An efficient interpolation for calculation of the response of composite layered material and its implementation in MUSIC imaging

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Research context

Main institutions involved

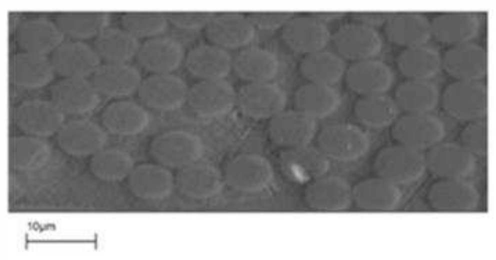
- **Département de Recherche en Électromagnétisme, Laboratoire des Signaux et Systèmes UMR8506**
- **Department of Electrical and Computer Engineering, National University of Singapore**

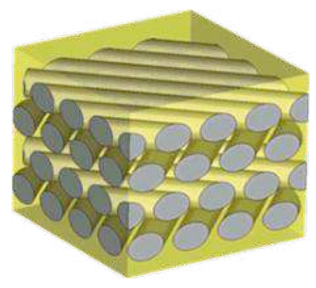
Main financial support

- Bilateral Merlion Project (2011–2012), projet N^{o} 8.14.10 "Fast 3-D electromagnetic imaging of anisotropic media and non-destructive evaluation"
- DIGITEO : PhD Scholarship jointly involving L2S and CEA-LIST (Département Imagerie et Simulation pour le Contrôle)
- Univ Paris-Sud : invited assistant-professor
- Internal ressources (DRÉ & NUS)

To build

- Accurate computational models of complex anisotropic multi-layer composite panels
- Robust, fast, end-user's friendly imaging procedures

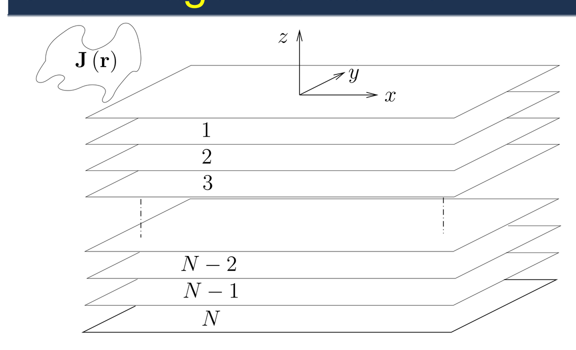




NdT-E from eddy-currents to microwaves in aeronautics and automotive industries

Electromagnetic modeling and preliminary numerical results

Undamaged structure

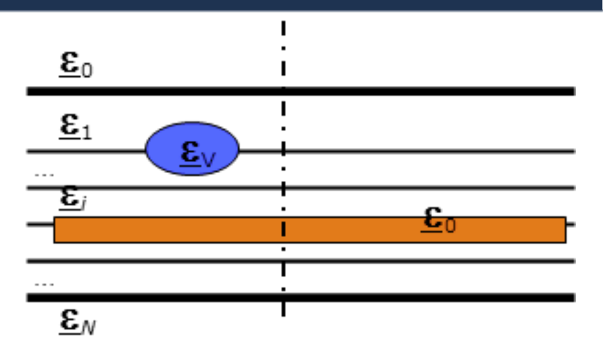


- Each layer: homogeneous *anisotropic* (different e.m. properties from layer to next)
- Uniaxial dielectric (glass-based) or conductive (graphite-based) effective media *large-scale hypothesis*
- *Diagonal* tensor in eigenframes along and orthogonal to fibers' axes
- *Dyadic Green* function in need as well as response to known distributed electric source anywhere in the structure

$$\bar{\bar{\varepsilon}}(\bar{r}) = \mathrm{diag}\left[\varepsilon_{n;11};\varepsilon_{n;22}(\bar{r});\varepsilon_{n;22}(\bar{r})\right] \quad \text{in local coordinate system}$$

Damaged structure

- E.M. parameters differ from background stratified panel within layers or at interfaces
- 3-D volumetric defects (voids, fluid-filled cavities, localized damaged zones, etc.), or delaminations (thin, air-type slabs)
- Method of Moments upon vector contrast-source integral formulations, or change of dyads via supplementary reflection/transmission



Constructing the spectral and spatial response of the laminate (forward modeling)

New recurrence relations based on the propagator matrix method [8]

