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Analysis of Outage Probability and Throughput for Half-Duplex Hybrid-ARQ Relay Channels

Behrouz Maham, *Member, IEEE*, Aydin Behnad, *Member, IEEE*, and Mérouane Debbah, *Senior Member, IEEE*

Abstract— We consider a half-duplex wireless relay network with hybrid-automatic retransmission request (HARQ) and Rayleigh fading channels. In this paper, we analyze the average throughput and outage probability of the multi-relay delay-limited HARQ system with opportunistic relaying scheme in decode-and-forward mode, in which the *best* relay is selected to transmit the source's regenerated signal. A simple and distributed relay selection strategy is considered for multi-relay HARQ channels. Then, we utilize the non-orthogonal cooperative transmission between the source and selected relay for retransmitting of the source data toward the destination if needed, using space-time codes. We analyze the performance of the system. We first derive the cumulative density function (CDF) and probability density function (PDF) of the selected relay HARQ channels. Then, the CDF and PDF are used to determine the *exact* outage probability in the l -th round of HARQ. The outage probability is required to compute the throughput-delay performance of this half-duplex opportunistic relaying protocol. The packet delay constraint is represented by L , the maximum number of HARQ rounds. An outage is declared if the packet is unsuccessful after L HARQ rounds. Furthermore, simple closed-form upper-bounds on outage probability are derived. Based on the derived upper-bound expressions, it is shown that the proposed schemes achieve the full spatial diversity order of $N + 1$, where N is the number of potential relays. Our analytical results are confirmed by simulation results. In addition, simulation show that our proposed scheme can achieve higher average throughput compared to the direct transmission and conventional two-phase relay networks.

I. INTRODUCTION

Cooperation among devices has been considered to provide diversity in wireless networks where fading may significantly affect single links [1]. Initial works have emphasized on relaying, where a cooperator node amplifies (or decodes) and forwards, possibly in a quantized fashion [2], the information from the source node in order to help decoding at the destination node [3], [4], [5]. The achieved throughput can be increased with the integration of cooperation and coding, i.e., by letting the cooperator send incremental redundancy to the destination [6]. In particular, it has been shown in [6] that coded cooperation achieves a diversity order of two, while decode-and-forward reaches only a diversity order one, when

the transmissions of source and cooperator are orthogonal. The capacity of cooperative networks using both the decode-and-forward and coded cooperation has been extensively studied [6], [7] for simple networks with simple medium access control (MAC) protocols. In [8], a system with two transmission phases that makes use of convolutional codes is analyzed and characterized by means of partner choice and performance regions. Resource allocation for space-time coded cooperative networks has been studied in [9], [10], where the analysis of bit error rate and outage probability are also derived. Unfortunately, in cooperative relaying the diversity gain is increased at the expense of throughput loss due to the half-duplex constraint at relay nodes. Different methods have been proposed to recover this loss. In [11], successive relaying using repetition coding has been introduced for a two relay wireless network with flat fading. In [12], relay selection methods have been proposed for cooperative communication with decode-and-forward (DF) relaying.

In cooperative relaying, communication between the source and the destination nodes is performed with the aid of multiple relays acting as the retransmitters. This technique presents a generalization of the classical automatic repeat request (ARQ) mechanisms. It provides a substantial increase in the diversity gain compared to conventional ARQ, especially in the case of the so-called long-term quasi-static ARQ channels, where the channel connecting the source and destination nodes varies very slowly from round to round (see, e.g., [13]). A prominent alternative to reducing the throughput loss in relay-aided transmission mechanisms is the combination of both ARQ and relaying. This approach would significantly reduce the half-duplex multiplexing loss by activating ARQ for rare erroneously decoded data packets, when they occur. Approaches targeting the joint design of ARQ and relaying in one common protocol have recently received more interest (see for instance [14], [15]). The author in [16] evaluated the expected waiting time and the sojourn time of packets in the network of queues of cooperative truncated HARQ. A cross-layer approach to design an adaptation scheme over the relay channel in the presence of HARQ is developed in [17] to enhance the system performance for data packet transmission over block-fading relay channels. In [18], two smart HARQ schemes with incremental redundancy were developed for a dual-hop network of two relays implementing cooperative communication. Motivated by the above suggestion, we investigate and analyze throughput efficient cooperative transmission techniques where both ARQ and relaying are jointly designed. To this end, a diversity effect can be introduced to a relay networks by simply allowing the nodes to main-

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tain previously received information concerning each active message. Each time a message is retransmitted, either from a new node or from the same node, every node in the relay network will increase the amount of resolution information it has about the message. Once a node has accumulated sufficient information it will be able to decode the message and can act as a relay and forward the message (as in decode-and-forward [3], [19]). This diversity effect can be viewed as a space-time generalization of the time-diversity effect of hybrid-automatic repeat request (HARQ) as described in [20]. Thus, the HARQ scheme which is used in this paper is a practical approach to designing wireless ad hoc networks that exploit the spatial diversity, which is achievable with relaying. The retransmitted packets could originate from any node that has overheard and successfully decoded the message. Current and future wireless networks based on packet switching use HARQ protocols at the link layer. Hence, the performance of HARQ protocols in relay channels has attracted recent research interest [21], [22], [14]. In [21], diversity-multiplexing tradeoffs for half-duplex single relay protocols with HARQ are computed as the signal-to-noise ratio (SNR) tends to infinity. The tradeoffs are computed from the high-SNR asymptotic behavior of the outage probability, defined as the probability of packet failure after L HARQ rounds. However, in this work, we derive the exact expressions for the outage probability and diversity-multiplexing tradeoffs in multiple relays protocols with HARQ.

In this paper, we propose an efficient HARQ multi-relay protocol which leads to full spatial diversity. We assume a delay-limited network with the maximum number of HARQ rounds L , which represents the delay constraint. The protocol uses a form of incremental redundancy HARQ transmission with assistance from the selected relay via non-orthogonal transmission in the second transmission phase if the relay decodes the message before the destination. Note that by non-orthogonal transmission, we mean that the source node and the selected relay simultaneously retransmit the source data using space-time codes or beamforming techniques.

Our main contributions can be summarized as follows:

- We introduce a distributed relay selection scheme for HARQ multi-relay networks by using acknowledgment (ACK) or non-acknowledgment (NACK) signals transmitted by destination.
- The exact and closed-form expressions are derived for the outage probability of the system, defined as the probability of packet failure after L HARQ rounds, in half-duplex.
- For sufficiently high SNR, we derive a simple closed-form average outage probability expression for a HARQ system with multiple cooperating branches, and it is shown that the full diversity is achievable in the proposed HARQ relay networks.
- The simulations show that the throughput of the HARQ-based relay channel is significantly larger than that of direct transmission and also conventional two-phase relay networks for a wide range of SNRs, target outage probabilities, delay constraints and relay numbers.

