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***Abstract*— We consider a decode-and-forward based wireless multihop network with a single source node, a single destination node, and N intermediate nodes. To increase the spectral efficiency and energy efficiency of the system, we propose a cooperative multihop communication with spatial reuse, in which interference is treated as noise. The performance of spatial-reused space-time coded cooperative multihop network is analyzed over Rayleigh fading channels. More specifically, the exact closed-form expression for the outage probability at the nth receiving node is derived when there are multiple interferences over non-i.i.d. Rayleigh fading channels. Moreover, in high SNR scenario, closed-form asymptotic formulas for the outage probability are derived, from which, we show that the full-spatial diversity is still achievable given interferences from the transmission of concurrent packets. In addition, we propose a simple power control scheme which is only dependent on the statistical knowledge of channels. Finally, the analytic results were confirmed by simulations. It is shown by simulations that the spatial-reused multihop transmission outperforms the interference-free multihop transmission in terms of energy efficiency in low and medium SNR scenarios.**

I. Introduction

Cooperative multihop wireless systems have been considered as the promising technique to extend coverage area and reduce power consumption [1], [2]. In [2], three cooperative multihop transmission protocols were proposed that compromise between spectral and energy efficiencies. To further increase of multiplexing gain and energy efficiency, in this paper, we consider a cooperative multihop transmission with interference due to the simultaneous transmission of multiple packets. The idea of multihop transmission with spatial reuse is proposed in [3]. To facilitate concurrent transmission of several packets in the network, the available bandwidth is reused among transmitters, with a minimum division of K nodes between simultaneously transmitting nodes. Therefore, we have to deal with a type of co-channel interference (CCI).

The performance analysis of multihop transmission in Rayleigh fading channels under CCI were recently studied in the number of literature such as [4]–[7]. However, to the best of our knowledge, this is the first work that investigate the performance analysis of multihop networks with multiple interferences over non-i.i.d. Rayleigh fading. This is of primarily importance for the study of interference due to spatial reused

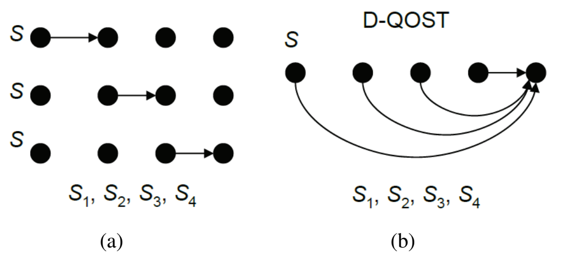

Fig. 1. Multihop-then-Cooperate Protocol: (a) Initial phases; (b) Subsequent phases using a distributed space-time code with $M = 4$.

cooperative multihop transmission.

In this paper, we study the performance analysis of the decode-and-forward based cooperative multihop transmission with interference due to the concurrent transmission of multiple data. We show that the capacity of the cooperative multihop transmission can be improved by using spatial reuse scheme. The achievable rate of the multihop transmission can be increased up to $\lfloor \frac{N+1}{K} \rfloor$ times, where K is the minimum separation of concurrently transmitting nodes in a network with N relays, in expense of performance degradation. Moreover, we derive a closed-form expression for the outage probability of the cooperative multihop system in presence of interferences due to the spatial reuse over Rayleigh fading channels. The simplicity of the calculated expression can give insights on performance of the system and ways to optimize the system. In addition, the asymptotic formulas for different signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) conditions are derived. Furthermore, we formulate the problem of minimizing the transmit power for an outage-restricted equal power multihop network under the the assumption of no instantaneous CSI knowledge at the transmitters.

II. System Model and Protocol Description

Consider a wireless communication network in which the source s intend to transmit its data to the destination d with the help of N cascaded intermediate nodes. Due to the broadcast nature of the wireless channel, some intermediate relays can overhear and retransmit the received packets. The channel between any two nodes in the network is assumed to be a Rayleigh fading. Similar to [1], each transmission could be either a broadcast transmission where one node transmits the signal that is heard by multiple receivers, or a cooperative transmission where multiple nodes concurrently transmit the signal to a single receiving node. Here, we employ the cooperation protocol proposed in [8] where consists of $N+1$ transmission phases. We assume there is no CSI knowledge at transmitters and only statistical CSI is available at the transmitters. Thus, distributed space-time coded transmissions like codes proposed in [9] is a feasible method to be employed for the cooperative transmission.