- To efficiently calculate the spectral response of the laminate
- Capable of stably dealing with distributed source along z
- More efficient compared to the traditional Green's function method
- To numerically solve the state equation $\frac{d}{dz}\bar{\psi}(z) = \bar{\bar{A}}_n \cdot \bar{\psi}(z) + \bar{f}(z)$ with $\bar{\psi}(z)$ containing the tangential components of the fields and $\bar{f}(z)$ being the source term

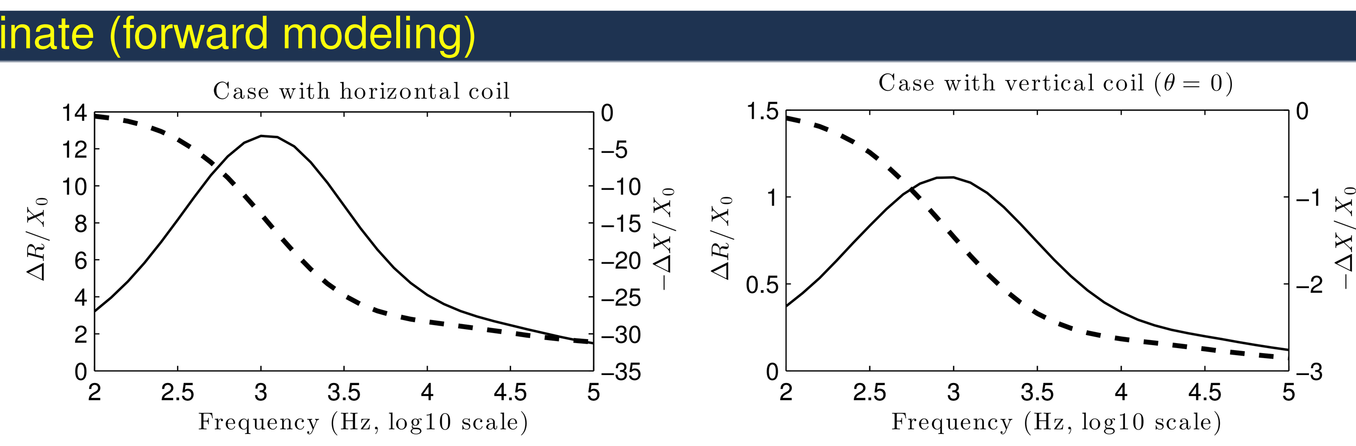


Figure : Reproduction of the results found in [2]

Fast algorithm to calculate the spatial response of the laminate [7]

- To efficiently calculate the spatial response on the rectilinear mesh
- No inverse Fourier transform involved
- Much faster than the inverse Fourier transform
- Incorporating the fast numerical interpolation and integration based on Padua points
- Especially useful when constructing the impedance matrix of the MoM

MUltiple SIgnal Classification (MUSIC) imaging with anisotropic layered media

Interpolation and integration using the Padua points

- Alternative representation as self intersections and boundary contacts of the generating curve $\gamma(t) = (-\cos((n+1)t), -\cos(nt)),\ t \in [0,\pi])$

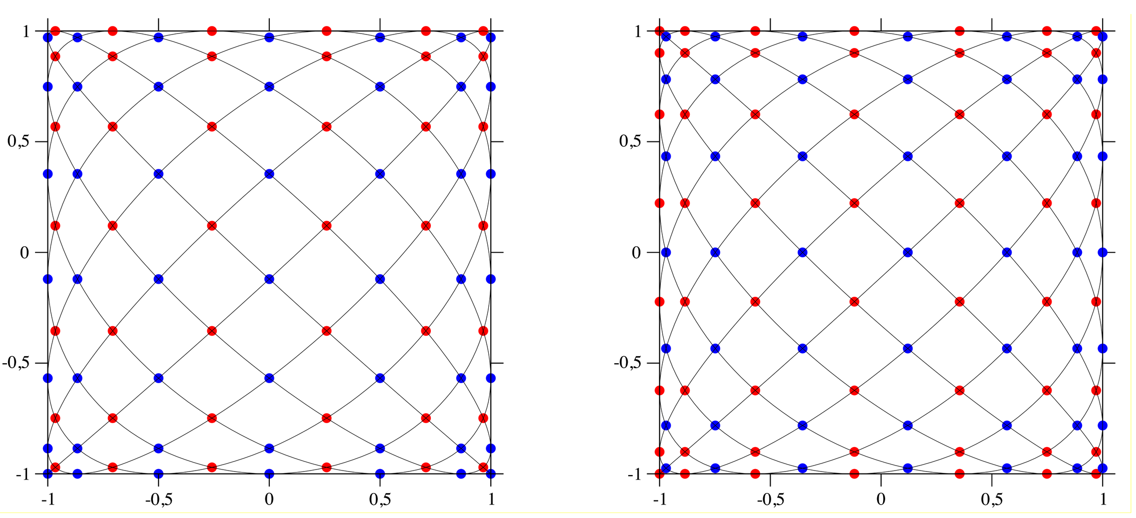


Figure : The Padua points with their generating curve for $n = 12$ (left, 91 points) and $n = 13$ (right, 105 points), also as union of two Chebyshev-Lobatto sub-grids (red and blue bullets). Image taken from [3].

