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***Abstract*—In this work we address the problem of channel probing in a multicarrier downlink wireless network where in order to collect CSI feedback from each user at a channel, a fraction of the available time for transmission is used. This means that the time left to transmit is getting smaller. We study the aspect of stability of such a system and we find a randomized algorithm which can guarantee an expansion of the stability region with respect to full probing and prior works. In addition, we investigate a special case of a probing scheme that does not require knowledge of the statistics of the channels and can still enlarge the stability region of the system. Simulations show the performance of the proposed scheme.**

I. Introduction

User scheduling has emerged as an attractive solution to improve the performance of wireless networks by allocating the resources (timeslots, frequencies) to the users depending on their channel states. Since each user in the network is associated with an incoming traffic process, stability is among the first-order desirable properties (performance metrics) of a scheduler. It roughly means that the mean of all the queue lengths (and consequently delays experienced by the users) in the network is finite. It was shown [1], [2] that MaxWeight types of scheduling policies are throughput optimal i.e. stabilizing the system if it can indeed be stabilized. However, these works assume that the realizations of the channel processes are known to the scheduler at each time slot, which can only be done by feedback from the receivers. The cost, in terms of resources, needed to acquire the instantaneous channel processes is neglected in these works.

On the other hand, in most works tackling the problem of limited feedback (e.g. see [3] and references therein) the focus is on maximizing the total throughput (i.e. assuming constantly backlogged transmitters). Adding the objective of attaining stability, the authors in [4] study the problem of deciding which subset of users to collect feedback from. Also in [5], a CSMA-based scheme is presented for channel state feedback. In the latter two cases however the authors do not take into account the fact that time (e.g. in TDD mode) or frequency (e.g. in FDD mode) resources need to be taken up by probing. Assuming channel statistics are known, the authors in [6] propose a heuristic feedback scheme with two feedback slots based on the idea of maximum quantile scheduling. More in this direction, in [7] it is shown that for a system of L carriers with FDD mode for feedback,the base station needs to acquire at least $\Theta(L)$ channel realizations each time slot to obtain the biggest achievable stability region. In [8], a TDD mode of probing is used: the base station probes the users to feed back their channel states but each such procedure is centralized and takes up a portion of the time slot. Based on optimal stopping theory and assuming that the distributions of the channel gains are known to the base station, the authors derive the general properties of the *centralized* optimal probing policy and completely characterize it in some special cases. Finally, for the same model, the authors in [9] propose a simple feedback scheme for a single channel system. This scheme requires no knowledge of channel and traffic statistics and is shown to guarantee greater stability region than a scheme where all channels are probed. In multi-carrier systems, the probing problem is more challenging since a user may be scheduled on a subset of channels and therefore each user needs to feed back the channel state informations CSIs of a subset (as small as possible) of its channels/subcarriers. Applying the aforementioned schemes to multi-carrier systems will not result in good performance (stability region) as one will see later in this paper.

In this paper, we focus on the downlink of a multi-carrier single cell system with feedback in TDD mode. We assume that the base station schedules the users using a MaxWeight scheduling policy where the weight of each user is its current queue state. We propose a randomized scheme where a threshold for the achievable rate of the channel is adjusted by the base station according to the queue lengths of the users and then users with rate above the threshold feed back in this subcarrier with some probability. We also provide a version of this randomized scheme where the probing probability does not require the channel statistics (which simplifies the implementation of the algorithm) and that still increases the stability region.

The rest of the paper is organized as follows: In Section II we present the system model and describe our proposed probing scheme. In Section III, we provide our stability analysis and prove the expansion of the region compared to existing probing schemes. In Section IV, we describe an approximate probing scheme where the probing probability does not depend on the statistics of the channels. In Section V, we present simulation results to illustrate the performance of the probing schemes and Section VI concludes the paper.

II. Preliminaries

A. Setting and basic notions

In this work we consider a single cell multi-carrier system where a base station serves K users using N subcarriers. Subcarriers are assumed to be randomly time varying, i.i.d.

across time. Let $R_{kn}(t)$ be this rate for user k at subcarrier n. Also they are independent from each other and across users, but not necessarily identically distributed. Time is slotted. Each user $i \in \{1, ..., K\}$ requests randomly incoming traffic with mean rate λ_i. Incoming traffic processes are i.i.d. across time, independent across users and independent with respect to the channel processes . For the MAC layer, the base station maintains a different queue for each user, whose queue length at time slot t is denoted $Q_i(t)$.