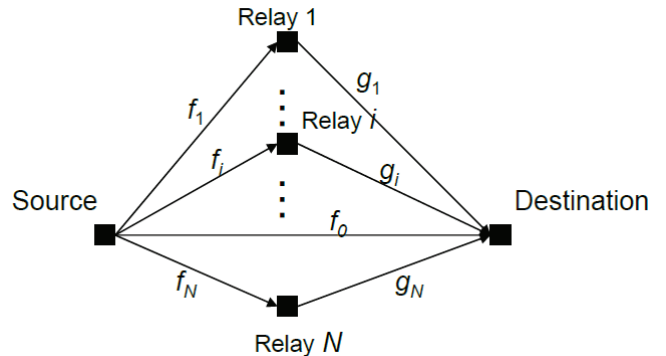


Fig. 1. Wireless relay network consisting of a source, a destination, and N relays.

The remainder of this paper is organized as follows: In Section II, the system model and protocol description are given. The performance analysis The closed-form expressions for the outage probability and asymptotic analysis of the system are presented in Section III, which are utilized for optimizing the system. In Section IV, the overall system performance is presented for different numbers of relays and channel conditions, and the correctness of the analytical formulas are confirmed by simulation results. Conclusions are presented in Section V.

Notations: The superscripts $(\cdot)^t$, $(\cdot)^H$, and $(\cdot)^*$ stand for transposition, conjugate transposition, and element-wise conjugation, respectively. The expectation operation is denoted by $\mathbb{E}\{\cdot\}$. The union and intersection of a collection of sets are denoted by \cup and \cap , respectively. The symbol $|x|$ is the absolute value of the scalar x , while $[x]^+$ denotes $\max\{x, 0\}$. The logarithms \log_2 and \log are the based two logarithm and the natural logarithm, respectively.

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

Consider a network consisting of a source, one or more relays denoted $i = 1, 2, \dots, N$, and one destination. The wireless relay network model is illustrated in Fig. 1. It is assumed that each node is equipped with a single antenna. We denote the source-to-destination, source-to- i th relay, and i th relay-to-destination links by f_0 , f_i , and g_i , respectively. Suppose each link has Rayleigh fading, independent of the others. Therefore, f_0 , f_i , and g_i are i.i.d. complex Gaussian random variables with zero-mean and variances σ_0^2 , $\sigma_{f_i}^2$, and $\sigma_{g_i}^2$, respectively. For any two nodes, $\sigma_x^2 = k_0/d_x^\nu$ is the path-loss coefficient, where d_x is the distance of the link x , k_0 depends on the operating frequency, and ν is the path-loss exponent, which typically lies in the range of $2 \leq \nu \leq 6$. As in [14], all links are assumed to be long-term quasi-static wherein all HARQ rounds of a single packet experience a single channel realization. Subsequent packets experience independent channel realizations. Note that such an assumption, applicable in low-mobility environments such as indoor wireless local area networks (WLANs), clearly reveals the gains due to HARQ since temporal diversity is not present.

A. Relay Selection Strategy

In this paper, we use selection relaying, a.k.a. opportunistic relaying [23], which selects the best relay among N available relays. Inspired by the distributed algorithm proposed in [23], which uses request-to-send (RTS) and clear-to-send (CTS) signals to select the best relay, we propose the following selection procedure for HARQ systems using ACK/NACK signals:

- In the first step, the source node broadcast its packet toward the relays and the destination. Thus, relays can estimate their source-to-relay channels.
- If the destination decodes the packet correctly, the relays would not cooperate. Otherwise, relays exploit the NACK signal which is transmitted by the destination to estimate their corresponding relay-to-destination channels.
- The i th relay, $i = 1, \dots, N$ has a timer T_i which its value is proportional to the inverse of $\min\{|f_i|^2, |g_i|^2\}$.
- The relay with maximum amount of $\min\{|f_i|^2, |g_i|^2\}$ has a smallest T_i . Whenever the first relay finished its timer, it broadcasts a flag packet toward the other relays to make them silent and announce itself as the selected relay.

The transmission of NACK from the destination allows for the estimation of the instantaneous wireless channel between relay and destination at each relay according to the reciprocity theorem [24]. We assume that the forward and backward channels between the relay and destination are the same due to the reciprocity theorem. Note that these transmissions occur on the same frequency band and same coherence interval. Therefore, the relay with the minimum of its backward and forward channels is selected as the best relay. In other words, this policy selects the "bottleneck" of the two paths. In [25], the request to an automatic repeat request (ARQ) is served by the relay closest to the destination, among those that have decoded the message. However, distance-dependent relay selection does not consider the fading effect of wireless networks and leads to a maximum diversity of two. Therefore, in this work, the request to an ARQ is served by the relay with the best instantaneous channel conditions. Similar to [23], we choose the relay with the maximum of $\min\{\gamma_{f_i}, \gamma_{g_i}\}$, $i = 1, \dots, N$, as the best relay, where $\gamma_{f_i} = |f_i|^2$ and $\gamma_{g_i} = |g_i|^2$. Thus, the index of the best relay is written by

$$r = \arg \max_{i=1, \dots, N} \{\min\{\gamma_{f_i}, \gamma_{g_i}\}\}, \quad (1)$$

and we define

$$\gamma_{\max} \triangleq \min\{\gamma_{f_r}, \gamma_{g_r}\} = \max_{i=1, \dots, N} \{\min\{\gamma_{f_i}, \gamma_{g_i}\}\}. \quad (2)$$

As it can be seen from (1), the index r depends on all relay channels γ_{f_i} and γ_{g_i} , $i = 1, 2, \dots, N$. Hence, although γ_{f_i} and γ_{g_i} , $i = 1, 2, \dots, N$ are independent random variables, γ_{f_r} and γ_{g_r} could not be independent.

B. Transmission Strategy

Let s and \hat{s} denote the transmitted signals from the source and the selected relay, respectively. As shown in Fig. 2, during the first HARQ round, the relays and destination listen to the source transmit block s . At the end of the transmission, the

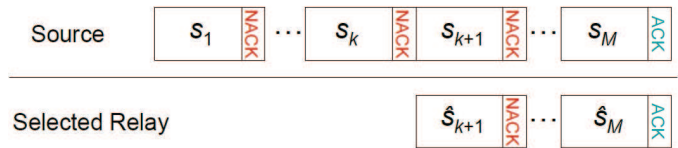


Fig. 2. Example of HARQ protocol for relay selection system. The selected relay decodes message after HARQ round k . The source and selected relay simultaneously transmit s_l and \hat{s}_l , respectively, for all HARQ rounds $l > k$. In this figure, the destination decodes the message after HARQ round M where $k < M \leq L$.

destination transmits both the source and relays a one-bit ACK or NACK indicating, respectively, the success or failure of the transmission. The NACK/ACK is assumed to be received error-free and with negligible delay. Then, with the procedure given above, the best relay is selected. As long as NACK is received after each HARQ round and the maximum number of HARQ rounds is not reached, the source successively transmits subsequent HARQ blocks of the same packet. As illustrated in Fig. 2, suppose the selected relay decodes the message after HARQ round k , while the destination has not yet decoded the message correctly. For all HARQ rounds $l > k$, the source and the selected relay simultaneously transmit s and \hat{s} , respectively. For this non-orthogonal transmission, the destination can be benefited from a spatial diversity technique like distributed space-time codes.