The transmission protocols have two major phases of non-cooperative and cooperative stages. Depending on the requirements, non-cooperative phase may contain one are multiple steps. The next phases employ space-time cooperated transmission. As an example, Fig. 1 depicts a protocol employing distributed quasi-orthogonal space time code (D-QOST) with $M = 4$, where M is the number of cooperating nodes. The detailed description of cooperative multihop protocols, i.e. Broadcast-then-Cooperate, Multihop-then-Cooperate, and Full-Cooperation, is studied in [10]. To be consistent through the paper, from now on, we consider the Multihop-then-Cooperate protocol illustrated in Fig. 2. However, the proposed procedure can be easily modified using two other protocols. Assuming the usage of full-rate distributed space-time codes, the number of cooperating nodes is equal to the transmitting packets. Hence, the source node intends to transmit M packets to the destination. The signals transmitted by the source terminal during the mth time slot of Phase 1 is denoted as $s_m(t)$, $m = 1, \ldots, M$ where t is the time index and is indicated as a group of M packets transmitted at a given time, and $\mathbb{E}\{s_m(t)\} = 0$ and $\mathbb{E}\{|s_m(t)|^2\} = 1$ for $m = 1, \ldots, M$. In Phase 1, the source transmits the information, and the signal received at the ith node in the first M time slots is given by

$$y_{i,m}(t) = \sqrt{P_{0,1}}\, h_{0,i} s_m(t) + v_{i,m}, \quad \text{for } m = 1, \ldots, M, \quad (1)$$

where $P_{0,1}$ is the average transmit power of the source symbol in the first phase, by assuming that the data symbols $s_m(t)$'s have zero mean and unit variance, and $v_{i,m}$ denotes complex zero-mean white Gaussian noise with variance $\mathcal{N}_0$. The link coefficients from the jth node to the ith node $h_{j,i}$, $j = 0, 1, \ldots, N$, $i = 1, 2 \ldots, N+1$, are complex Gaussian random variables with zero-mean and variances $\sigma^2_{j,i}$, where the $(N+1)$th node is the destination d. We assume coherence times of the channels are such that channel coefficients $h_{j,i}$ are not varying during M consecutive time slots. Note that $\sigma^2_{j,i} = (d_0/d_{j,i})^{\delta}$ is the path-loss coefficient, where $d_{j,i}$ is the distance between nodes j and i, δ is the path-loss exponent, which typically lies in the range of $2 \leq \delta \leq 6$, and d_0 depends on the operating frequency.

For the first $M-1$ phases, we repeat the non-cooperative transmission described above for the nth relay, $1 \leq n < M-1$ to retransmit the source data. After initial stages, the cooperative transmission is utilized for routing the source packets to the destination. In contrast to [1] where transmitters

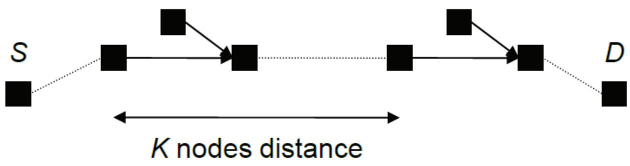

Fig. 2. Spatial-reused cooperative multihop network with $M = 2$.

are able to modify their phases, here, the *instantaneous CSI* is not known at the transmitter nodes. This assumption is realistic for most wireless systems. Hence, space-time coded cooperation is the appropriate choice to achieve the spatial diversity gain. In Phase n, $M \leq n$, the previous M nodes transmit their signals concurrently toward the next node using an appropriate distributed space-time code, with a simple power allocation.

To facilitate simultaneous transmission of several packets in the network, the available bandwidth is reused among transmitters, with a minimum separation of K nodes between concurrently transmitting nodes. Fig. 4 shows a spatial-reused cooperative multihop network with $M = 2$. Since in Phase $n \geq M$, M nodes transmit the intended data to the nth node, and by assuming half-duplex transmission, the minimum value of the spatially-reused factor K is $M+1$, and thus, $M+1 \leq K \leq N+1$. For the message detection, the nth node consider all received signals not coming from the M previous nodes as Gaussian interference. In this work, we treat the interference as additive Gaussian noise. In the presence of inter-network interference from the spatial-reused nodes, for $n \geq M$ the received signal at the n-th receiving node can be represented as

$$y_{n,m} = \sum_{m=1}^{M} P_{n-m,n} h_{n-m,n}\, x_{n,m}(t) + \sum_{u \in \mathcal{U}_n} \sum_{m=1}^{M} P_{u-m,u} h_{u-m,n}\, x_{u,m}(t-\tau) + v_{n,m}, \quad (2)$$

where $x_{n,m}(t)$ is the zero-mean space-time coded signal, normalized as $\mathbb{E}\{|x_{n,m}(t)|^2\} = 1$ during the whole packet transmission, and $P_{n-m,n}$, $m = 1, \ldots, M$, is the average transmit power of node $n-m$ during the mth time slot of Phase n. In (2), $\mathcal{U}_n$ denotes the set of nodes transmitting simultaneously with Nodes $n-m$, $m = 1, \ldots, M$, due to spatial reuse, i.e.,

$$\mathcal{U}_n = \{u \in \{1, 2, \ldots, N+1\} | u \neq n \text{ and } K \text{ divides } n-u\}, \quad (3)$$

and $\tau = \frac{n-u}{K}$. The objective of the system In this system, we are interested in the reliable delivery of messages at a rate of R bits/second/Hertz by consuming the minimum total transmit power.