Central in our case is the notion of stability of the system. We say that the system is (strongly) stable if for every queue i it holds $\lim_{T\to\infty} \sup \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\{Q_i(t)\} < +\infty$. This implies that the process of queue lengths converges to an ergodic distribution and that the queues (therefore delays) for each user will be finite.

Definition 1 (Stability Region)**.** *The stability region Λ of an algorithm is defined as the set of vectors of the arrival rates for which the system is stable under this algorithm.*

B. Probing scheme

We consider feedback in TDD mode. At the beginning of each timeslot of duration T_s, the users feed back their CSIs and once the feedback procedure is finished the base station schedules a user and transmits in the rest of the slot. This procedure is done per subcarrier. It takes βT_s for a user to feed back its channel state. Also, the base station can broadcast other signalling information but still taking up time of βT_s.

The scheme we propose is essentially a randomized version of the scheme proposed in [9] initially for a single carrier downlink. At each time slot t:

1) At the beginning of the slot, the base station broadcasts pilot signals (of duration that is assumed negligible).
2) The base station requests the user with maximum queue length, say user k^*, to report its subcarriers. After this is done, it broadcasts the channel states at the corresponding subcarriers. This implies that if $U_n(t)$ users in total (that is *including* the user with the maximum queue whose channel states have been requested by the base station) feed back on subcarrier n, transmission will be done for the remaining duration of $(1-\beta(1+U_n(t)))T_s$
3) At each subcarrier n, each user k compares the channel state of this subcarrier with the broadcasted channel state $R_{k^*n}(t)$. If $R_{kn}(t) < R_{k^*n}(t)$, the user does not report its channel state for subcarrier n. Otherwise, he reports the channel state with some probability p.
4) At each subcarrier n, as soon as users have finished reporting, the base station selects the user to schedule using a MaxWeight type of criterion, i.e. is scheduled the user that maximizes the quantity $Q_k(t)(1-\beta(1+U_n(t)))R_{kn}(t)$.

The intuition of introducing this feedback probability in the scheme in [9], termed ”SDF” for the rest of the paper, is that it can be tuned in a way so that fewer users will feed back while still scheduling good users for transmission on each subcarrier. In the remainder of the paper, the proposed scheme will be denoted as ”pSDF”. Also, we will refer to the quantity $Q_i(t)R_{in}(t)$ as ”weight” of user i in subcarrier n.

C. Preliminary results and definitions

Define $Z_{kn}(t)$ the scheduling decision at time slot t (i.e. $Z_{kn}(t) = 1$ if user k is scheduled on channel n at time slot t and zero otherwise) for the SDF scheme. Note that $Z_{kn}(t)$ is the same schedule as MaxWeight scheduling when all the channels were known [9]. Since at most one user can be scheduled on a subcarrier, $Z_{kn}(t) = 1$ only for the user with the maximum weight at subcarrier n.

A tilde over the variables will indicate that they correspond to the pSDF scheme. Also, boldface letters will denote vectors. Unless stated otherwise, all expectations in the remainder of the paper are taken over the stationary distribution of the channel states and the feedback decisions taken (for the case of pSDF).

Define the following quantities, which essentially correspond to the average total utility function of the system under the SDF and pSDF schemes (in other words, the MaxWeight scheduler assign the users such that the instantaneous total utility is maximized):

$$f(\mathbf{Q}(t)) =$$
$$\mathbb{E}\{\sum_{n=1}^{N}(1-\beta(U_n(t)+1))\sum_{i=1}^{K} Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\},$$
$$\tilde{f}(\mathbf{Q}(t)) =$$
$$\mathbb{E}\{\sum_{n=1}^{N}(1-\beta(\tilde{U}_n(t)+1))\sum_{i=1}^{K} Q_i(t)R_{in}(t)\tilde{Z}_{in}(t)|\mathbf{Q}(t)\}.$$

Notice also that these quantities correspond to negative part of the drift of the quadratic Lyapunov function under the SDF and pSDF schemes. The expectation is taken with respect to the randomness of channel variation and scheduling decisions. Then, the following holds (see [10], also e.g. [9], [7]):

Theorem 2. *If there exists an $\epsilon > 0$ such that for every queue length vector $\mathbf{Q}(t)$ it holds*

$$\frac{\tilde{f}(\mathbf{Q}(t))}{f(\mathbf{Q}(t))} \geq 1+\epsilon,$$

then the stability region of pSDF is guaranteed to increase at least to $(1+\epsilon)$ times the stability region of SDF.