The Alamouti code can be used to transmit the coded packets, hence, no interference occurs due to the simultaneous transmissions of the source and relay. The effective coding rate after l HARQ rounds is R/l bps/Hz, where R is the spectral efficiency (in bps/Hz) of the first HARQ round. Let x and \hat{x} denote the Alamouti code transmitted signals from the source and the selected relay, respectively. The received signal y at the destination can be written as follows:

$$y = \begin{cases} f_0x + g_r\hat{x} + n, & \text{if } l > k, \\ f_0x + n, & \text{if } l \leq k. \end{cases} \quad (3)$$

where the index r refers to the index of the selected relay and n is a complex white Gaussian noise sample with variance N_0 .

III. PERFORMANCE ANALYSIS

A. Average Throughput

Two definitions of throughput are considered. A frequently used metric for throughput analysis is the long-term (LT) average throughput, given by [13]

$$\tilde{G}_{LT} = \frac{R}{\mathbb{E}\{l\}} = \frac{R}{\sum_{l=0}^{L-1} P_{\text{out}}(l)}, \quad (4)$$

where $\mathbb{E}\{l\}$ is the average number of HARQ rounds spent transmitting an arbitrary message and $P_{\text{out}}(l)$ denotes the probability that the packet is incorrectly decoded at the destination after l HARQ rounds. In the next section, we calculate closed-form solutions for the outage probability terms $P_{\text{out}}(l)$ used in (4). The definition in (4) relies on the steady-state behavior of several message transmissions. During this time, the probabilities $P_{\text{out}}(l)$ are assumed to be constant. This

assumption is removed by considering the delay-limited (DL) throughput, which is the throughput of a single packet, defined by [14]

$$\bar{G}_{DL} = \sum_{l=1}^L \frac{R}{l} [P_{\text{out}}(l-1) - P_{\text{out}}(l)], \quad (5)$$

An advantage of definition (5), which does not resort to long-term behavior, is the ability to track slow time variations in the channels.

In this subsection, we calculate the outage probability of the HARQ relay selection system proposed in the previous section. Besides achieving a performance metric, outage probability expression is needed in both throughput definitions in (4) and (5).

B. Exact Outage Probability

The outage probability in the first HARQ transmission round, i.e., $P_{\text{out}}(1)$ is similar to the outage probability of the direct transmission. For direct transmission, the mutual information between source and destination at each HARQ round is

$$I_{f_0} = I_{s,d} = \log_2 \left(1 + \frac{P}{N_0} \gamma_{f_0} \right), \quad (6)$$

where P is the average transmit power from the source. Since the total mutual information after first HARQ round for long-term quasi-static channels is I_{f_0} , the outage probability after first HARQ round is

$$P_{\text{out}}(1) = \Pr[I_{f_0} < R] = 1 - \exp \left(-\frac{N_0(2^R - 1)}{P \sigma_0^2} \right). \quad (7)$$

Next, we calculate $P_{\text{out}}(l)$, $l = 2, 3, \dots, L$. Let χ denote the earliest HARQ round after which the relay stops listening to the current message. The outage probability for the relay channel after l HARQ rounds, where $l > 1$, is given by [21]

$$P_{\text{out}}(l) = \sum_{k=1}^{l-1} P_{\text{out}}(l|l > k) \Pr[\chi = k] + \sum_{k=l}^L P_{\text{out}}(l|l \leq k) \Pr[\chi = k]. \quad (8)$$

To compute $\Pr[\chi = k]$, the mutual information between source and relay for each HARQ round is given by

$$I_{f_r} = \log_2 \left(1 + \frac{P}{N_0} \gamma_{f_r} \right), \quad (9)$$

where γ_{f_r} is an exponentially distributed random variable with mean $\sigma_{f_r}^2$. The event $\chi = 1$ means that the relay is able to decode the message in the first transmission from the source, and thus, we have

$$\Pr[\chi = 1] = \Pr[I_{f_r} > R] = \Pr[\gamma_{f_r} > \mu_1], \quad (10)$$

where $\mu_1 = \frac{N_0}{P} (2^R - 1)$. For $k = 2, \dots, l-1$, $\chi = k$ if the message is successfully decoded by the relay at the k th HARQ round, and we have

$$\begin{aligned} \Pr[\chi = k] &= \Pr[(k-1)I_{f_r} < R, k I_{f_r} > R] \\ &= \Pr[(k-1)I_{f_r} < R] - \Pr[k I_{f_r} < R] \\ &= \Pr[\gamma_{f_r} < \mu_{k-1}] - \Pr[\gamma_{f_r} < \mu_k], \end{aligned} \quad (11)$$

where

$$\mu_k = \frac{N_0}{P} (2^{R/k} - 1). \quad (12)$$

For $k = l, \dots, L$, $\chi = k$ if the relay did not decode the message successfully after $(l-1)$ HARQ rounds, and thus, we have

$$\Pr[\chi = k] = \Pr[(l-1)I_{f_r} < R] = \Pr[\gamma_{f_r} < \mu_{l-1}]. \quad (13)$$

From (11) and (13), $\Pr[\chi = k]$, i.e. the probability that the number of retransmission needed for the relay to decode is equal to k , can be calculated as

$$\Pr[\chi = k] = \begin{cases} \Pr[\gamma_{f_r} > \mu_1], & \text{if } k = 1, \\ \Pr[\gamma_{f_r} < \mu_{k-1}] - \Pr[\gamma_{f_r} < \mu_k], & \text{if } 1 < k < l, \\ \Pr[\gamma_{f_r} < \mu_{l-1}], & \text{if } k \geq l. \end{cases} \quad (14)$$

Since the index r given in (1) is dependent on channels, γ_{f_r} and γ_{g_r} are *not independent* for $N > 1$. Thus, obtaining a closed-form for PDF is not straightforward. As it is seen from (14), for computing $\Pr[\chi = k]$, the CDF of random variable γ_{f_r} is required. In the following, the CDF of the random variable γ_{f_r} is derived.

