An efficient interpolation for calculation of the response of composite layered material and its implementation in MUSIC imaging

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***Abstract*—There is always the need to calculate the response of a layered composite material to a source that is not close to the domain of interest when dealing with the imaging of an anomaly that might be affecting such a background medium. If this medium is anisotropic, the availability of an efficient and accurate method to calculate this response becomes essential. A novel interpolation and integration method that is taking care of fast oscillating spectral response due to a source that is not close to the domain of interest is proposed herein. The implementation of such a technique to the multiple signal classification (MUSIC) imaging method is presented also.**

***Index Terms*—Planar-layered media, anisotropy, Green's function, numerical interpolation, imaging, MUSIC**

I. Introduction

Availability of accurate computational models of complex multi-layer composite materials and robust, fast, end-user's friendly imaging procedures for problems of quality, viability, and safety of complex systems is getting quite essential nowadays. From eddy currents to microwaves and beyond, a good example is the non-destructive testing-evaluation (often referred with acronym NdT-E) of manufactured parts in aeronautics and in automotive industry [1], [2].

Often, the structures under investigation can be considered, at least at some first level of modeling, as a succession of planar slabs (panels) one over the other; each slab is usually formed from a bundle of fibers within some polymer matrix, the orientation of the bundles being parallel with the interfaces and usually differing from one slab to the next [3]. The fibers themselves or the way they are organized might lead to either electromagnetic isotropy or electromagnetic anisotropy of the layered material: in the isotropic case, the material parameters in any given layer are described by scalar quantities, while the anisotropic case leads to tensor quantities.

Disorders that might affect these structures are of many kinds: internal cracks and voids, delaminations, fiber breakings, etc., with impact on their electromagnetic behaviors. So, making available to end-users images of the structures in order to indicate the presence, position, and geometric and electromagnetic parameters of a defect is needed. Imaging techniques are then involved, those being usually tailored to the expected electric size of the defect (to be appraised vs. the local wavelengths or skin-depths), and in most cases, the response of the background medium is needed.

A fast algorithm to construct the impedance matrix used in the method of moments is proposed in [5], where the response to a current source of layered anisotropic media on a rectilinear mesh can be efficiently and accurately computed. A sophisticated numerical interpolation and integration method is adopted, based on the recently proposed Padua points [4] involving one-dimensional Chebyshev polynomials.

Given such a fast algorithm, the response of the background layered media on a rectilinear mesh (often used in imaging) to a dipole source of arbitrary orientation and located anywhere is possible. Yet, directly using the fast algorithm in the imaging problem may not be so smart. Indeed, if in [5], since the challenge is to construct the impedance matrix, the current source can be set at the origin of the $x-y$ plane (z as vertical coordinate, all slab interfaces parallel to $x-y$), and the spectrum of the response is a smoothly varying one easily interpolated from a small number of polynomials [4]. However, in imaging, the measurement could be a few wavelengths away from the origin. This means a fast oscillating spectrum of the response, and the interpolation and integration method of [4] is not efficient anymore, since the required number of the polynomials could become quite large.

Applying the Fourier transform property, if the source is shifted away from the origin of the $x-y$ plane, the spectrum of the response is simply the multiplication of the original spectrum for the source at the origin and a sinusoidal term. Using [6], one then extends the method of [4] in order to deal with the oscillating spectrum with a sinusoidal type of oscillation, yielding an efficient and accurate computation of the response of the layered media to a current source not close to the origin. Further on, the technique is implemented in the multiple signal classification (MUSIC) imaging to locate small defects affecting the anisotropic layering.

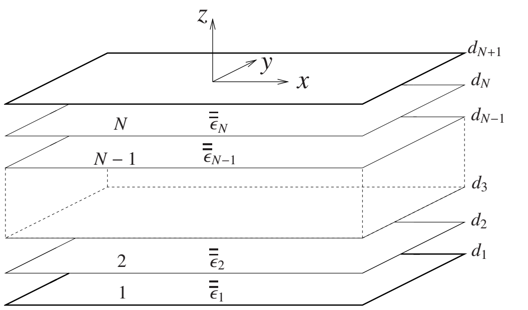

Fig. 1: Planar structure of anisotropic media.

II. Methodology

From now, one considers the physical scenario as sketched in Fig. 1, where each anisotropic slab is characterized by a permittivity tensor. In [5], the response on a rectilinear mesh of such a layering is given when the current source is located close to the origin of the $x - y$ plane as already commented upon. Since the background slabs are invariant along the $x - y$ plane, the electromagnetic response is invariant as well. If one shifts the source from the origin of the $x - y$ plane to a new position, say (x_s, y_s), the spatial response of the layering is not changed referring to the source, and so it is just laterally shifted by (x_s, y_s).

The corresponding change in the spectral domain of the response is a phase shift according to the Fourier transform property, i.e.,

$$\tilde{\eta}(k_x, k_y) = \tilde{\eta}_0(k_x, k_y) \exp\left(-ik_x x_s - ik_y y_s\right) \tag{1}$$

where $\tilde{\eta}(k_x, k_y) = \text{FT}\{\eta_0(x - x_s, y - y_s)\}$ is the spectral response of the layering after shifting the source, and $\tilde{\eta}_0(k_x, k_y) = \text{FT}\{\eta_0(x, y)\}$ is its spectral response when the source is located at the origin of the $x - y$ plane, with $\text{FT}\{\cdot\}$ being the Fourier transform. From this equation, one notices that the new spectrum can be factorized into two terms: the original spectrum and an extra sinusoidally-oscillating term.

In [4], the entire integrand of the integral is interpolated by means of the Chebyshev polynomials and the weights are calculated based on the samplings at Padua points. This facilitates the integration since integrals of the Chebyshev polynomials can be easily calculated as

$$\int_{-1}^{1} \hat{T}_n(x) dx = \begin{cases} 2 & n = 0 \\ 0 & n \ \text{odd} \\ \frac{2\sqrt{2}}{1-n^2} & n \ \text{even} \end{cases} . \tag{2}$$

But, once the integrand is fast oscillating, one needs to separate the oscillating term from other smooth functions, such that the interpolation with the smooth functions can still be efficient. To deal with the integration itself, one needs to manage the Chebyshev polynomials as

$$\int_{-1}^{1} \hat{T}_n(x) \exp(-ixa) dx. \tag{3}$$

Such integrations can be carried out by using the recipe given in [6]. With such a new interpolation and integration technique, the oscillating spectrum can thus be efficiently and accurately taken care of, and the response of the background layered medium obtained subsequently.

III. Implementation into MUSIC

The complex multi-layered composite structures of interest here are as said affected by many kinds of defects. Among the imaging methods which are developed in the literature, the so-called MUSIC algorithm [7] [8] is a good imaging method to locate small inclusions, that is, whose dimensions are much smaller than the local wavelength (in the undamaged material), or skin depth if this medium is essentially conductive.

With MUSIC, one needs to have at hand the dyadic Green's functions that take from the domain of interest (the one in which the defect is sought) to the domain of measurements (where a multistatic response matrix is to be collected). To obtain such functions, the reciprocity theorem means that one can assume the dipole sources being set within the domain of measurements, and then calculate the responses within the domain of interest.

However, when the dipole source is not close to the origin of the $x - y$ plane, the spectrum of the response to such a source could be fast oscillating as aforementioned. So, the method proposed in the previous section could be implemented in such a MUSIC imaging to generate a reliable result as it will be discussed in more depth and illustrated in the extended paper.

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