to the reference signals.

### 3.3 Decentralized predictive control for the followers

In this subsection, we present a control strategy which is a combination of MPC and Potential Field control approach. The goal is to control the agents to achieve a formation while following the specified trajectory. The repulsive potential functions introduced in (9) produce a potential field. The negative gradient of this potential assures a collision free behavior for the agents. Globally, an attractive component of the potential function aims at maintaining a given formation. In this context, we provide a practical control design method which enables the decentralized decision making for a leader/followers group of agents. The proposed method exhibits effective trajectory tracking performances while avoiding the centralized design which can be computationally demanding.

**Corollary 1.** Consider the agents  $i$  and  $j$  with the associated safety regions  $S_i$ ,  $S_j$  as defined in (1). The agent  $i$  with the associated position  $p^i$  does not intersect agent  $j$  with the position  $p^j$  if and only if  $p^i \notin S_{ij}(p^j)^2$ , where

$$S_{ij}(p^j) \triangleq \{p^j\} \oplus S_j \oplus \{-S_i\}, \quad (18)$$

with  $i = 1, \dots, N_a$ ,  $i \neq j$ .

Let us now assume the steering policy for each follower agent (i.e.  $i \neq l$  in (13)) based only on local state information from its nearest neighbors.

**Definition 3** (Neighboring graph (Tanner et al., 2007)). An undirected graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$  represents the nearest neighboring relations and consists of:

- a set of vertices (nodes)  $\mathcal{V} = \{n_1, n_2, \dots, n_{N_a}\}$  indexed by the agents in the group;
- a set of edges  $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} : n_i \leftrightarrow n_j\}$ , containing unordered pairs of nodes that represent neighboring relations.

The set of neighbors of agent  $i$  with  $i = 1, \dots, N_a$  and  $i \neq l$  can be defined as follows:

$$\mathcal{N}_i(k) \triangleq \{j = 1, \dots, N_a : \|p^i(k) - p^j(k)\| \leq r, i \neq j\}, \quad (19)$$

where  $r$  is the radius of the ball centered in  $p^i$ . Since the agents are in motion, their relative distances can change with time, affecting their neighboring sets (19). For each agent  $i$ , we define an inter-agent potential function which aims to accomplish the collision avoidance between agents, the convergence to a group formation and following the leader. To be specific, the following inter-agent potential function is used:

$$V_i(p^i, v^i) = \beta_r V_i^r(p^i) + \beta_a V_i^a(p^i, v^i), \quad \forall i \in \mathcal{N}_i. \quad (20)$$

<sup>2</sup>This is implied by the requirement that the safety regions of the agents do not intersect.

The two components of the potential function account for the objectives presented above and  $\beta_r, \beta_a$  are weighting coefficients for each objective. For the  $i^{th}$  agent the total potential is formed by summing the potentials terms corresponding to each of its neighbors. Consequently, in our approach, the potential functions are designed as follows:

- 1)  $V_i^r(p^i)$  denotes the repulsive potentials that agent  $i$  senses from its neighbors:

$$V_i^r(p^i) = \sum_{j \in \mathcal{N}_i} V_{ij}^r(p^i) \quad (21)$$

To implement this, the concepts introduced in Subsection 2.2, specifically the potential functions (9) is taken into account:

$$V_{ij}^r(p^i) = c_1 e^{-(\mu_{ij}(p^i) - c_2)^2}, \quad i \neq j, \quad i \neq l, \quad (22)$$

where  $\mu_{ij}(p^i)$  is the polyhedral function (3) induced by the polyhedral set defined in (18). Note that the repulsive component (22) takes into account the safety regions (18) associated to both the followers and the leader.

- 2)  $V_i^a(p^i, v^i)$  denotes the attractive component between agents in order to achieve a formation and to follow the leader:

$$V_i^a(p^i, v^i) = \sum_{j \in \mathcal{N}_i} V_{ij}^a(p^i, v^i) + \|p^l - p^i\|, \quad (23)$$

for all  $i \in \mathcal{N}_i$  and  $i \neq l$ .