III. Performance Analysis of Spatial Reused Cooperative Multihop Transmission

In the following, the outage probability $\rho_n^{\text{out}} \triangleq \Pr\{r_n < R\}$ of the nth receiving node at the nth hop in spatial-reused system is derived, which describes the probability that the transmit rate R is larger than the supported rate r_n. This probability depends on the fixed transmission parameters and the channel condition within the hops.

In the cooperative multihop transmission with spatial reuse factor of K, from (1) and (2), the instantaneous achieved rate at the nth hop becomes

$$r_n = \frac{1}{K}\log\left(1+\frac{\sum_{m=1}^{M} P_{n-m,n}|h_{n-m,n}|^2}{\mathcal{N}_0 W + \sum_{u\in\mathcal{U}_n}\sum_{m=1}^{M} P_{u-m,u}|h_{u-m,n}|^2}\right), \tag{4}$$

where $P_{n-i,n} = 0$ for $n = 1,\ldots,M-1$, $i = 2,\ldots,M$.

A. Outage Probability

Now, we calculate an exact closed-form expression for the outage probability at the nth receiving node in presence of interference from the multiple-antenna secondary BS. By defining $\gamma_{\text{th}} \triangleq 2^{RK} - 1$, the outage probability of non-cooperative transmission can be represented as

$$\rho_n^{\text{out}} = \Pr\left\{\frac{\sum_{m=1}^{M} P_{n-m,n}|h_{n-m,n}|^2}{\mathcal{N}_0 W + \sum_{u\in\mathcal{U}_n}\sum_{m=1}^{M} P_{u-m,u}|h_{u-m,n}|^2} < \gamma_{\text{th}}\right\}. \tag{5}$$

Thus, the receiver can reliably decode the source data whenever $r_n \geq R$. For decoding the message correctly, the outage probability must be less than a desired end-to-end outage probability $\rho_{\max}$.

Lemma *1:* Considering a set of independent exponential random variables $\mathcal{X} = \{X_1,\ldots,X_M\}$ with mean of $\sigma_{x_m}^2$, $m = 1,\ldots,M$, the cumulative distribution function (CDF) of the summation of independent-not-identical exponentially distributed random variables, i.e., $X = \sum_{m=1}^{M} X_m$ is given by

$$\Pr\{X < x\} = \sum_{m=1}^{M} \alpha_m \left(1 - e^{\frac{-x}{\sigma_{x_m}^2}}\right) \tag{6}$$

where

$$\alpha_m = \prod_{\substack{j=1\\ j\neq m}}^{M} \frac{\sigma_{x_m}^2}{\sigma_{x_m}^2 - \sigma_{x_j}^2}. \tag{7}$$

Proof: The proof is given in [11]. ■

Using the inductive reasoning, the following lemma can be obtained:

Lemma *2:* For α_m defined in (7), the following properties hold:

$$\sum_{m=1}^{M} \alpha_m = 1, \tag{8}$$

$$\sum_{m=1}^{M} \frac{\alpha_m}{\sigma_{x_m}^{2k}} = 0, \text{ for } k = 1,\ldots,M-1, \tag{9}$$

$$\sum_{m=1}^{M} \frac{\alpha_m}{\sigma_{x_m}^{2M}} = \frac{(-1)^{M+1}}{\prod_{m=1}^{M}\sigma_{x_m}^2}. \tag{10}$$

Proposition *1:* Given finite sets of independent random variables $\mathcal{X} = \{X_1,\ldots,X_M\}$ and $\mathcal{Y} = \{Y_1,\ldots,Y_Q\}$ with non-identical exponential distribution and mean of $\sigma_{x_m}^2$, $m = 1,\ldots,M$, and $\sigma_{y_q}^2$, $q = 1,\ldots,Q$, respectively, the CDF of

$$\text{SINR} = \frac{\sum_{m=1}^{M} X_m}{1 + \sum_{q=1}^{Q} Y_q}$$

can be calculated as

$$P\{\text{SINR} < \gamma\} = 1 - \sum_{m=1}^{M} \alpha_m\, e^{-\frac{\gamma}{\sigma_{x_m}^2}} \prod_{q=1}^{Q}\left(\frac{\sigma_{y_q}^2}{\sigma_{x_m}^2}\gamma + 1\right)^{-1}. \tag{11}$$