We denote the expectations of the maximum weights at channel n as $W_n(t) = \mathbb{E}\left\{\sum_{i=1}^{K} Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\right\} = \mathbb{E}\left\{max(Q_i(t)R_{in}(t))|\mathbf{Q}(t)\right\}$. Also, let $W(t) := \sum_{n=1}^{N} W_n(t)$. For the algorithm where every user is probed at every slot (referred to as ”full feedback scheme” hereafter) define $\hat{f}(\mathbf{Q}(t)) = \mathbb{E}\{\sum_{n=1}^{N}(1-\beta K)\sum_{i=1}^{K} Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\}$. Finally, we denote the number of users not feeding back at time slot t and subcarrier n as $M_n(t)$ in the SDF scheme and $\tilde{M}_n(t)$ in the pSDF scheme. Then, $M_n(t) = K - U_n(t)$ and similarly for $\tilde{M}_n(t)$. Finally, note that for the whole analysis to make sense there must be $\beta < 1/3$. This can be justified because if $\beta = 1/3$ then there is no use to broadcast the rate of the user with the maximum queue length: If at least a user reports back then no time is left for transmission and if no user reports back then we lose time to send data to the user probed first.

III. Increasing the Stability Region with pSDF

In this section we work on the case where there is enough time in the slot for each user to probe every channel, i.e. $(1-\beta K)>0$. An important intermediate result follows:

Lemma 1. *For any vector of queue lengths, the following holds:*

$$\frac{\tilde{f}(\mathbf{Q}(t))}{f(\mathbf{Q}(t))} \geq \frac{1+r(\mathbf{Q}(t),p)\epsilon}{1+\epsilon} \tag{1}$$

where $\epsilon>0$ is the increase of the stability region guaranteed by SDF with respect to full probing and

$$\begin{aligned} r(\mathbf{Q}(t),p) = &(1-(K-2)S(\mathbf{Q}(t)))\,p^2 \\ &+\frac{1-2\beta}{\beta}S(\mathbf{Q}(t))p - \frac{1-\beta K}{\beta}S(\mathbf{Q}(t)). \end{aligned} \tag{2}$$

In the above,

$$S(t)=\frac{W(t)}{\sum_{n=1}^{N}\left(\mathbb{E}(M_n(t)|\mathbf{Q}(t))-1\right)W_n(t)}. \tag{3}$$

Proof:

Note that the schedule decided in SDF and full probing schemes (after probing has been done) is the same, picking the user with the maximum product $R_{in}(t)Q_i(t)$ in every subchannel n [9]. Note also that this value does not depend on the number of users probing each channel in SDF algorithm, which implies that its expectation is independent of the expectation of the number of users probing. Then, we have $f(\mathbf{Q}(t)) = \hat{f}(\mathbf{Q}(t)) + \beta\sum_{n=1}^{N}(\mathbb{E}\left\{M_n(t)|\mathbf{Q}(t)\right\}-1)W_n(t)$,and therefore

$$\frac{f(\mathbf{Q}(t))}{\hat{f}(\mathbf{Q}(t))} = 1+\frac{\beta\sum_{n=1}^{N}(\mathbb{E}\left\{M_n(t)|\mathbf{Q}(t)\right\}-1)W_n(t)}{\hat{f}(\mathbf{Q}(t))} := 1+\epsilon \tag{4}$$

with $\epsilon>0$.

Now we will do the same procedure for the quantities in pSDF. Since now the user with the maximum weight is not guaranteed to probe the channel, we cannot proceed as above. However, a lower bound can be found considering the following: For every channel n, if the user with the maximum weight has probed then is scheduled, otherwise no user is scheduled. This is a lower bound since even if the user with the maximum weight is not probed there will be some other user with nonzero rate scheduled with some probability.. Denoting $\tilde{\mathcal{U}}_n(t)$ the set of the users that have probed the channel at slot t and by $\tilde{\mathcal{M}}_n(t)$ the set of users that have not, we have

$$\begin{aligned} &\tilde{f}(\mathbf{Q}(t)) \geq \\ &\mathbb{E}\left\{\sum_{n=1}^{N}(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{U}}_n(t)}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\right\} \\ &=\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i=1}^{K}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\right\}- \\ &\sum_{n=1}^{N}\mathbb{E}\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{M}}_n(t)}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\}- \\ &\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{M}}_n(t)}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\right\} \\ &\geq \hat{f}(\mathbf{Q}(t))+\beta\sum_{n=1}^{N}(\mathbb{E}\left\{\tilde{M}_n(t)|\mathbf{Q}(t)\right\}-1)W_n(t)- \\ &\sum_{n=1}^{N}\mathbb{E}\left\{(1-\beta(\tilde{U}_n(t)+1))\sum_{i\in\tilde{\mathcal{M}}_n(t)}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t})\right\} \end{aligned} \tag{5}$$