The second component denotes the relative distance between the leader and the followers. The first component  $V_{ij}^a(x_i)$  has the following form:

$$V_{ij}^a(p^i, v^i) = \log(\mu_{ij}^2(p^i)) + \beta_v(v^i - v^j), \quad (24)$$

where  $\beta_v$  denotes a weighting coefficient for which the agents velocities are synchronizing.

Similar with other methods from the literature, the parameters of the potential field have to be determined experimentally. It will be seen in the simulations that the collision avoidance is realized for the chosen parameters.

In the following, we reformulate the optimization problem (16) for the followers, by using the potential-based cost function described in (20). A control sequence  $\mathbf{u}^i = \{u^i(k|k), u^i(k+1|k), \dots, u^i(k+N_f-1|k)\}$  which minimizes the finite horizon nonlinear objective function:

$$\mathbf{u}^* = \arg \min_{\mathbf{u}^i} \left( \sum_{s=0}^{N_f} V_i(p^i(k+s|k), v^i(k+s|k)) \right). \quad (25)$$

Here  $N_f$  denotes the prediction horizon for the followers. In the optimization problem (25) we need to

know the future values of the neighboring graph and the values of the state for the corresponding neighbors. All these elements are time-varying and difficult to estimate. For the ease of computation we assume the following:

- The neighboring graph is considered to be constant along the prediction horizon, that is,

$$\mathcal{N}_i(k+s|k) \triangleq \mathcal{N}_i(k) \quad (26)$$

- The future values of the followers state are considered constant

$$x^j(k+s|k) \triangleq x^j(k) \quad (27)$$

- An estimation of the leader's state is provided by the equation (15)

$$x^l(k+s|k) \triangleq \hat{x}_{ref}^l(k+s) \quad (28)$$

The equations (26)–(28) represent only rough approximations of the future state of the agents. Obviously, the MPC formulation can be improved by using prediction of the future state of the neighboring agents. Where feasible, this prediction may be provided by the agents themselves (Dunbar and Murray, 2006). Here a simplified approach was implemented for the followers (by assuming constant predictions) and using the reference trajectory for the leader.

## 4 SIMULATIONS

Consider a set of  $N_a = 5$  heterogeneous agents in two spatial dimensions with the dynamics described by:

$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{v_i}{m_i} & 0 \\ 0 & 0 & 0 & -\frac{v_i}{m_i} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{m_i} \end{bmatrix} \quad (29)$$

where  $[x^i \ y^i \ v_x^i \ v_y^i]^T$ ,  $[u_x^i \ u_y^i]^T$  are the state and the input of each system. The components of the state are: the position  $(x^i, y^i)$  and the velocity  $(v_x^i, v_y^i)$  of the  $i^{th}$  agent,  $i = 1, \dots, N_a$ . The parameters  $m_i, v_i$  are the mass of the agent  $i$  and the damping factor, respectively:  $m_1 = 45\text{kg}$ ,  $m_2 = 60\text{kg}$ ,  $m_3 = 30\text{kg}$ ,  $m_4 = 50\text{kg}$ ,  $m_5 = 75\text{kg}$ ,  $v_1 = 15\text{Ns/m}$ ,  $v_2 = 20\text{Ns/m}$ ,  $v_3 = 18\text{Ns/m}$ ,  $v_4 = 35\text{Ns/m}$ ,  $v_5 = 23\text{Ns/m}$ . The initial positions and velocities of the agents are chosen randomly. For the sake of illustration, an identical polyhedral safety region as in (1) is associated to each agent. We take arbitrarily  $l = 1$  to be the leader which has to be followed by the rest of the agents  $i = 1, \dots, 4$ .