Proof: The proof is given in Appendix I. ■

From Proposition 1 and by defining $X_m = \frac{P_{n-m,n}|h_{n-m,n}|^2}{\mathcal{N}_0 W}$, $m = 1,\ldots,M$, $Y_q = \frac{P_{u-m,u}|h_{u-m,n}|^2}{\mathcal{N}_0 W}$, $q = 1,\ldots,Q$, and $Q = |\mathcal{U}_n|M$ where $|\mathcal{U}_n|$ denotes the cardinality of the set $\mathcal{U}_n$, the outage probability in (5) can be written as

$$\rho_n^{\text{out}} = P\{\text{SINR} < \gamma_{\text{th}}\} = 1 - \sum_{m=1}^{M} \alpha_{m,n}\, e^{-\frac{\gamma_{\text{th}}\mathcal{N}_0 W}{P_{n-m,n}\sigma_{n-m,n}^2}} \times \prod_{u\in\mathcal{U}_n}\prod_{i=1}^{M}\left(\frac{P_{u-i,u}\sigma_{u-i,n}^2}{P_{n-m,n}\sigma_{n-m,n}^2}\gamma_{\text{th}} + 1\right)^{-1}, \tag{12}$$

where

$$\alpha_{m,n} = \prod_{\substack{j=1\\ j\neq m}}^{M} \frac{P_{n-m,n}\sigma_{n-m,n}^2}{P_{n-m,n}\sigma_{n-m,n}^2 - P_{n-j,n}\sigma_{n-j,n}^2}. \tag{13}$$

The outage probability ρ_n at the nth receiver is affected by all previous n nodes. An upper-bound expression for the outage probability at the destination, i.e., at the $(N+1)$th hop can be found as [10, Eq. (29)]

$$\rho_{\text{out}} \leq 1 - \prod_{\nu=0}^{N}\left(1 - \rho_{N-\nu+1}^{\text{out}}\right)^{\Omega_M(\nu)}, \tag{14}$$

where $\Omega_M(\nu) = 1$, $0 \leq \nu < M$, and $\Omega_M(\nu) = \sum_{i=1}^{M}\Omega_M(\nu - i)$. Note that for the case of $M = 2$, $\{\Omega_2(\nu)\}$ is a Fibonacci sequence, i.e., $\Omega_2(\nu) = \Omega_2(\nu-1) + \Omega_2(\nu-2)$. In addition, for the extreme case of $M = N+1$, we have $\Omega_{M+1}(\nu) = 2^{\nu-1}$. In addition, when $M = 1$, i.e., in the non-cooperative multihop transmission scenario, we have $\Omega_M(\nu) = 1$, for $\nu = 0,\ldots,M$. It is important to note that assuming the equality in (14) implies that the outage at the destination happens even if one intermediate node experience an error. This guarantees that by using the power control strategies proposed in the next section, the outage probability QoS at the destination is satisfied. To get an insight into the relationship between the end-to-end outage probability of $\rho_{\text{des}} \triangleq \rho_{N+1}$ and ρ_0, we have

$$\rho_{\text{des}} = 1 - \prod_{\nu=0}^{N}(1-\rho_0)^{\Omega_m(\nu)} = 1 - (1-\rho_0)^{\sum_{\nu=0}^{N}\Omega_m(\nu)}. \tag{15}$$

Thus, the target outage probability at each hop ρ_0 can be represented in terms of the desired probability of error at the destination ρ_{des}.

Furthermore, assuming $\rho_{\text{out}} \ll 1$, the outage probability at the destination in (14) can be approximated as follows:

$$\rho_{\text{out}} \approx \sum_{\nu=0}^{N} \Omega(\nu)\, \rho_{N-\nu+1}^{\text{out}}. \tag{16}$$

B. Asymptotic Analysis

Proposition *2:* In high SNR conditions, i.e., when $\text{SNR}_{n,m} \triangleq \frac{P_{n-m,n}\sigma^2_{n-m,n}}{\mathcal{N}_0 W} \gg 1$, and medium or low interference scenario due to spatial-reuse where interference terms is defined as $\text{INR}_{n,u,m} \triangleq \frac{P_{u-m,u}\sigma^2_{u-m,n}}{\mathcal{N}_0 W}$, the outage probability at the nth receiving node can be stated as

$$\rho_n^{\text{out}} \approx \frac{\gamma_{\text{th}}^M}{\prod_{m=1}^{M} \text{SNR}_{n,m}} \sum_{u\in\mathcal{U}_n} \sum_{m=1}^{M} \alpha'_{m,u,n} \sum_{i=0}^{M} \frac{\text{INR}^i_{n,u,m}}{(M-i)!}. \tag{17}$$

where $\alpha'_{m,u,n}$ is defined as

$$\alpha'_{m,u,n} = \prod_{\substack{i\in\mathcal{U}_n \\ (i,j)\neq(u,m)}} \prod_{j=1}^{M} \frac{P_{u-m,u}\sigma^2_{u-m,n}}{P_{u-m,u}\sigma^2_{u-m,n} - P_{i-j,i}\sigma^2_{i-j,n}}. \tag{18}$$