Proposition 1: Let γ_{f_i} and γ_{g_i} , $i = 1, \dots, N$, be independent exponential random variables with means $\sigma_{f_i}^2$ and $\sigma_{g_i}^2$, respectively. The CDF and PDF of γ_{f_r} , where r is defined as (1), are given by (15) and (16), respectively.

$$\begin{aligned} F_{\gamma_{f_r}}(\gamma) &= \prod_{i=1}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\gamma} \right) - \sum_{j=1}^N \frac{1}{\sigma_{f_j}^2} e^{-\frac{\gamma}{\sigma_{f_j}^2}} \int_0^{\gamma} e^{-\frac{\beta}{\sigma_{g_j}^2}} \\ &\quad \times \prod_{\substack{i=1 \\ i \neq j}}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\beta} \right) d\beta, \quad \gamma \geq 0, \end{aligned} \quad (15)$$

$$\begin{aligned} f_{\gamma_{f_r}}(\gamma) &= \sum_{j=1}^N \left(\frac{1}{\sigma_{f_j}^2} e^{-\left(\frac{1}{\sigma_{f_j}^2} + \frac{1}{\sigma_{g_j}^2}\right)\gamma} \prod_{\substack{i=1 \\ i \neq j}}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\gamma} \right) \right. \\ &\quad \left. + \frac{1}{\sigma_{f_j}^2 \sigma_{g_j}^2} e^{-\frac{\gamma}{\sigma_{f_j}^2}} \int_0^{\gamma} e^{-\frac{\beta}{\sigma_{g_j}^2}} \prod_{\substack{i=1 \\ i \neq j}}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\beta} \right) d\beta \right). \end{aligned} \quad (16)$$

Proof: The proof is given in Appendix I. ■

It is noteworthy that the integrals in (15) and (16) can be easily calculated, as all terms in the expansion of integrands are of the exponential form.

Corollary 1: When $\sigma_{f_i}^2 = \sigma_f^2$ and $\sigma_{g_i}^2 = \sigma_g^2$ for all $i = 1, 2, \dots, N$, CDF and PDF of γ_{f_r} are respectively simplified as follows:

$$\begin{aligned} F_{\gamma_{f_r}}(\gamma) &= \left(1 - e^{-\left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}\right)\gamma} \right)^N \\ &\quad - \frac{N \sigma_f^2}{\sigma_f^2 + \sigma_g^2} e^{-\frac{\gamma}{\sigma_f^2}} \mathcal{B} \left(1 - e^{-\left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}\right)\gamma}; N, \frac{\sigma_f^2}{\sigma_f^2 + \sigma_g^2} \right) \end{aligned} \quad (17)$$

$$\begin{aligned} f_{\gamma_{f_r}}(\gamma) &= \frac{N}{\sigma_f^2} e^{-\left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}\right)\gamma} \left(1 - e^{-\left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}\right)\gamma} \right)^{N-1} \\ &\quad + \frac{N}{\sigma_f^2 + \sigma_g^2} e^{-\frac{\gamma}{\sigma_f^2}} \mathcal{B} \left(1 - e^{-\left(\frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}\right)\gamma}; N, \frac{\sigma_f^2}{\sigma_f^2 + \sigma_g^2} \right) \end{aligned} \quad (18)$$

where $\mathcal{B}(x; a, b) = \int_0^x t^{a-1}(1-t)^{b-1}dt$ is the incomplete beta function [26].

Corollary 2: For high values of SNR, i.e. when $\gamma = \mu_k \rightarrow 0$, the closed form solution for (15) can be obtained as

$$F_{\gamma_{f_r}}(\gamma) \cong \gamma^N \left(\prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) \right) \left(\frac{1}{N} \sum_{i=1}^N \frac{\sigma_{g_i}^2}{\sigma_{f_i}^2 + \sigma_{g_i}^2} \right). \quad (19)$$

From (15), $\Pr[\chi = k]$ in (14) can be written as

$$\Pr[\chi = k] = \begin{cases} 1 - F_{\gamma_{f_r}}(\mu_1), & \text{if } k = 1, \\ F_{\gamma_{f_r}}(\mu_{k-1}) - F_{\gamma_{f_r}}(\mu_k), & \text{if } 1 < k < l, \\ F_{\gamma_{f_r}}(\mu_{l-1}), & \text{if } k \geq l. \end{cases} \quad (20)$$

Next, the conditional probabilities $P_{\text{out}}(l|l > k)$ and $P_{\text{out}}(l|l \leq k)$ in (11) will be calculated. After correct decoding of the source packet at the relay, the relay helps the source by simultaneous transmission according to the Alamouti code. Hence, assuming the relay transmits the same power P as the source, the mutual information of the effective channel is given by

$$I_{s,r,d} = \log_2 \left(1 + \frac{P}{N_0} \gamma_{f_0} + \frac{P}{N_0} \gamma_{g_r} \right). \quad (21)$$

Let $I_{\text{tot},k,l}$ denote the total mutual information accumulated at the destination after l HARQ rounds and when $\chi = k$. For $k < l$, the relay listens for k HARQ rounds and transmits the message simultaneously with the source using the Alamouti code for the remaining $(l - k)$ HARQ rounds. For $k \geq l$, the relay does not help the source during the l HARQ rounds. Hence,

$$I_{\text{tot},k,l} = \begin{cases} kI_{f_0} + (l - k)I_{s,r,d}, & \text{if } k = 1, \dots, l - 1, \\ lI_{f_0}, & \text{if } k = l, \dots, L, \end{cases} \quad (22)$$

where I_{f_0} is the mutual information between the source and destination at each HARQ round and can be written as $I_{f_0} = \log_2 \left(1 + \frac{P}{N_0} \gamma_{f_0} \right)$.

Therefore, for $k \geq l$, we have

$$P_{\text{out}}(l|l \leq k) = \Pr[lI_{f_0} < R] = 1 - \exp \left(\frac{-\mu_l}{\sigma_0^2} \right). \quad (23)$$

From (22), the conditional probability $P_{\text{out}}(l|l > k)$ can be calculated as

$$\begin{aligned} P_{\text{out}}(l|l > k) &= \Pr[I_{\text{tot},k,l} < R] \\ &= \Pr \left\{ \log_2 \left[\left(1 + \frac{P}{N_0} \gamma_{f_0} \right)^k \left(1 + \frac{P}{N_0} \gamma_{f_0} + \frac{P}{N_0} \gamma_{g_r} \right)^{l-k} \right] < R \right\} \\ &= \Pr \left\{ \gamma_{g_r} < \frac{2^{R/(l-k)}}{\frac{P}{N_0} \left(1 + \frac{P}{N_0} \gamma_{f_0} \right)^{k/(l-k)} - \gamma_{f_0} - \frac{N_0}{P}} \right\} \\ &= \int_{\gamma_{f_0}=0}^{\mu_l} \int_{\gamma_{g_r}=0}^{\beta(\gamma_{f_0})} \frac{e^{-\frac{\gamma_{f_0}}{\sigma_0^2}}}{\sigma_0^2} f_{\gamma_{g_r}}(\gamma_{g_r}) d\gamma_{f_0} d\gamma_{g_r} \triangleq \Upsilon(l, k), \end{aligned} \quad (24)$$

where $\beta(\gamma_{f_0}) = \frac{2^{R/(l-k)} N_0}{P \left(1 + \frac{P}{N_0} \gamma_{f_0} \right)^{k/(l-k)}} - \gamma_{f_0} - \frac{N_0}{P}$. Due to symmetry, the PDF of random variable γ_{g_r} , i.e., $f_{\gamma_{g_r}}(\gamma)$, is same as the PDF of γ_{f_r} , with perhaps different mean. Thus, the PDF of γ_{g_r} can be found from (16) as