To proceed further, we use that in pSDF a user among the $U_n(t)-1$ (i.e. excluding the user polled by the base station) whose channel is better than the broadcasted feeds back with probability p independently of anything else, therefore the average number of users that feed back after the threshold has been set will be $p\mathbb{E}\left\{U_n(t)-1|\mathbf{Q}(t)\right\}$. So

$$\mathbb{E}\left\{\tilde{U}_n(t)|\mathbf{Q}(t)\right\} = 1+p(K-1-\mathbb{E}\left\{M_n(t)|\mathbf{Q}(t)\right\}). \tag{6}$$

Now consider the second sum in (5) and denote $\mathcal{X}_n$ the event that the user with the maximum queue is *not* the user with the maximum weight in subcarrier n. If this event happens, the user with the maximum weight has not been probed so the sum over $i\in\tilde{\mathcal{M}}_n(t)$ is the maximum weight over this subcarrier. Also, note that if $\mathcal{X}_n$ does happen, the probability that the sum $i\in\tilde{\mathcal{M}}_n(t)$ being nonzero is $1-p$, since the user with the maximum weight will not feed back with this probability. Denote thus $\mathcal{X}'_n$) the event where the user with the maximum weight does not feed back given the event $\mathcal{X}_n$ does happens. There is $\mathbb{P}(\mathcal{X}'_n|\mathcal{X}_n)=1-p$. Then, the sum (denoted S_2) can be written as $S_2=\sum_{n=1}^{N}\mathbb{P}(\mathcal{X}_n)\mathbb{P}(\mathcal{X}'_n|\mathcal{X}_n)\mathbb{E}\{(1-\beta(\tilde{U}_n(t)+1))$ $\sum_{i\in\tilde{\mathcal{M}}_n(t)}Q_i(t)R_{in}(t)Z_{in}(t)|\mathbf{Q}(\mathbf{t}),\mathcal{X}_n,\mathcal{X}'_n\}$ $=$ $\sum_{n=1}^{N}(1-p)\mathbb{P}(\mathcal{X}_n)(1-\beta(\mathbb{E}\left\{\tilde{U}_n(t)\right\}+1))W_n(t)$ $\leq$ $(1-p)\sum_{n=1}^{N}(1-\beta(\mathbb{E}\left\{\tilde{U}_n(t)\right\}+1))W_n(t)$. Here, we have used that the expectation is conditioned on the fact that the user with the maximum weight does not feed back, therefore is contained in the set $\tilde{M}_n(t)$ and that each user feeds back independently.

Therefore, applying the above in (5) and using (6), we obtain:

$$\tilde{f}(\mathbf{Q}(t)) \geq \hat{f}(\mathbf{Q}(t)) + \left(\sum_{n=1}^{N}(\mathbb{E}\left\{M_n(t)|\mathbf{Q}(t)\right\}-1)W_n(t)-(K-2)W(t)\right)\beta p^2 + p(1-2\beta)W(t)-(1-\beta K)W(t). \tag{7}$$

The stated result follows combining the above with (4). ■

Using Lemma 1 and Theorem 2 we get the following:

Theorem 3. *If $r(\mathbf{Q}(t),p)>1, \forall \mathbf{Q}(t)$ then the stability region is guaranteed to increase with respect to the SDF algorithm. Moreover, this guaranteed increase is the biggest for feedback probability*

$$p^* = \min\left\{1, \frac{1-2\beta}{2\beta}\frac{S(\mathbf{Q}(t))}{(K-2)S(\mathbf{Q}(t))-1}\right\}. \tag{8}$$

Proof: Assume that for every $\mathbf{Q}(t)$, $r(\mathbf{Q}(t),p) \geq 1+\delta(p)>1$. Then, denoting $\epsilon' = \frac{\epsilon\delta(p)}{1+\epsilon}$, from Lemma 1 it follows that $\frac{\tilde{f}(\mathbf{Q}(t))}{f(\mathbf{Q}(t))} > 1+\epsilon'$, and using Theorem 1 we conclude that the stability region of pSDF is $(1+\epsilon')$ times bigger the stability region of SDF. Also, note that the ratio (1) is increasing in $r(\mathbf{Q}(t),p)$, which in turn is concave in p. Therefore optimizing over it we get the stated result. We skip the detailed derivation of p^* due to space limitations. ■

It is has to be noted that optimizing according to the above result implies that $S(\mathbf{Q}(t))$ is known at every time slot. This assumes that the probability distributions of the channels are known and requires some complexity computation since this quantity $S(\mathbf{Q}(t))$ should be frequently updated. Therefore, we will present in the next section a simple version of our algorithm that guarantees an expansion of the stability region at least with respect to full probing when the distribution of the channels is identical among users. The interest in this case is that the probability p will be independent of $S(\mathbf{Q}(t))$.