Proof: The proof is given in Appendix II. ■

From Proposition 2, and by using the definition of diversity order $G_d = \lim_{\text{SNR}\to\infty} \frac{-\log(\rho_n^{\text{out}})}{\log(\text{SNR})}$ [12, Eq. (1.19)], we have the following corollary:

Corollary *1:* In a spatial-reused multihop network with cooperation order of M, even in existence of inter-network interference due to spatial reuse, we can achieve the full diversity order of M.

Corollary *2:* In interference-free conditions and high SNR regime, the outage probability in (17) can be modified as

$$\rho_n^{\text{out}} \approx \frac{\gamma_{\text{th}}^M}{M! \prod_{m=1}^{M} \text{SNR}_{n,m}}. \tag{19}$$

From Proposition 1 and by using the facts that $e^{-x} \approx 1-x$ and $\frac{1}{1+x} \approx 1-x$, for $x \ll 1$, we have

Corollary *3:* In high SNR regime, for a non-cooperative spatial-reused multihop transmission, i.e., when $M = 1$, the outage probability can be approximated as

$$\rho_n^{\text{out}} \approx \frac{\gamma_{\text{th}}}{\text{SNR}_{n,1}} \left(1 + \sum_{u\in\mathcal{U}_n} \text{INR}_{n,u,1}\right). \tag{20}$$

If the interference due to spatial reuse is strong, the following corollary can be obtained from Proposition 2:

Corollary *4:* In high SNR and high interference scenario, i.e., when $\text{SNR}_{n,m} \gg 1$ and $\text{INR}_{n,u,m} \gg 1$, the system becomes interference-limited due to an error floor, and the outage probability can be approximated as

$$\rho_n^{\text{out}} \approx 1 - \sum_{m=1}^{M} \prod_{\substack{j=1 \\ j\neq m}}^{M} \frac{\text{SNR}_{n,m}}{\text{SNR}_{n,m} - \text{SNR}_{n,j}} \times \prod_{u\in\mathcal{U}_n} \prod_{i=1}^{M} \left(\frac{\text{INR}_{n,u,i}}{\text{SNR}_{n,m}}\gamma_{\text{th}} + 1\right)^{-1}. \tag{21}$$

IV. Power Allocation for Multihop Transmission with Spatial-Reuse Interference

In the previous section, the impact of spatial-reuse interference on the performance of cooperative multihop system is studied. Here, we introduce a power control strategy to improve the system performance. In this section, we derive the required power for the multihop transmission scheme discussed in Section II in order to achieve a certain rate R with a given an outage probability QoS. Finding the optimum value of the transmit powers is complicated due to the complexity of outage probability $\Pr\{r_n < R\}$ derived in (12) and (17). However, by assuming an equal power in every nodes, in the following, a suboptimal power allocation strategy is proposed.

In the case of interference-free transmission, as stated in [10, Theorem 1], the cooperative transmit power coefficients should be equal in each transmission phase. To get a more accurate result, we further assume equal transmission power in all phases to achieve a target outage probability QoS. Thus, assuming the equal transmit power, i.e., $P_{n-m,n} = P_{u-m,u} = P_0$, for $m = 1,\ldots,M$, $n = 1,\ldots,N+1$, and $u \in \mathcal{U}_n$, we have

$$\rho_n^{\text{out}} = 1 - e^{-\frac{\gamma_{\text{th}}\mathcal{N}_0 W}{P_n \sigma^2_{n-1,n}}} \prod_{u\in\mathcal{U}_n} \prod_{i\in\mathcal{M}_u} \left(\frac{\sigma^2_{u-i,n}}{\sigma^2_{n-1,n}}\gamma_{\text{th}} + 1\right)^{-1}, \tag{22}$$

for $n = 1,\ldots,M-1$, where $\mathcal{M}_u = \{1\}$ if $u < M$, and $\mathcal{M}_u = \{1,2,\ldots,M\}$, if $u \geq M$. For $n = M,\ldots,N+1$, the outage probability can be rewritten as

$$\rho_n^{\text{out}} = 1 - \sum_{m=1}^{M} A_{m,n}\, e^{-\frac{\gamma_{\text{th}}\mathcal{N}_0 W}{P_n \sigma^2_{n-m,n}}}, \tag{23}$$

where

$$A_{m,n} = \prod_{\substack{j=1 \\ j\neq m}}^{M} \frac{\sigma^2_{n-m,n}}{\sigma^2_{n-m,n} - \sigma^2_{n-j,n}} \prod_{u\in\mathcal{U}_n} \prod_{i\in\mathcal{M}_u} \left(\frac{\sigma^2_{u-i,n}}{\sigma^2_{n-m,n}}\gamma_{\text{th}} + 1\right)^{-1}. \tag{24}$$