$$\begin{aligned} f_{\gamma_{g_r}}(\gamma) &= \sum_{j=1}^N \left(\frac{1}{\sigma_{f_j}^2} e^{-\left(\frac{1}{\sigma_{f_j}^2} + \frac{1}{\sigma_{g_j}^2}\right)\gamma} \prod_{\substack{i=1 \\ i \neq j}}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\gamma} \right) \right. \\ &\quad \left. + \frac{1}{\sigma_{f_j}^2 \sigma_{g_j}^2} e^{-\frac{\gamma}{\sigma_{g_j}^2}} \int_0^\gamma e^{-\frac{\beta}{\sigma_{f_j}^2}} \prod_{\substack{i=1 \\ i \neq j}}^N \left(1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)\beta} \right) d\beta \right). \end{aligned} \quad (25)$$

By substituting $f_{\gamma_{g_r}}(\gamma)$ from (16) into (24), $P_{\text{out}}(l|l > k)$ is obtained. Therefore, using (14), (23), and (24), the outage probability in the l th stage of HARQ process can be achieved as

$$\begin{aligned} P_{\text{out}}(l) &= (1 - F_{\gamma_{f_r}}(\mu_1)) \Upsilon(l, 1) + \sum_{k=2}^{l-1} (F_{\gamma_{f_r}}(\mu_{k-1}) - F_{\gamma_{f_r}}(\mu_k)) \\ &\quad \times \Upsilon(l, k) + \sum_{k=l}^L F_{\gamma_{f_r}}(\mu_{l-1}) \left(1 - e^{-\frac{\mu_l}{\sigma_0^2}} \right), \end{aligned} \quad (26)$$

where $\Upsilon(l, k)$ is defined in (24).

IV. ASYMPTOTIC ANALYSIS

In this section, we will study the achievable diversity gains in a HARQ-based parallel relay network containing N relays. In particular, we consider relay selection network with selection strategy as stated in (1) and HARQ channels with the maximum number of HARQ rounds of L . An outage is declared if the packet is unsuccessful after L . also relays with partial CSI of f_r with the scaling factor presented in (32).

A tractable definition of the diversity gain is [27, Eq. (1.19)]

$$G_d = - \lim_{\rho \rightarrow \infty} \frac{\log(P_{\text{out}})}{\log(\rho)}, \quad (27)$$

where $\rho = \frac{P}{N_0}$. Thus, in the following, we investigate the asymptotic behavior and diversity order of $P_{\text{out}}(l)$ in (8).

For calculating the minimum diversity gain of HARQ wireless relay networks when selection strategy in (1) is used, it is enough to derive an upper-bound on the outage probability $P_{\text{out}}(l)$.

The random variable γ_{f_r} , which is corresponding the source-relay channel of the selected relay, can be bounded as

$$\gamma_{\text{max}} \leq \gamma_{f_r} \leq \gamma_{\text{max}}^s, \quad (28)$$

where γ_{max} is given in (2) and γ_{max}^s is defined as

$$\gamma_{\text{max}}^s = \max_{i=1, \dots, N} \{\gamma_{f_i}\}. \quad (29)$$

The CDF of γ_{max}^s can be written as

$$\begin{aligned} \Pr\{\gamma_{\text{max}}^s < \gamma\} &= \Pr\{\gamma_{f_1} < \gamma, \gamma_{f_2} < \gamma, \dots, \gamma_{f_N} < \gamma\} \\ &= \prod_{i=1}^N \left(1 - e^{-\frac{\gamma}{\sigma_{f_i}^2}} \right). \end{aligned} \quad (30)$$

The CDF of γ_{max} can be also written as

$$\Pr\{\gamma_{\text{max}} < \gamma\} = \Pr\{\gamma_1 < \gamma, \gamma_2 < \gamma, \dots, \gamma_N < \gamma\} \quad (31)$$

where $\gamma_i = \min\{\gamma_{f_i}, \gamma_{g_i}\}$ is again an exponential random variable (RV) with the parameter equal to the sum of parameters of exponential RV γ_{f_i} and γ_{g_i} , i.e., $1/\sigma_{f_i}^2$ and $1/\sigma_{g_i}^2$, respectively.

Thus, assuming that all channel coefficients are independent of each others, we can rewrite (31) as

$$\Pr\{\gamma_{\max} < \gamma\} = \prod_{i=1}^N \left(1 - e^{-\gamma \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right)} \right). \quad (32)$$

Thus, it is easy to show that the CDF of γ_{f_r} can be bounded as

$$\Pr\{\gamma_{\max}^s < \gamma\} \leq \Pr\{\gamma_{f_r} < \gamma\} \leq \Pr\{\gamma_{\max} < \gamma\}. \quad (33)$$

Therefore, combining (14) and (33), an upper-bound on $\Pr[\chi = k]$ will be obtained as follows

$$\Pr[\chi = 1] \leq 1 - \Pr[\gamma_{\max}^s < \mu_1], \quad (34)$$

and

$$\Pr[\chi = k] \leq \Pr[\gamma_{\max} < \mu_{k-1}] - \Pr[\gamma_{\max}^s < \mu_k], \quad (35)$$

for $1 < k < l$. From (32), (30), and (35), we have

$$\Pr[\chi = 1] \leq 1 - \prod_{i=1}^N \left(1 - e^{-\frac{\mu_1}{\sigma_{f_i}^2}} \right) \triangleq \Lambda_1(1). \quad (36)$$

and $\Pr[\chi = k]$ for $1 < k < l$ can be calculated as

$$\begin{aligned} \Pr[\chi = k] &\leq \prod_{i=1}^N \left(1 - e^{-\mu_{k-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right)} \right) - \prod_{i=1}^N \left(1 - e^{-\frac{\mu_k}{\sigma_{f_i}^2}} \right) \\ &\triangleq \Lambda_1(k). \end{aligned} \quad (37)$$

For $k \geq l$, by combining (14), (32), and (33), we have

$$\begin{aligned} \Pr[\chi = k] &\leq \Pr[\gamma_{\max} < \mu_{l-1}] \\ &= \prod_{i=1}^N \left(1 - e^{-\mu_{l-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right)} \right) \triangleq \Lambda_2(l). \end{aligned} \quad (38)$$

Next, $P_{\text{out}}(l|l > k)$ in (24) can be upper-bounded as

$$\begin{aligned} &P_{\text{out}}(l|l > k) \\ &\leq \Pr \left\{ \gamma_{\max} < \frac{2^{R/(l-k)}}{\frac{P}{N_0} \left(1 + \frac{P}{N_0} \gamma_{f_0} \right)^{k/(l-k)} - \gamma_{f_0} - \frac{N_0}{P}} \right\} \\ &= \int_{\gamma_{f_0}=0}^{\mu_l} \int_{\gamma_{\max}=0}^{\beta(\gamma_{f_0})} \frac{e^{-\frac{\gamma_{f_0}}{\sigma_0^2}}}{\sigma_0^2} f_{\gamma_{\max}}(\gamma_{\max}) d\gamma_{f_0} d\gamma_{\max}. \end{aligned} \quad (39)$$