In the above analysis we have implicitly assumed that $\mathbb{E}\{M_n(t)|\mathbf{Q}(t)\}>1$ for each subcarrier. Recall that K, being the number of users, can take only positive integer values. From [9] we have that for every subcarrier, $\mathbb{E}\{M_n(t)\} \geq \frac{1}{2}(1+\frac{1}{L})(K-1)$. Therefore, the assumption holds for every $K \geq 3$, i.e. whenever there are at least three users in the system. This is the case where it actually makes sense to use SDF/pSDF kind of schemes. Indeed, for the case where $K=1$ there is essentially no scheduling problem. For $K=2$, a scheme where every user feeds back is always better than the proposed one and SDF since both require a fraction of timeslot for the base station to broadcast the channel states of the user with maximum queue.

IV. Approximate pSDF

In order to simplify the implementation of our probing algorithm, we provide in this section an algorithm where the probing probability p does not depend on quantity $S(\mathbf{Q}(t))$. We will consider the case where the channels are homogeneous, that is when the distribution of the rates at a subcarrier n is the same for all users. Further, let us consider the case where the achievable rates are uniformly distributed. Denote by $M_{uni} := \mathbb{E}\left\{M_n(t)|\mathbf{Q}(t), \text{uniform channel distribution}\right\} = (1/2+1/(2L))(K-1)$ (relation given in [9]). By (7), (8) and (3), it follows that the increase in the stability region *guaranteed* by pSDF with respect to full probing in the case with homogeneous is increasing as $\mathbb{E}\{M_n(t)|\mathbf{Q}(t)\}$ increases. For each subcarrier, we can prove that $M_{uni} \leq \mathbb{E}\left\{M_n(t)|\mathbf{Q}(t)\right\}$ for any possible distribution of the channel states, in other words the case with uniform channel distribution has the worst lower bound gain with respect to the full probing case. The detailed proof of this result is not provided here since it can be obtained directly from the results in [9]. Therefore, examining this case gives a lower bound on the *guaranteed* achievable improvement with respect to full probing in the case of homogeneous channels .

From the analysis in the previous Section it follows:

Corollary 4. *When channel rates are uniformly distributed the feedback probability that maximizes the guaranteed enlargement of the stability region is given as* $p^*_{uni} = \min\left\{1, \frac{1-2\beta}{2\beta}\frac{2L}{2KL+K-3L-1}\right\}$.

Proof: Since the achievable rates are now identically distributed among subcarriers and users, it will also hold that $W_n(t)=W'(t)$, therefore $W(t)=\sum_{n=1}^{N}W_n(t)=NW'(t)$. So in the uniform distribution case, $S(t)=\frac{NW'(t)}{W'(t)N(M_{uni}-1)} = \frac{1}{(K-1)\left(\frac{L+1}{2L}-1\right)}$. Replacing in (8) we get the stated result. ■

An attractive property of the feedback probability in this case is that it reduces the implementation complexity of pSDF in practice. Therefore, and even if the distributions of the channel states are not homogeneous, we propose in this section to simplify the implementation of the pSDF by using p^*_{uni} given above instead of p^* given by (8). We call this algorithm "approximate pSDF". Also, note that for homogeneous but not uniform channel there will be $p^* > p^*_{uni}$, which implies that the approximate method will probe fewer channels. This follows from the fact that p^* is decreasing in $\mathbb{E}\{M_n(t)|\mathbf{Q}(t)\}$ (see equations (3) and (8)), which is the smallest possible in the case of uniform distribution.

V. Simulation Results and Discussion

In order to illustrate the gains and operation of the algorithm we will consider a single cell downlink with $N=15$ channels. The channels are assumed to be i.i.d. across users, frequencies, and time slots and the achievable rates (in bits per time slot) are as in Table I (the rates are calculated according to the LTE specifications, with $T_s = 1ms$).