Since ρ_n^{out} is an decreasing function of the power coefficient P_0 for $P_0 \geq 0$, to find the minimum value of the problem in P_0, the constraint $\rho_n^{\text{out}} \leq \rho_n$ is turned into the equality. Thus, the positive root of $\rho_n^{\text{out}} - \rho_n = 0$ should be calculated. Hence, from (22), for $n = 1,\ldots,M-1$, we have

$$P_n = \frac{-\gamma_{\text{th}}\mathcal{N}_0 W\, \sigma^{-2}_{n-1,n}}{\ln(1-\rho_n) + \sum_{u\in\mathcal{U}_n}\sum_{i\in\mathcal{M}_u} \ln\left(\frac{\sigma^2_{u-i,n}}{\sigma^2_{n-1,n}}\gamma_{\text{th}} + 1\right)}. \tag{25}$$

For $n = M,\ldots,N+1$, and for a given initial value, P_n can be calculated from (23) using the following recursive equation:

$$P_n^{(t+1)} = \frac{-\gamma_{\text{th}}\mathcal{N}_0 W\, \sigma^{-2}_{n-1,n}}{\ln\left[\frac{1-\rho_n}{A_{1,n}} - \sum_{i\in\mathcal{M}_u-\{1\}} \frac{A_{m,n}}{A_{1,n}}\, e^{-\frac{\gamma_{\text{th}}}{P_n^{(t)}\sigma^2_{n-m,n}}}\right]} \tag{26}$$

where $P_n^{(t)}$ is the updated version of the power coefficient in the t-th iteration. Since ρ_n^{out} is a decreasing function of P_n, to guarantee that $\rho_n^{\text{out}} \leq \rho_n$ where ρ_n is a target outage probability per hop, we have

$$P_0 = \max\{P_n^*\}. \tag{27}$$

where P_n^* is the solution of recursive equation in (26). Assuming a fixed per-hop outage target of $\rho_n = \rho_0$ and using (15), we can represent P_0 in terms of ene-to-end outage probability of ρ_{des} by replacing ρ_n with $\rho_0 = 1 - [1-\rho_{\text{des}}]^{\frac{1}{\sum_{\nu=0}^{N}\Omega_m(\nu)}}$ in (26).

Proposition *3:* In the spatial-reused multihop transmission, the minimum allowed target outage requirement at the destination is given by

$$\rho_{\text{des}} \geq 1 - \left[\prod_{n=1}^{M-1}\prod_{u\in\mathcal{U}_n}\prod_{i\in\mathcal{M}_u}\left(\frac{\sigma^2_{u-i,n}}{\sigma^2_{n-1,n}}\gamma_{\text{th}}+1\right)^{-1}\right] \times \prod_{n=M}^{N+1}\left(\sum_{m=1}^{M}A_{m,n}\right)^{\Omega_M(N-n+1)}. \tag{28}$$

Proof: The minimum amount of permissable target outage per hop can be obtained by putting $P_n \to \infty$ in (22) and (23) to get

$$\rho_n \geq 1 - \prod_{u\in\mathcal{U}_n}\prod_{i\in\mathcal{M}_u}\left(\frac{\sigma^2_{u-i,n}}{\sigma^2_{n-1,n}}\gamma_{\text{th}}+1\right)^{-1}, \tag{29}$$

for $n = M,\ldots,N+1$, and

$$\rho_n \geq 1 - \sum_{m=1}^{M}A_{m,n}, \quad \text{for } n = M,\ldots,N+1. \tag{30}$$

Combining (14), (29), and (30), the minimum feasible outage probability QoS at the destination is obtained as (28). ■

In addition, for a given desired outage probability ρ_{des} at the destination, one can find the minimum spatial-reused factor, i.e., nodes distance K, using Proposition 3. Moreover, it can be observed from (8), (28), and (30) that when there is no interference, we have $\rho_{\text{des}} \geq 0$, and thus, there is no limitation in choosing ρ_{des}.