The PDF of random variable γ_{\max} , i.e., $f_{\gamma_{\max}}(\gamma)$ can be found by the derivative of $\Pr\{\gamma_{\max} < \gamma\}$ in (32). Thus, we have

$$\begin{aligned} f_{\gamma_{\max}}(\gamma) &= \sum_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) e^{-\gamma \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right)} \\ &\quad \times \prod_{\substack{j=1 \\ j \neq i}}^N \left(1 - e^{-\gamma \left(\frac{1}{\sigma_{f_j}^2} + \frac{1}{\sigma_{g_j}^2} \right)} \right). \end{aligned} \quad (40)$$

Therefore, by substituting $\Pr[\chi = k]$ from (37) and (38), and $P_{\text{out}}(l|l \leq k)$ and $P_{\text{out}}(l|l > k)$ from (23) and (39), respectively, in (8), an upper-bound on outage probability the l th stage of HARQ process, i.e., $P_{\text{out}}(l)$ can be achieved.

To find the diversity order of $P_{\text{out}}(l)$ from (40), an upper-bound for $f_{\gamma_{\max}}(\gamma)$ can be found as

$$f_{\gamma_{\max}}(\gamma) \leq N \gamma^{N-1} \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right), \quad (41)$$

which is a tight bound when $\gamma \rightarrow 0$. Note that in high SNR scenario, the behavior of the fading distribution around zero is important (see, e.g., [28]).

Using (41) and the fact that the exponential distribution is a decreasing function of γ_{f_0} , $P_{\text{out}}(l|l > k)$ in (39) can be further upper-bounded as

$$\begin{aligned} &P_{\text{out}}(l|l > k) \\ &\leq \int_{\gamma_{f_0}=0}^{\mu_l} \int_{\gamma_{\max}=0}^{\beta(\gamma_{f_0})} \frac{N}{\sigma_0^2} \gamma_{\max}^{N-1} \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) d\gamma_{f_0} d\gamma_{\max} \\ &\leq \frac{\mu_l}{\sigma_0^2} \mu_l^{N-1} \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) \triangleq \Psi(l, k). \end{aligned} \quad (42)$$

Combining (8), (23), (37), (38), and (43), a closed-form upper-bound for the outage probability after l HARQ round can be obtained as

$$P_{\text{out}}(l) \leq \sum_{k=1}^{l-1} \Lambda_1(k) \Psi(l, k) + \sum_{k=l}^L \Lambda_2(l) \left(1 - e^{-\frac{\mu_l}{\sigma_0^2}} \right). \quad (43)$$

Proposition 2: Assuming a HARQ system with N potential relays nodes, the relay selection strategy based on (1) can achieve the full diversity order of $N + 1$ for $l > 1$, and the outage probability can be upper-bounded as

$$P_{\text{out}}(l) \leq \frac{\Delta(l)}{\rho^{N+1}}, \quad (44)$$

where

$$\Delta(l) = \left(2^{\frac{R}{l}} - 1 \right) \left(2^{\frac{R}{l-1}} - 1 \right)^N \frac{L-l+2}{\sigma_0^2} \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right).$$

Proof: The proof is given in Appendix II. ■

Hence, similar to the DF selective relaying [3] or amplify-and-forward based opportunistic relaying [23], the full spatial diversity is achievable when HARQ-based selection relaying is used.

Note that, at high SNR, a Taylor series expansion of $P_{\text{out}}(1)$ in (7) around zero in the variable $1/\rho$ can be used. Thus, we have $P_{\text{out}}(1) = \frac{2^R-1}{\rho \sigma_0^2} + O(1/\rho^2)$. Hence, the high-SNR diversity order of the system with $l = 1$ is one, as expected.

V. NUMERICAL ANALYSIS

In this section, the performance of the proposed relay-selection HARQ system is studied through numerical results. We used the equal power allocation among the source and the selected relay. Assume the relays and the destination have the same value of noise power, and all the links have unit-variance Rayleigh flat fading, i.e., $\sigma_{f_i}^2 = \sigma_{g_i}^2 = \sigma_0^2 = 1$. It is also assumed that rate R is normalized to 1. We compare the transmit SNR $\frac{P}{N_0}$ versus outage probability performance. The block fading model is used, in which channel coefficients changed randomly in time to isolate the benefits of spatial diversity. The simulation result is averaged over 3'000'000 transmitted symbols (channel realization trials).

Fig. 3 confirms that the analytical results attained in Section III for the outage probability have an accurate performance as the simulation results. We consider the maximum number of HARQ rounds to be $L = 5$. The outage probability at the 2nd HARQ round, i.e., $P_{\text{out}}(l = 2)$, is compared for two different number of relays $N = 2, 4$. One can see the exact outage probability derived in (26) has the similar performance as the simulated curved for all values of SNR. In addition, the closed-form outage probability expressions in (43) and (44)

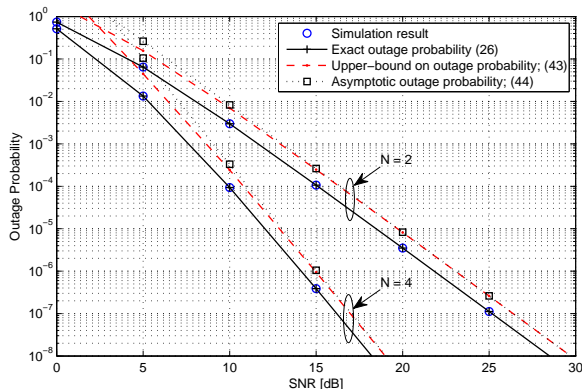


Fig. 3. The outage probability $P_{\text{out}}(l)$ curves of delay-limited HARQ networks employing opportunistic relaying with 2 and 4 relays, when $R = 1$ bits/sec, $L = 5$ is the maximum number of rounds, and we consider HARQ round of $l = 2$.

well approximate the simulated results, especially in medium and high SNR conditions. Furthermore, Fig. 3 shows that the upper-bound expression in (43) is a tight upper-bound. The asymptotic outage probability derived in (44) is also depicted in Fig. 3 which confirms the full-diversity order of the proposed scheme.

