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To cite this version:
William Pasillas-Lépine. Recent results on wheel slip control: Hybrid and continuous algorithms. TU Delft’s DCSC Mini-symposium on Automotive control, Mar 2011, Delft, Netherlands. hal-00832532

HAL Id: hal-00832532
https://hal-supelec.archives-ouvertes.fr/hal-00832532
Submitted on 10 Jun 2013

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Recent results on wheel slip control:
Hybrid and continuous algorithms

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Collaboration with
Mathieu Gerard (TU Delft), Edwin de Vries (TU Delft), and Antonio Loria (L2S)
Contents of the talk

- Two main families of ABS algorithms
- Why doing research on ABS today?
- Other recent approaches
- Continuous wheel slip control algorithms
- Experimental results
- Hybrid five-phase ABS algorithms
- Experimental results
- Conclusions and future work perspectives
- Publications
Why do we want to control wheel slip?

Tyre forces are generated by the wheel slip in the contact patch:

\[ \lambda = \frac{R \omega - v_x}{v_x}. \]

They have a nonlinear characteristics with a coupling between longitudinal and lateral forces.

Controlling the wheel-slip improves safety: it reduces the braking distance and maintains steerability.
Two main families of ABS algorithms

Algorithms based on **wheel slip** control:

- it is supposed (implicitly) that vehicle speed is measured (or estimated);
- the brake torque converges to a specific value (no oscillations);
- mainly present in an *academic* context...
- and in specific applications (ESP, motorcycles, tyre research).

Algorithms based on **angular acceleration** thresholds:

- do not need the vehicle speed, neither the value of optimal wheel slip;
- quite robust with respect to road conditions and tyre parameters;
- the brake torque oscillates around the optimal value (limit cycle);
• mainly present in an *industrial* context;

• widely diffused on actual vehicles, but completely heuristic.
Why doing research on ABS today?

Integrated chassis control:

• black box algorithms are difficult to integrate;
• open algorithms might clarify the architecture of ICC;
• decoupling the observation problem (for vehicle speed) from control.

Electric vehicles, In-wheel motors, EMB:

• standard ABS algorithms are not adapted to regenerative braking (Toyota Prius);
• these heuristic algorithms need the hydraulic lag in order to work properly...
• they loose performance or do not work at all with electric actuators.

Fault management:

• useful to have algorithms with a stability proof.

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Comparison of our work with other approaches


- Exponential stability in both the stable and unstable tyre domains — Tanelli et al. Robust nonlinear output feedback control for brake by wire control systems. *Automatica*, 2008.


- Other hybrid approaches that use only wheel acceleration information (Bosch) are based on heuristics, we propose a method based on the analysis of limit cycles.
Wheel dynamics

The angular velocity $\omega$ of a given wheel of the vehicle has the following dynamics:

$$I \dot{\omega} = -RF_x + T,$$

where $I$ denotes the inertia of the wheel, $R$ its radius, $F_x$ the longitudinal tyre force, and $T$ the torque applied to the wheel.

The torque $T = T_e - T_b$ is composed of the engine torque $T_e$ and the brake torque $T_b$. 

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Tyre force modelling

The longitudinal tyre force $F_x$ is often modeled as a function

$$F_x = \mu(\lambda)F_z,$$

of the wheel’s slip

$$\lambda = \frac{R\omega - v_x}{v_x}.$$

The curve $\mu(\cdot)$ can be approximated by a second order rational function

$$\mu(\lambda) = \frac{a_1\lambda - a_2\lambda^2}{1 - a_3\lambda + a_4\lambda^2}.$$
Experimental validation

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Wheel slip and acceleration offsets

Define the variables $x_1$ and $x_2$ by

\[
\begin{align*}
x_1(t) &= \lambda(t) \\
x_2(t) &= R \frac{d\omega(t)}{dt} - a_x(t),
\end{align*}
\]

where $a_x(t)$ is the vehicle’s acceleration. Derivating these variables we obtain:

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{1}{v_x(t)} (-a_x(t)x_1 + x_2) \\
\frac{dx_2}{dt} &= \frac{c\mu'(x_1)}{v_x(t)} (-a_x(t)x_1 + x_2) + \frac{u}{v_x(t)} - \frac{da_x(t)}{dt},
\end{align*}
\]

where

\[
c = \frac{R^2}{I} F_z \quad \text{and} \quad u = v_x(t) \frac{R}{I} \frac{dT}{dt}.
\]
Wheel-slip filtered setpoint

For a given wheel-slip reference $\lambda^*(t)$, we will define a filtered setpoint

\[
\begin{align*}
\frac{d\lambda_1}{dt} &= \frac{\lambda_2}{v_x(t)} \\
\frac{d\lambda_2}{dt} &= -\gamma_1 (\lambda_1 - \lambda^*) - \gamma_2 \lambda_2
\end{align*}
\]

where $\gamma_1$ and $\gamma_2$ are two positive real numbers.