For the case of non-cooperative multihop transmission, i.e., when $M = 1$, the closed-form solution for P_0 is obtained in the following proposition:

Proposition *4:* Assuming the equal power transmission from all nodes, the minimum transmit power P_0^* per node to achieve a per-hop outage probability of ρ_n in a non-cooperative spatial-reused multihop system over Rayleigh fading channels can be expressed as

$$P_0^* = \max_n\left\{\frac{\gamma_{\text{th}}\mathcal{N}_0 W\,\sigma^{-2}_{n-1,n}}{\rho_n - \gamma_{\text{th}}\sum_{u\in\mathcal{U}_n}\frac{\sigma^2_{u-1,n}}{\sigma^2_{n-m,n}}}\right\} \tag{31}$$

Proof: From the approximation given in Corollary 3, which is actually an upper-bound, and by the fact that $P_n = P_0$, for $n = 1,\ldots,N+1$, we have

$$\rho_n^{\text{out}} \leq \frac{\gamma_{\text{th}}\mathcal{N}_0 W}{P_n\sigma^2_{n-m,n}} + \gamma_{\text{th}}\sum_{u\in\mathcal{U}_n}\frac{\sigma^2_{u-1,n}}{\sigma^2_{n-m,n}}. \tag{32}$$

Then, combining (27) and (32), the result in (31) can be yielded. ■

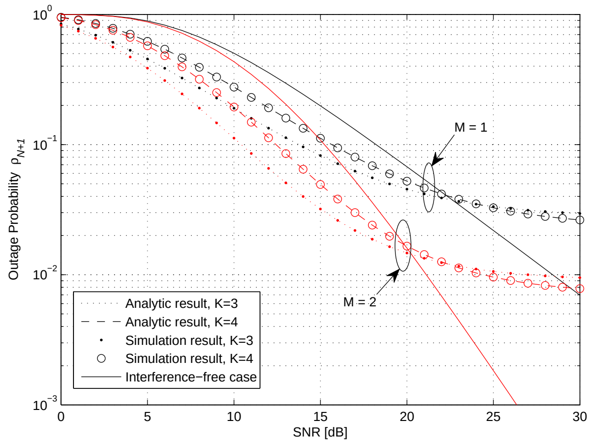

Fig. 3. The outage probability curves versus the transmit SNR in a wireless multihop network with $N = 5$ for non-cooperative ($M = 1$) and cooperative ($M = 2$) cases, different spatial reuse factors, and $R = \frac{1}{2}$ bits/sec/Hz.

V. Numerical Analysis

In this section, numerical results are provided to analyze the performance of the the proposed spatial-reused cooperative multihop scheme. A regular line topology is considered where nodes are located at *unit* distance from each other on a straight line. The optimal non-cooperative transmission in this network is to send the signal to the next closest node in the direction of the destination. Assume that rate R is $\frac{1}{2}$, bandwidth W is normalized to 1, and the path-loss exponent is assumed to be 3.

In Fig. 3, we compare the outage probability curves of the spatial-reused multihop transmission with respect to the interference-free multihop scenario. The depicted curves are outage probabilities at the last transmission phase, i.e., ρ_{N+1}, and the non-cooperative ($M = 1$) and cooperative ($M = 2$) cases with different spatial reuse factors ($K = 3,4$) were compared with interference-free case. As it can be seen, in low and medium SNR regimes, spatial-reused multihop transmission outperforms the interference-free case. For instance, for the cooperative transmission case, when the outage probability of 10^{-1} is required at each step, using the spatial-reused scheme with $K = 3$, around 5 dB saving in transmit power is achievable compared to the interference-free case. However, in high SNR conditions, one can observe that the interference-free transmission performs better than spatial-reused schemes. Since higher K means lower concurrent transmissions, and thus, lower interferences, as K increases, curves get closer to the interference-free case. Furthermore, Fig. 3 confirms the correctness of our analytical results derived in (12), since the curves are exactly match the simulations results.

Appendix I
Proof of Proposition 1

The PDF of Y_q is given as $p_q(y_q) = \frac{e^{-\frac{y}{\sigma^2_{y_q}}}}{\sigma^2_{y_q}}$. Moreover, the CDF of $X = \sum_{m=1}^{M}X_m$ is calculated in Lemma 1. By marginalizing over the independent random variables Y_q, the CDF of SINR can be calculated as

$$
\begin{aligned}
&P\{\text{SINR} < \gamma\} \\
&= \int_{0;Q-\text{fold}}^{\infty} \Pr\left\{X < \gamma + \gamma \sum_{q=1}^{Q} y_q\right\} \prod_{q=1}^{Q} p_q(y_q)\, dy_q \\
&= \sum_{m=1}^{M} \alpha_m \int_{0;Q-\text{fold}}^{\infty} \left(1 - e^{-\frac{\gamma(1+\sum_{q=1}^{Q} y_q)}{\sigma_{x_m}^2}}\right) \prod_{q=1}^{Q} p_q(y_q)\, dy_q \\
&= 1 - \sum_{m=1}^{M} \alpha_m\, e^{-\frac{\gamma}{\sigma_{x_m}^2}} \prod_{q=1}^{Q} \int_0^{\infty} e^{-y_q\left(\frac{\gamma}{\sigma_{x_m}^2} + \frac{1}{\sigma_{y_q}^2}\right)} \frac{dy_q}{\sigma_{y_q}^2}, \quad (33)
\end{aligned}
$$

where in the third equality, we used the first property of Lemma 2 in (8). Thus, the closed-form solution for integral in (33) is obtained as (11).