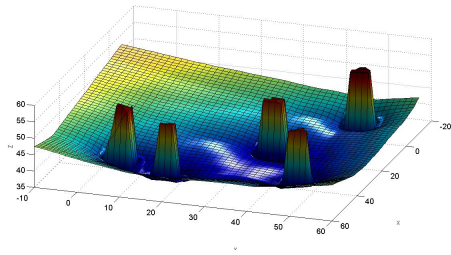


Figure 2: Potential filed in a workspace with 5 agents.

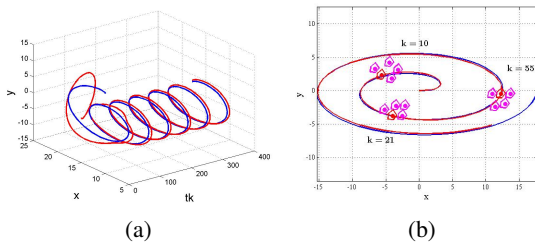


Figure 3: (a) The reference trajectory and the time evolution of the leader along the trajectory. (b) Trajectory tracking of the leader/followers formation at different time instances with their safety regions (leader in red, followers in magenta).

Figure 2 illustrates the potential filed generated for 5 agents.

For the leader we generate through flatness methods, state and input references (12) and for both types of agents we use MPC in order to construct the control action. A quadratic cost function as defined in (16) is used for the leader. Figure 3.a illustrates the reference trajectory (in blue) and the time evolution of the leader (in red) along the trajectory. Satisfactory tracking performances for the given reference trajectory are obtained with a prediction horizon  $N_l = 10$ . For the followers we consider a potential function as the cost function in the optimization problem (25), with a prediction horizon  $N_f = 2$ . The potential will be constructed such that both, the following of the leader and the maintaining of a formation are respected. The neighborhood radius is set to  $r = 8\text{m}$ , the weighting coefficients are  $\beta_r = 1$ ,  $\beta_a = 10$ ,  $c_3 = 1$ ,  $c_4 = 0.25$ ,  $\beta_v = 15$ . The effectiveness of the present algorithm is confirmed by the simulation depicted in Figure 3.b, where the evolution of the agents is represented at three different time instances. The agents successfully reach a formation and follow the leader without trespassing each other safety regions.

## 5 CONCLUSIONS

This paper presents the trajectory tracking problem of multiple agents. Convex safety regions are associated to each agent in order to solve the collision avoidance problem. First, the notion of polyhedral function is recalled and further introduced in a potential function which accounts for the associated safety region. Second, in real-time, a receding horizon control design and a leader/followers strategy are adopted for driving the agents into a formation with collision free behavior.

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## REFERENCES

- Blanchini, F. (1995). Nonquadratic Lyapunov functions for robust control. *Automatica*, 31(3):451–461.
- De Doná, J., Suryawan, F., Seron, M., and Lévine, J. (2009). A flatness-based iterative method for reference trajectory generation in constrained NMPC. *Nonlinear Model Predictive Control*, pages 325–333.
- Dunbar, W. and Murray, R. (2006). Distributed receding horizon control for multi-vehicle formation stabilization. *Automatica*, 42(4):549–558.
- Fliess, M., Lévine, J., Martin, P., and Rouchon, P. (1995). Flatness and defect of non-linear systems: introductory theory and examples. *International Journal of Control*, 61(6):1327–1361.
- Jadbabaie, A., Lin, J., and Morse, A. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *Automatic Control, IEEE Transactions on*, 48(6):988–1001.
- Maciejowski, J. (2002). *Predictive control: with constraints*. Pearson Education.
- Schneider, J. (2009). Pathway Toward a Mid-Infrared Interferometer for the Direct Characterization of Exoplanets. *Arxiv preprint arXiv:0906.0068*.
- Tanner, H., Jadbabaie, A., and Pappas, G. (2007). Flocking in fixed and switching networks. *Automatic Control, IEEE Transactions on*, 52(5):863–868.