This setpoint filter gives:

- A smooth reference setpoint (that one can differentiate twice) even if the original setpoint is discontinuous (for example, piecewise constant).
- A system for which all equations are divided by the vehicle’s velocity. This homogeneity allows an analysis of the system in a new (nonlinear) time-scale in which...
the dependence on speed disappears.
Changing the time-scale

In order to have $dt = v_x(t) ds$, we will use a new time-scale

$$s(t) = \int_0^t \frac{d\tau}{v_x(\tau)}.$$  

We use a dot to denote the new time-derivative

$$\dot{\varphi}(s) = \frac{d\varphi(s)}{ds}.$$  

When the acceleration $a_x$ is constant, in the new time-scale the system is simpler:

$$\begin{align*}
\dot{x}_1 &= -a_x x_1 + x_2 \\
\dot{x}_2 &= -c\mu'(x_1)(-a_x x_1 + x_2) + u \\
\dot{\lambda}_1 &= \lambda_2 \\
\dot{\lambda}_2 &= -\gamma_1(\lambda_1 - \lambda^*) - \gamma_2 \lambda_2.
\end{align*}$$
Choice of the operating point

Let \( x_1^* = \lambda_1 \) be the desired operating point for \( x_1 \). Define the error coordinates by

\[
\begin{align*}
  z_1 & = x_1 - x_1^* \\
  z_2 & = x_2 - x_2^*,
\end{align*}
\]

where

\[
x_2^* = \lambda_2 + a_x x_1 - \alpha z_1 \quad \text{and} \quad \alpha > 0.
\]

The closed-loop equation for \( z_1 \) reads

\[
\dot{z}_1 = -\alpha z_1 + z_2,
\]

which is exponentially stable if \( z_2 = 0 \). The objective is thus to design a control \( u \) such that \( x_2 \) converges towards \( x_2^* \) asymptotically.
Our cascaded control law

Driving \( x_2 \) towards the dynamic setpoint

\[
x_2^* = a_x x_1 + \lambda_2 - \alpha z_1
\]

is achieved using the control law

\[
u = -\gamma_1 (\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + a \mu'(x_1)) \lambda_2 - k_1 z_1 - k_2 z_2.
\]

The dynamic setpoint \( x_2^* \) is the core of the cascade:

- The steady state is \( a_x x_1 \).
- Other terms to reduce error \( z_1 \) using cascaded feedback \((-\alpha z_1)\) and cascaded feedforward \((\lambda_2)\).
Global exponential stability

Theorem 1  Consider an arbitrary piecewise-continuous wheel slip reference \( \lambda^*(t) \). If \( \lambda^*(t) \) is injected into the filtered setpoint equations and the control law

\[
u = -\gamma_1 (\lambda_1 - \lambda^*) + (\gamma_2 - a_x^* \mu' (x_1)) \lambda_2 - k_1 z_1 - k_2 z_2
\]

is introduced into the system, then a time-varying closed-loop system

\[
\dot{z} = \begin{bmatrix}
-\alpha & 1 \\
-k_1 + a_x \alpha - \alpha^2 + \alpha \eta(t) & -k_2 + \alpha - a_x - \eta(t)
\end{bmatrix} z,
\]

is obtained. If the control gains \( k_1 \) and \( k_2 \) satisfy

\[
k_1 > a_x \alpha - \alpha^2 \quad \text{and} \quad k_2 > \alpha - a_x + \eta_m
\]

then the origin of this closed loop system is globally exponentially stable.
Robustness

Corollary 1  Consider a constant wheel slip reference $\lambda^*$. If $\lambda^*$ is injected into the filtered setpoint equations and the control law

$$u = -\gamma_1 (\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + c\hat{\mu}'(x_1)) \lambda_2 - k_1 z_1 - k_2 z_2$$

is introduced into the system, then a time-varying closed-loop system

$$\dot{z} = A(t)z + B(t)w \quad \dot{w} = C(t)w$$

is obtained, with the same matrix $A(t)$ as in Theorem 1, and $w = (\lambda_1 - \lambda^*, \lambda_2)$. If the control gains $k_1$ and $k_2$ satisfy the bounds

$$k_1 > a_x \alpha - \alpha^2 \quad \text{and} \quad k_2 > \alpha - a_x + \eta_m$$

of Theorem 1, then the closed loop system is globally exponentially stable.
Simulations — Pure feedforward control
Simulations — Pure feedback control

![Graph showing wheel slip over time](image)

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Simulations — With both feedback and feedforward control

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Simulations — With a delay of $15 \text{ ms}$
Simulations — With a perturbation of $\mu(\cdot)$
TU Delft’s Tyre Setup

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Experimental validation, with Mathieu Gerard (TU Delft)
Experimental validation, with Mathieu Gerard (TU Delft)
Hybrid five-phase algorithms (WPL – VSD 2006)

\[ \dot{P}_b = -\frac{u_1}{R\omega} \]

1. \( x_2 < 0 \) and \( x_1 < 0 \)
2. \( x_2 \geq \epsilon_1 \)
3. \( x_2 \geq \epsilon_2 \)
4. \( x_2 \leq \epsilon_1 \)
5. \( x_2 \leq -\epsilon_3 \)

\[ \dot{P}_b = \frac{u_5 x_2}{R\omega} \]

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A method based on first integrals

\[ I_a(x) = x_2 + a \mu(x_1) \]
\[ I_b(x) = x_1 - \frac{1}{2u_0} x_2^2 \]
\[ I_c(x) = x_2 + a \mu(x_1) - u_0 x_1 \]

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Conclusion

- We proposed a new cascaded wheel slip controller.
- It uses a feedforward to speed up convergence, but a perfect knowledge of the tyre is not required (the feedback part does not use it).
- It leads to a proof of global exponential stability of the closed-loop system.
- Robust to practical phenomena (delays, relaxation length, tyre parameters).
- Validated experimentally with a tyre in-the-loop, by Mathieu Gerard (TU Delft).

Perspectives

- A controller that takes into account actuation delays is currently developed.
- The algorithms for computing angular wheel acceleration need to be improved.

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Publications


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