Dealing with fast oscillating integrals

- Goal is to compute the I-FT of fast oscillating spectrum in the $k_x - k_y$ plane

$$G(x,y) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} \tilde{G}_0(k_x,k_y) e^{(ik_x x + ik_y y)}\, dk_x\, dk_y \tag{1}$$

- Interpolation of the non-oscillating part at the Padua points with Chebyshev's polynomial interpolant

$$\mathcal{L}_n \tilde{G}_0(k_x,k_y) = \sum_{k=0}^{n}\sum_{j=0}^{k} c_{j,k-j}\hat{T}_j(k_x)\hat{T}_{k-j}(k_y) - \frac{1}{2}c_{n,0}\hat{T}_n(k_x)T_0(k_y) \tag{2}$$

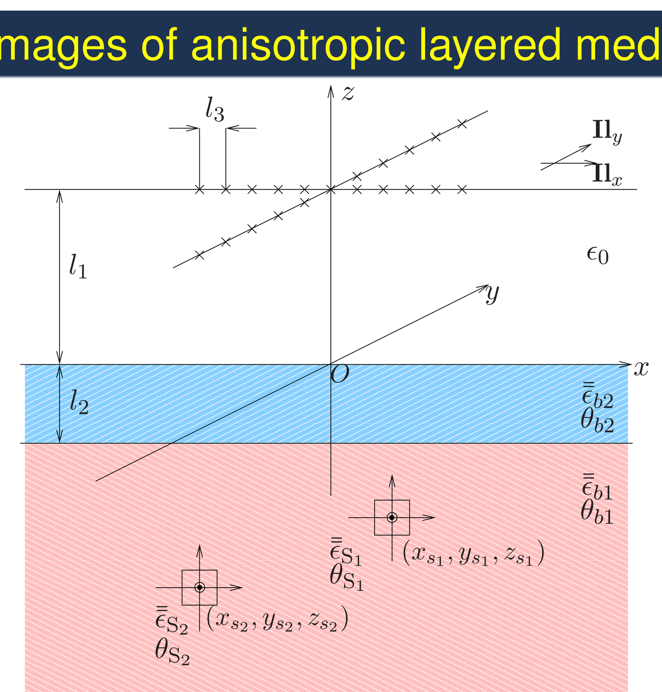
with weights $c_{j,k-j}$ computed using [4]

- Fourier transform of Chebyshev polynomials given by

$$\int_{-1}^{1}\hat{T}_n(k_x)\exp(-ik_x x)\, dk_x \tag{3}$$

are managed using [6] among other good options.

MUSIC images of anisotropic layered media affected by two defects



- A cross type antenna array with 5 antennas on each arm.
- Each antenna as transceivers with x and y polarizations.
- Two small inclusions with dimension of $0.1\lambda_0 \times 0.1\lambda_0 \times 0.1\lambda_0$.

$\bar{\bar{\epsilon}}_{b1} = \mathrm{diag}\,[4.5 + i0.2, 6 + i0.05, 6 + i0.05]$; $\theta_{b1} = 45^\circ$

$\bar{\bar{\epsilon}}_{b2} = \mathrm{diag}\,[2 + i0.3, 3 + i0.1, 3 + i0.1]$ $\quad \theta_{b2} = 60^\circ$

$\bar{\bar{\epsilon}}_{S_1} = \bar{\bar{I}}\epsilon_0 \quad \theta_{S_1} = 0^\circ \quad \theta_{S_2} = 120^\circ$

$\bar{\bar{\epsilon}}_{S_2} = \mathrm{diag}\,[4.5 + i0.2, 6 + i0.05, 6 + i0.05] \quad l_1 = 0.5\lambda_0$

$(x_{S_2}, y_{S_2}, z_{S_2}) = (-0.2\lambda_0, -0.1\lambda_0, -0.55\lambda_0)$ $\quad l_2 = 0.2\lambda_0$

