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Recent results on wheel slip control: Hybrid and continuous algorithms

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Collaboration with
Mathieu Gerard (TU Delft), Edwin de Vries (TU Delft), and Antonio Loria (L2S)
Contents of the talk

- Two main families of ABS algorithms
- Why doing research on ABS today?
- Other recent approaches
- Continuous wheel slip control algorithms
- Experimental results
- Hybrid five-phase ABS algorithms
- Experimental results
- Conclusions and future work perspectives
- Publications
Why do we want to control wheel slip?

Tyre forces are generated by the wheel slip in the contact patch:

$$\lambda = \frac{R\omega - v_x}{v_x}.$$ 

They have a nonlinear characteristics with a coupling between longitudinal and lateral forces. Controlling the wheel-slip improves safety: it reduces the braking distance and maintains steerability.
Two main families of ABS algorithms

Algorithms based on wheel slip control:

- it is supposed (implicitly) that vehicle speed is measured (or estimated);
- the brake torque converges to a specific value (no oscillations);
- mainly present in an academic context...
- and in specific applications (ESP, motorcycles, tyre research).

Algorithms based on angular acceleration thresholds:

- do not need the vehicle speed, neither the value of optimal wheel slip;
- quite robust with respect to road conditions and tyre parameters;
- the brake torque oscillates around the optimal value (limit cycle);
• mainly present in an *industrial* context;

• widely diffused on actual vehicles, but completely heuristic.
Why doing research on ABS today?

Integrated chassis control:

- black box algorithms are difficult to integrate;
- open algorithms might clarify the architecture of ICC;
- decoupling the observation problem (for vehicle speed) from control.

Electric vehicles, In-wheel motors, EMB:

- standard ABS algorithms are not adapted to regenerative braking (Toyota Prius);
- these heuristic algorithms need the hydraulic lag in order to work properly...
- they loose performance or do not work at all with electric actuators.

Fault management:

- useful to have algorithms with a stability proof.
Comparison of our work with other approaches


- Exponential stability in both the stable and unstable tyre domains — Tanelli et al. Robust nonlinear output feedback control for brake by wire control systems. *Automatica*, 2008.


- Other hybrid approaches that use only wheel acceleration information (Bosch) are based on heuristics, we propose a method based on the analysis of limit cycles.
Wheel dynamics

The angular velocity $\omega$ of a given wheel of the vehicle has the following dynamics:

$$I \dot{\omega} = -RF_x + T,$$

where $I$ denotes the inertia of the wheel, $R$ its radius, $F_x$ the longitudinal tyre force, and $T$ the torque applied to the wheel.

The torque $T = T_e - T_b$ is composed of the engine torque $T_e$ and the brake torque $T_b$. 
Tyre force modelling

The longitudinal tyre force $F_x$ is often modeled as a function

$$F_x = \mu(\lambda) F_z,$$

of the wheel’s slip

$$\lambda = \frac{R\omega - v_x}{v_x}.$$

The curve $\mu(\cdot)$ can be approximated by a second order rational function

$$\mu(\lambda) = \frac{a_1 \lambda - a_2 \lambda^2}{1 - a_3 \lambda + a_4 \lambda^2}.$$
Experimental validation

Wheel slip [-]  Tyre characteristics [-]

- Experimental ABS braking
- Second order rational fraction

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Wheel slip and acceleration offsets

Define the variables $x_1$ and $x_2$ by

$$
x_1(t) = \lambda(t)
$$

$$
x_2(t) = R \frac{d\omega(t)}{dt} - a_x(t),
$$

where $a_x(t)$ is the vehicle’s acceleration. Derivating these variables we obtain:

$$
\frac{dx_1}{dt} = \frac{1}{v_x(t)} (-a_x(t)x_1 + x_2)
$$

$$
\frac{dx_2}{dt} = -\frac{c\mu'(x_1)}{v_x(t)} (-a_x(t)x_1 + x_2) + \frac{u}{v_x(t)} - \frac{da_x(t)}{dt},
$$

where

$$
c = \frac{R^2}{I} F_z \quad \text{and} \quad u = v_x(t) \frac{R}{I} \frac{dT}{dt}.
$$
Wheel-slip filtered setpoint

For a given wheel-slip reference $\lambda^*(t)$, we will define a filtered setpoint

$$\frac{d\lambda_1}{dt} = \frac{\lambda_2}{v_x(t)}$$

$$\frac{d\lambda_2}{dt} = -\gamma_1 (\lambda_1 - \lambda^*) - \gamma_2 \lambda_2 \frac{v_x(t)}{v_x(t)},$$

where $\gamma_1$ and $\gamma_2$ are two positive real numbers.

This setpoint filter gives:

- A smooth reference setpoint (that one can differentiate twice) even if the original setpoint is discontinuous (for example, piecewise constant).