It is straightforward to show that the outage probability for direct transmission after l HARQ round is [14, Eq. (7)]

$$P_{\text{out},d}(l) = 1 - \exp\left(-\frac{2^{R/l} - 1}{\rho \sigma_0^2}\right). \quad (45)$$

In Fig. 4, the outage probability at the $l = L = 5$ th HARQ round of the system with different number of relays are considered. After selecting the best relay, Alamouti code is employed in the second transmission phase since the source and the best selected relay are viewed as a two-antenna virtual MIMO. Compared to the single HARQ relaying system proposed in [14], the proposed HARQ opportunistic relaying system with $N = 2, 3, 4$ relays outperforms considerably for all SNR conditions. For example, it can be seen that in outage probability of 10^{-3} , the system with two relays saves around 8 dB in SNR compared to the single relay HARQ system. Furthermore, it can be checked that the system with N relays can achieve the diversity order of $N + 1$.

In Fig. 5, we compare the opportunistic scheme based on (1), called best bottleneck link, with two other opportunistic relaying schemes. The first scheme is based on the selection of a relay with the best source-relay link. This opportunistic relaying can be easily implemented by relays. Thus, unlike the selection based on (1), which needs the reciprocity of channels, this selection scheme can be implemented in frequency-division duplex (FDD) channels as well as time-division duplex (TDD) channels. The index of the selected relay for this strategy becomes

$$\alpha = \arg \max_{i=1, \dots, N} \{\gamma_{f_i}\}. \quad (46)$$

Another scheme that we investigate is a fixed DF relaying with the selection of the best relay-destination path. That is, the best relay can be selected by the destination and the index of the best relay is

$$\omega = \arg \max_{i=1, \dots, N} \{\gamma_{g_i}\}. \quad (47)$$

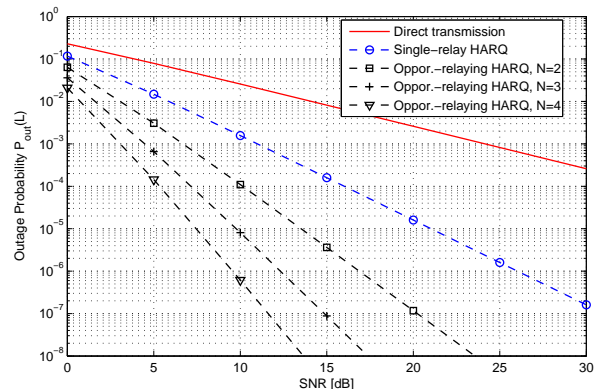


Fig. 4. The outage probability performance of the proposed relay selection HARQ system versus transmit SNR in a network with different number of relays, $R = 1$ bits/sec, $L = l = 5$ HARQ rounds, and $\sigma_{f_i}^2 = \sigma_{g_i}^2 = \sigma_0^2 = 1$.

This opportunistic relaying can be also exploited in FDD channels as well as TDD channels (unlike method based on (1) which requires the reciprocity of relay-destination channels). As it can be observed from Fig. 5, our proposed HARQ-based relay selection outperforms two other schemes in which only one of the source-relay or relay-destination links is taken into account in the decision of the selected relay. It can be also seen that in high SNR conditions both relay selections based on (46) and (47) can only achieve the diversity of 2. However, using the selection strategy based on (1), full diversity of $N + 1$ is obtainable which confirms the validity of our analytic result obtained in Proposition 2.

A delay-limited throughput where defined in (5) explicitly accounts for finite delay constraints and associated non-zero packet outage probabilities. It can be shown that for small outage probabilities, this delay-limited throughput is greater than the conventional long-term average throughput defined in (4). In addition to finite delay constraint, represented by the maximum number L of HARQ rounds, higher-layer applications usually require that $P_{\text{out}} \leq \rho_{\text{max}}$, where ρ_{max} is a target outage probability. The total LT and DL throughput are studied in Fig. 6 subject to user QoS constraints, represented by outage probability target ρ_{max} and delay constraint L . In Fig. 6, the total LT and DL throughputs of opportunistic relaying HARQ system with $N = 2, 4$ relays are plotted as a function of SNR and compared with the direct transmission HARQ system with $L = 3$, $\rho_{\text{max}} = 10^{-3}$, and the following linear relay geometry: $\sigma_{f_r}^2 = \sigma_{g_r}^2 = \sigma_0^2 = 1$. As expected, the presence of the relays significantly increases the throughput. Furthermore, in agreement with [14, Eq. (5)], it can be seen that the delay-limited throughput is greater than the long-term average throughput. An interesting observation is that as well as the diversity gain achieved by the opportunistic relaying HARQ system, which is previously shown in Fig. 4, obtaining higher throughputs are possible. This behavior underscores the importance of the proposed system.

VI. CONCLUSION

In this paper, we proposed a throughput-efficient relay selection HARQ system over Rayleigh fading. The throughput-delay performance of a half-duplex multi-branch relay sys-

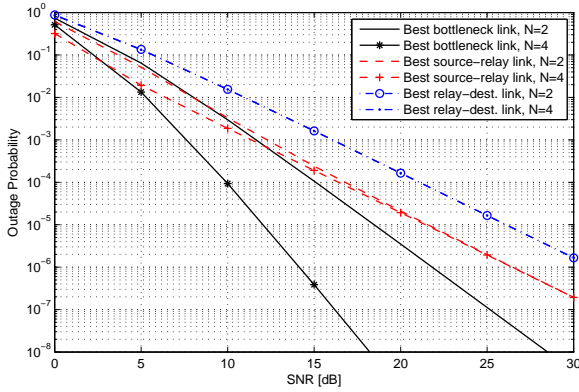


Fig. 5. The outage probability $P_{\text{out}}(l)$ curves of multiple-relays HARQ networks employing different opportunistic relaying schemes based on (1), (46), and (47) with 2 and 4 relays, when $R = 1$ bits/sec, $L = 5$ is the maximum number of rounds, and we consider HARQ round of $l = 2$.

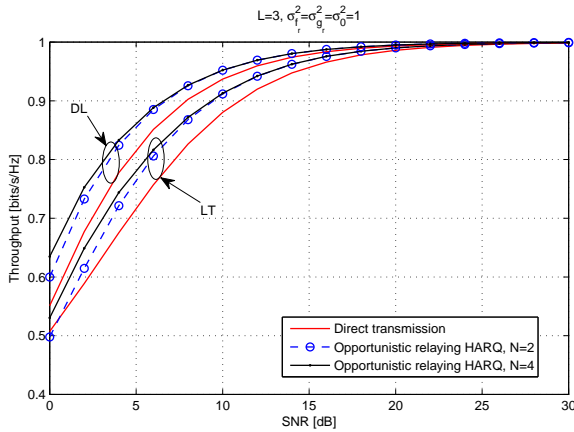


Fig. 6. The delay-limited (DL) and long-term (LT) throughputs of direct transmission and relay selection HARQ system versus transmit SNR for target outage probability 10^{-3} , $L = 3$ HARQ rounds, physical layer rate $R = 1$ bits/sec, and $\sigma_{f_r}^2 = \sigma_{g_r}^2 = \sigma_0^2 = 1$, $\sigma_{f_i}^2 = \sigma_{g_i}^2 = \sigma_0^2 = 1$.

tem with HARQ was analyzed. A distributed relay selection scheme was introduced for HARQ multi-relay networks by using ACK/NACK signals transmitted by destination. We evaluated the average throughput and outage error probability performance and showed that the proposed technique significantly reduces the multiplexing loss due to the half-duplex constraint while providing attractive outage error probability performance. The closed-form expressions outage probability were derived, defined as the probability of packet failure after L HARQ rounds, in half-duplex. For sufficiently high SNR, we derived a simple closed-form average outage probability expression for a HARQ system with multiple cooperating branches. Based on the derived upper-bound expressions, it was shown that the proposed scheme achieves the full spatial diversity order of $N+1$ in a non-orthogonal relay network with N parallel relays. The analysis presented here allows quantitative evaluation of the throughput-delay performance gain of the relay selection channel compared to direct transmission. The numerical results confirmed that the proposed schemes can bring diversity and multiplexing gains in the wireless relay networks.