$(x_{S_1}, y_{S_1}, z_{S_1}) = (0.15\lambda_0, 0.2\lambda_0, -0.35\lambda_0)$ $\quad l_3 = 0.2\lambda_0$

- Standard MUSIC imaging method [1]

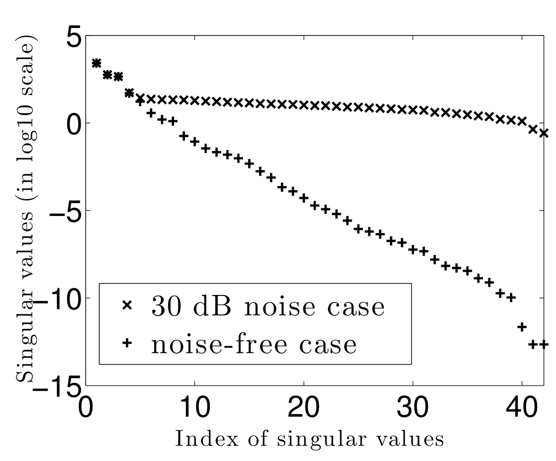
$$\phi(\bar{r}) = \frac{1}{\sum_{\sigma_j<\sigma_L}\sum_{v=1}^{3}\left|\bar{u}_j^* \cdot \bar{G}_v(\bar{r})\right|^2}$$

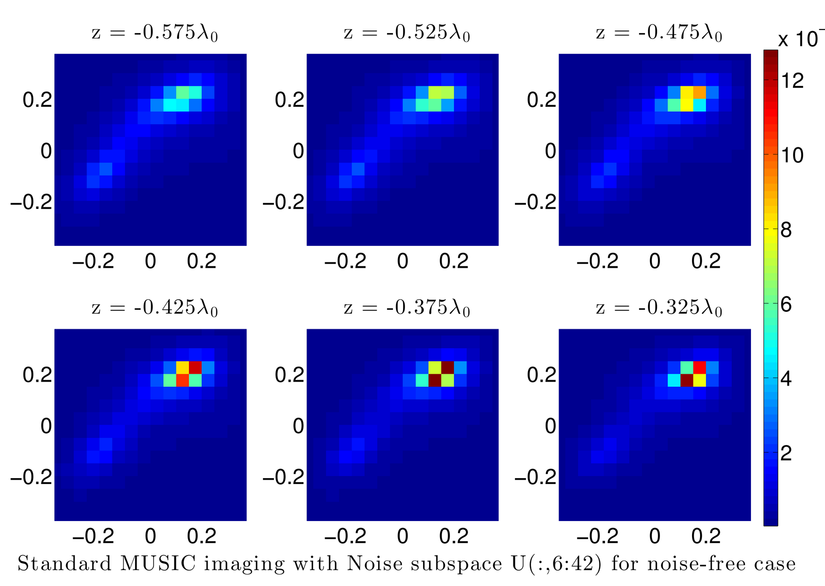
- Enhanced MUSIC imaging method [5]

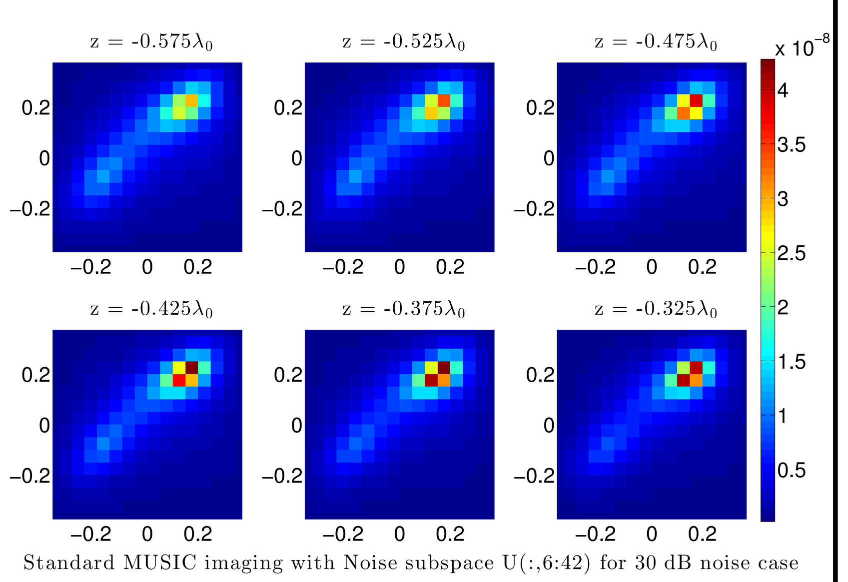
$$\phi(\bar{r}) = \frac{1}{1 - \sum_{\sigma_j>\sigma_L}\left|\bar{u}_j^* \cdot \bar{\bar{G}}(\bar{r}) \cdot \bar{a}_{\mathrm{test}}\right|^2}$$