- A system for which all equations are divided by the vehicle’s velocity. This homogeneity allows an analysis of the system in a new (nonlinear) time-scale in which
the dependence on speed disappears.
Changing the time-scale

In order to have $dt = v_x(t)ds$, we will use a new time-scale

$$s(t) = \int_0^t \frac{d\tau}{v_x(\tau)}.$$  

We use a dot to denote the new time-derivative

$$\dot{\varphi}(s) = \frac{d\varphi(s)}{ds}.$$  

When the acceleration $a_x$ is constant, in the new time-scale the system is simpler:

$$\begin{align*}
\dot{x}_1 &= -a_x x_1 + x_2 \\
\dot{x}_2 &= -c\mu'(x_1)(-a_x x_1 + x_2) + u \\
\dot{\lambda}_1 &= \lambda_2 \\
\dot{\lambda}_2 &= -\gamma_1(\lambda_1 - \lambda^*) - \gamma_2 \lambda_2.
\end{align*}$$

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Choice of the operating point

Let $x_1^* = \lambda_1$ be the desired operating point for $x_1$. Define the error coordinates by

$$
\begin{align*}
    z_1 &= x_1 - x_1^* \\
    z_2 &= x_2 - x_2^*,
\end{align*}
$$

where

$$
x_2^* = \lambda_2 + a_x x_1 - \alpha z_1 \quad \text{and} \quad \alpha > 0.
$$

The closed-loop equation for $z_1$ reads

$$
\dot{z}_1 = -\alpha z_1 + z_2,
$$

which is exponentially stable if $z_2 = 0$. The objective is thus to design a control $u$ such that $x_2$ converges towards $x_2^*$ asymptotically.
Our cascaded control law

Driving $x_2$ towards the dynamic setpoint

$$x_2^* = a_x x_1 + \lambda_2 - \alpha z_1$$

is achieved using the control law

$$u = -\gamma_1 (\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + a\mu'(x_1)) \lambda_2 - k_1 z_1 - k_2 z_2.$$  

The dynamic setpoint $x_2^*$ is the core of the cascade:

- The steady state is $a_x x_1$.
- Other terms to reduce error $z_1$ using cascaded feedback $(-\alpha z_1)$ and cascaded feedforward ($\lambda_2$).
Global exponential stability

**Theorem 1** Consider an arbitrary piecewise-continuous wheel slip reference \( \lambda^*(t) \).

If \( \lambda^*(t) \) is injected into the filtered setpoint equations and the control law

\[
u = -\gamma_1(\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + c\mu'(x_1)) \lambda_2 - k_1 z_1 - k_2 z_2
\]

is introduced into the system, then a time-varying closed-loop system

\[
\dot{z} = \begin{bmatrix}
-\alpha & 1 \\
-k_1 + a_x \alpha - \alpha^2 + \alpha \eta(t) & -k_2 + \alpha - a_x - \eta(t)
\end{bmatrix} z,
\]

is obtained. If the control gains \( k_1 \) and \( k_2 \) satisfy

\[
k_1 > a_x \alpha - \alpha^2 \quad \text{and} \quad k_2 > \alpha - a_x + \eta_m
\]

then the origin of this closed loop system is globally exponentially stable.
Robustness

**Corollary 1** Consider a constant wheel slip reference $\lambda^*$. If $\lambda^*$ is injected into the filtered setpoint equations and the control law

$$u = -\gamma_1 (\lambda_1 - \lambda^*) + (-\gamma_2 + a_x + c\hat{\mu}'(x_1)) \lambda_2 - k_1 z_1 - k_2 z_2$$

is introduced into the system, then a time-varying closed-loop system

$$\dot{z} = A(t) z + B(t) w \quad \dot{w} = C(t) w$$

is obtained, with the same matrix $A(t)$ as in Theorem 1, and $w = (\lambda_1 - \lambda^*, \lambda_2)$. If the control gains $k_1$ and $k_2$ satisfy the bounds

$$k_1 > a_x \alpha - \alpha^2 \quad \text{and} \quad k_2 > \alpha - a_x + \eta_m$$

of Theorem 1, then the closed loop system is globally exponentially stable.
Simulations — Pure feedforward control

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Simulations — Pure feedback control

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Simulations — With both feedback and feedforward control
Simulations — With a delay of 15 ms
Simulations — With a perturbation of $\mu(\cdot)$
TU Delft’s Tyre Setup

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Experimental validation, with Mathieu Gerard (TU Delft)
Experimental validation, with Mathieu Gerard (TU Delft)
Hybrid five-phase algorithms (WPL – VSD 2006)

\[ \dot{P}_b = -\frac{u_1}{R\omega} \]

1. \( x_2 < 0 \) and \( x_1 < 0 \)

2. \( x_2 \geq \epsilon_1 \)

3. \( x_2 \geq \epsilon_2 \)

4. \( x_2 \leq \epsilon_3 \)

5. \( x_2 \leq -\epsilon_4 \)

\[ \dot{P}_b = -\frac{u_5 x_2}{R\omega} \]

\[ \dot{P}_b = \frac{u_4}{R\omega} \]

\[ \dot{P}_b = 0 \]

\[ \dot{P}_b = \frac{u_3}{R\omega} \]
A method based on first integrals

\[ I_a(x) = x_2 + a \mu(x_1) \]
\[ I_b(x) = x_1 - \frac{1}{2 u_0} x_2^2 \]
\[ I_c(x) = x_2 + a \mu(x_1) - u_0 x_1 \]
Simulation

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Conclusion

- We proposed a new cascaded wheel slip controller.
- It uses a feedforward to speed up convergence, but a perfect knowledge of the tyre is not required (the feedback part does not use it).
- It leads to a proof of global exponential stability of the closed-loop system.
- Robust to practical phenomena (delays, relaxation length, tyre parameters).
- Validated experimentally with a tyre in-the-loop, by Mathieu Gerard (TU Delft).

Perspectives

- A controller that takes into account actuation delays is currently developed.
- The algorithms for computing angular wheel acceleration need to be improved.
Publications


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