Appendix II
Proof of Proposition 2

We express the CDF of $X = \sum_{m=1}^{M} X_m$ in Lemma 1 in terms of its Taylor series as

$$
\Pr\{X < x\} = \sum_{m=1}^{M} \alpha_m \sum_{k=1}^{\infty} \frac{-1}{k!}\left(\frac{-x}{\sigma_{x_m}^2}\right)^k. \quad (34)
$$

In addition, $Y = \sum_{q=1}^{Q} Y_q$, where Y_q is defined in Lemma 1, has a distribution similar to X with different parameters, and its PDF can be represented as

$$
p_y(y) = \sum_{q=1}^{Q} \frac{\alpha'_q}{\sigma_{y_q}^2} e^{-\frac{y}{\sigma_{y_q}^2}}, \quad (35)
$$

where $\alpha'_q = \prod_{\substack{j=1\\ j\neq q}}^{Q} \frac{\sigma_{y_q}^2}{\sigma_{y_q}^2 - \sigma_{y_j}^2}$. By marginalizing over the random variable Y and using (34), the integral in (33) can be rewritten as

$$
\begin{aligned}
\Pr\{\text{SINR} < \gamma\} &= \int_0^{\infty} \Pr\{X < \gamma(1+y)\}\, p_y(y)\, dy \\
&= \int_0^{\infty} \sum_{m=1}^{M} \alpha_m \sum_{k=1}^{\infty} \frac{-1}{k!}\left(\frac{-\gamma(1+y)}{\sigma_{x_m}^2}\right)^k p_y(y)\, dy \\
&= \sum_{k=1}^{\infty} \Psi_k \sum_{m=1}^{M} \frac{\alpha_m}{\sigma_{x_m}^{2k}}, \quad (36)
\end{aligned}
$$

where Ψ_k is defined as

$$
\Psi_k = \int_0^{\infty} \frac{(-1)^{k+1}\gamma^k(1+y)^k}{k!}\, p_y(y)\, dy. \quad (37)
$$

Now, by replacing $p_y(y)$ from (35) in (37), we have

$$
\begin{aligned}
\Psi_k &= \int_0^{\infty} \frac{(-1)^{k+1}\gamma^k(1+y)^k}{k!} \sum_{q=1}^{Q} \frac{\alpha'_q}{\sigma_{y_q}^2} e^{-\frac{y}{\sigma_{y_q}^2}}\, dy \\
&= \frac{(-1)^{k+1}\gamma^k}{k!} \sum_{q=1}^{Q} \frac{\alpha'_q}{\sigma_{y_q}^2} \int_0^{\infty} (1+y)^k\, e^{-\frac{y}{\sigma_{y_q}^2}}\, dy \\
&= \frac{(-1)^{k+1}\gamma^k}{k!} \sum_{q=1}^{Q} \frac{\alpha'_q}{\sigma_{y_q}^2} \int_0^{\infty} \sum_{i=0}^{k} \binom{k}{i} y^i\, e^{-\frac{y}{\sigma_{y_q}^2}}\, dy \\
&= (-1)^{k+1}\gamma^k \sum_{q=1}^{Q} \alpha'_q \sum_{i=0}^{k} \frac{\sigma_{y_q}^{2\,i}}{(k-i)!}. \quad (38)
\end{aligned}
$$

where in the third equality, the binomial series expansion of $(1+y)^k$ is used. Combining (36) in (38) and the closed-form solution for the integral is obtained.

Then, by using the second property of Lemma 2, i.e., (9), the first $M-1$ terms in (36) becomes zero, and the outage probability is simplified as

$$
\Pr\{\text{SINR} < \gamma\} = \sum_{k=M}^{\infty} \Psi_k \sum_{m=1}^{M} \frac{\alpha_m}{\sigma_{x_m}^{2k}}, \quad (39)
$$

Finally, by the fact that $\sigma_{x_m}^2 \gg 1$ is equivalent to $\text{SNR}_{n,m} \gg 1$, we can ignore higher order terms, and thus, we have $\Pr\{\text{SINR} < \gamma\} \approx \Psi_M \sum_{m=1}^{M} \frac{\alpha_m}{\sigma_{x_m}^{2M}}$. Using (10), the outage probability can be further simplified as $\Pr\{\text{SINR} < \gamma\} \approx \Psi_M \frac{(-1)^{M+1}}{\prod_{m=1}^{M} \sigma_{x_m}^2}$ Hence, the result in (17) is obtained.

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