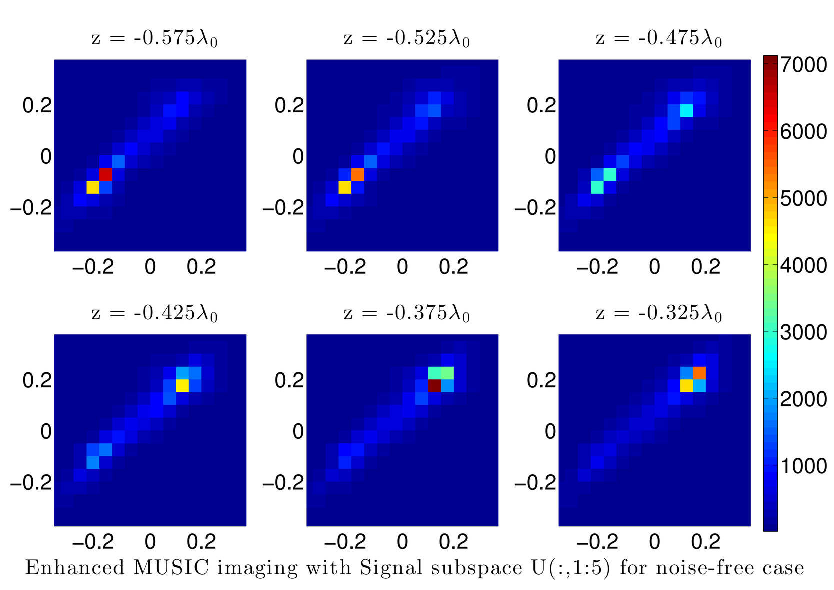
with

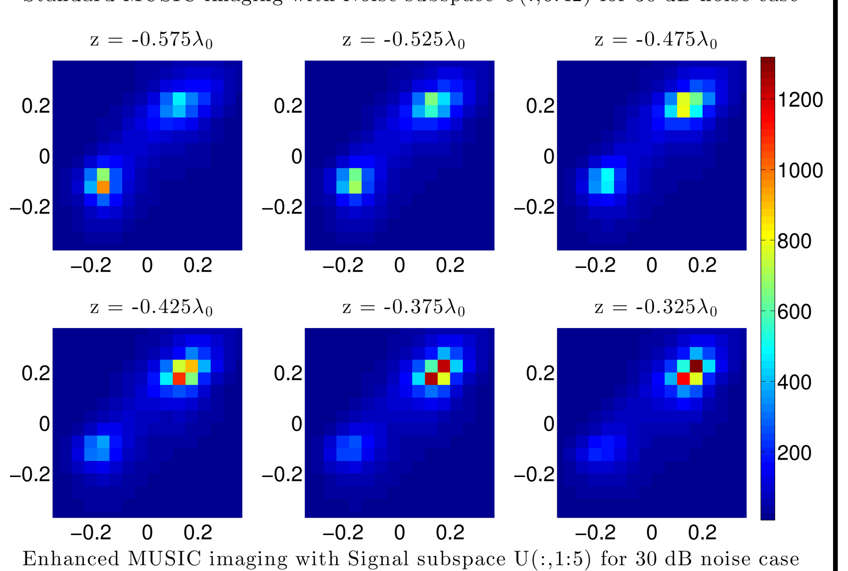
$$\bar{a}_{\mathrm{test}} = \arg\max_{\bar{a}} \frac{\sum_{\sigma_j>\sigma_L}\left|\bar{u}_j^* \cdot \bar{\bar{G}}(\bar{r}) \cdot \bar{a}\right|^2}{\left|\bar{\bar{G}}(\bar{r}) \cdot \bar{a}\right|^2}$$











Conclusions & perspectives

- Numerical integration method based on Padua points is proposed to avoid directly interpolating on the fast oscillating function.
- The approach is validated by comparison with configurations found in the literature
- Green's function constructed by the proposed method is applied in MUSIC imaging.
- Preliminary numerical results show the efficiency of the proposed method in a fully complex anisotropic configuration

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