APPENDIX I PROOF OF PROPOSITION 1

First, we define the auxiliary random variables $\gamma_i \triangleq \min\{\gamma_{f_i}, \gamma_{g_i}\}$ for $i = 1, 2, \dots, N$. Since $\{\gamma_{f_i}, \gamma_{g_i}\}_{i=1}^N$ are independent exponential random variables, γ_i s are also independent exponential random variables with the following CDF:

$$F_{\gamma_i}(x) = 1 - e^{-\left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2}\right)x}, \quad x \geq 0 \quad (48)$$

Also, using partitioning theorem, we have

$$\Pr\{\gamma_{f_r} \leq \gamma\} = \sum_{j=1}^N \Pr\{(\gamma_{f_r} \leq \gamma) \cap (r = j)\}. \quad (49)$$

For $j = 1, 2, \dots, N$, the summands of (49) can be obtained as follows

$$\begin{aligned} \Pr\{(\gamma_{f_r} \leq \gamma) \cap (r = j)\} &= \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \cap (\gamma_{f_j} \leq \gamma)\} \\ &= \Pr\{(\gamma_j \leq \gamma) \cap \Gamma\} - \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \cap (\gamma_{f_j} > \gamma)\}, \end{aligned} \quad (50)$$

where $\Gamma = \bigcap_{i=1, i \neq j}^N (\gamma_i < \gamma_j)$. By substituting (50) in (49), we obtain

$$\begin{aligned} \Pr\{\gamma_{f_r} \leq \gamma\} &= \sum_{j=1}^N \Pr\{(\gamma_j \leq \gamma) \cap \Gamma\} \\ &\quad - \sum_{j=1}^N \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \cap (\gamma_{f_j} > \gamma)\} \\ &= \Pr\{\gamma_{\max} \leq \gamma\} - \sum_{j=1}^N \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \cap (\gamma_{f_j} > \gamma)\}, \end{aligned} \quad (51)$$

where γ_{\max} is defined in (2). Since γ_i s are independent, the first term on the right side of (51) is given by

$$\begin{aligned} \Pr\{\gamma_{\max} \leq \gamma\} &= \Pr\left\{\bigcap_{i=1}^N (\gamma_i \leq \gamma)\right\} \\ &= \prod_{i=1}^N \Pr\{\gamma_i \leq \gamma\} = \prod_{i=1}^N F_{\gamma_i}(\gamma). \end{aligned} \quad (52)$$

Also, the summand of the summation on the right side of (51) is obtained as follows

$$\begin{aligned} \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \cap (\gamma_{f_j} > \gamma)\} &= \Pr\{\gamma_{f_j} > \gamma\} \Pr\{(\gamma_j \leq \gamma) \cap \Gamma \mid \gamma_{f_j} > \gamma\} \\ &= \Pr\{\gamma_{f_j} > \gamma\} \Pr\{(\gamma_{g_j} \leq \gamma) \cap \Gamma\} \\ &= e^{-\frac{\gamma}{\sigma_{f_j}^2}} \int_0^{\gamma} \frac{1}{\sigma_{g_j}^2} e^{-\frac{\beta}{\sigma_{g_j}^2}} \Pr\{\Gamma\} d\beta \\ &= \frac{1}{\sigma_{g_j}^2} e^{-\frac{\gamma}{\sigma_{f_j}^2}} \int_0^{\gamma} e^{-\frac{\beta}{\sigma_{g_j}^2}} \prod_{i=1, i \neq j}^N F_{\gamma_i}(\beta) d\beta. \end{aligned} \quad (53)$$

Substituting from (48) into (52) and (53), one can obtain the CDF in (15) using (51) to (53). Also, taking derivative of (15) with respect to γ , results in the PDF of γ_{f_r} , given by (16).

APPENDIX II
PROOF OF PROPOSITION 2

From a Taylor series expansion, it can be shown that $P_{\text{out}}(l)$ in (43) can be rewritten as

$$P_{\text{out}}(l) \leq \Psi(l, 1) \left(1 - \prod_{i=1}^N \frac{\mu_1}{\sigma_{f_i}^2} \right) + \sum_{k=2}^{l-1} \left[\prod_{i=1}^N \mu_{k-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) - \prod_{i=1}^N \frac{\mu_k}{\sigma_{f_i}^2} \right] \times \Psi(l, k) + \sum_{k=l}^L \frac{\mu_l}{\sigma_0^2} \prod_{i=1}^N \mu_{l-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right). \quad (54)$$

By replacing $\Psi(l, k)$ from (42) into (54), we have

$$P_{\text{out}}(l) \leq \left[\frac{\mu_l}{\sigma_0^2} \mu_{l-1}^N \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) \right] \left(1 - \prod_{i=1}^N \frac{\mu_1}{\sigma_{f_i}^2} \right) + \sum_{k=2}^{l-1} \left[\prod_{i=1}^N \mu_{k-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) - \prod_{i=1}^N \frac{\mu_k}{\sigma_{f_i}^2} \right] \times \left[\frac{\mu_l}{\sigma_0^2} \mu_{l-k}^N \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right) \right] + \sum_{k=l}^L \frac{\mu_l}{\sigma_0^2} \prod_{i=1}^N \mu_{l-1} \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right). \quad (55)$$

From (43), and by representing the factor μ_k in terms of the SNR ratio ρ , i.e., $\mu_k = \frac{2^{R/k} - 1}{\rho}$, the outage probability in high SNR can be written as

$$P_{\text{out}}(l) \leq \frac{\Delta(l)}{\rho^{N+1}}, \quad (56)$$

where

$$\Delta(l) = \left(2^{\frac{R}{l}} - 1 \right) \left(2^{l \frac{R}{l-1}} - 1 \right)^N \frac{L - l + 2}{\sigma_0^2} \prod_{i=1}^N \left(\frac{1}{\sigma_{f_i}^2} + \frac{1}{\sigma_{g_i}^2} \right).$$

Hence, observing (56), the diversity order defined in (27) is equal to $N + 1$, which is the full spatial diversity for $N + 1$ transmitting nodes.

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