We set the traffic patterns to be i.i.d. Poisson, with the same arrival rate, λ bits per slot for each user. We run simulations lasting 10000 time slots each for different arrival rates and plot the average total queue length at each simulation for SDF, randomized SDF with probing probability as derived in Section III (denoted "Optimized pSDF") and the approximate probability as set in Section IV.

At first we simulate the system with $\beta=0.1$ and $K=9$ users. In this case full probing is possible. The results are plotted in Fig. 1. We can see that the randomized version of the algorithm obtained via optimizing the upper bound is the same as SDF here, while the probability of probing in the approximate algorithm is smaller. Also, the performance of the approximate algorithm is better from the other two.

In Fig. 2 we present the results of a scenario with $K = 25$ users and two different values of β, namely 0.05 and 0.01. Note that in both of these cases full probing is not possible. Again, the approximate version of the algorithm has a lower probing probability than the version that optimizes the upper bound and performs better. In turn, the latter version performs better than SDF. Also, from Figures 1 and 2 we can see that the stability region of the system shrinks under all algorithms as β and/or the number of users K grow larger. In the case of SDF this happens because as these parameters grow larger, more time needs to be devoted to channel reporting, leaving fewer time for transmission. However, in the randomized versions the main reason for the rate decrease is that it becomes more possible that the user with maximum weight will not report his channel and subsequently another user will be scheduled instead of him. As we can see in the figures, the decrease in the stability region is slower in the case of the randomized algorithms. This demonstrates that there is a gain with respect to SDF algorithm and moreover that the relative gains of randomizing the SDF algorithm are bigger when there are more users and/or channel probing is more costly.

The main reason why the approximate algorithm outperforms the other pSDF is that the bound to which the optimized probability corresponds to is not tight. In fact, the theoretical analysis in this paper has been done in terms of region increase guarantee and this has been studied using the lower bounds developed in the previous sections. These lower bounds are not necessarily tight which means that the real expansion is higher than the lower bound. Recall that in the course of derivation of equation (2), the quantity was bounded assuming implicitly that (i) if the user with the maximum weight has not probed a channel then no user is scheduled in the channel and (ii) the user polled by the base station is never the user with the maximum weight. As seen previously the approximate probability p^*_{uni} is less or equal that the one obtained through full optimization.

VI. Conclusions

We examined a randomized channel feedback algorithm that expands the stability region in a multi-carrier system. We have obtained a lower bound of the expansion of the region and found the optimal feedback probability that maximizes this bound. Our probing scheme ensures thus a region increase guarantee. We provided also a simple version of our algorithm that simplifies the implementation of the feedback scheme by finding a feedback probability that does not depend on the system statistics and can achieve also a very good performance. Further issues to be studied include finding the maximum stability region with distributed probing as well as looking at practical ways to implement the probing scheme taking into account delay constraints.

Rate (bits/slot):	0	25	39	63	101	147	197	248
Probability:	0.03	0.04	0.05	0.05	0.06	0.06	0.09	0.09
Rate (bits/slot):	321	404	458	558	655	759	859	933
Probability:	0.1	0.1	0.09	0.06	0.06	0.05	0.04	0.03

TABLE I

ACHIEVABLE RATES AND PROBABILITIES USED FOR THE SIMULATIONS

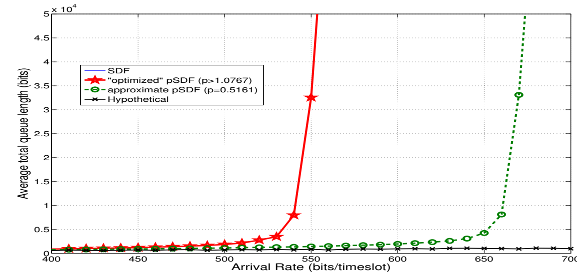

Fig. 1. Average Total Queue Length for Different Mean Arrival Rates for 9 users and $\beta = 0.1$

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References

[1] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936 –1948, Dec. 1992.

[2] A. L. Stolyar, "Maxweight scheduling in a generalized switch: State space collapse and workload minimization in heavy traffic," *Annals of Applied Probability*, vol. 14, no. 1, pp. 1–53, Nov. 2004.

[3] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1341 –1365, Oct. 2008.

[4] A. Gopalan, C. Caramanis, and S. Shakkottai, "On wireless scheduling with partial channel-state information," *IEEE Transactions on Information Theory*, vol. 58, no. 1, pp. 403 –420, Jan. 2012.

[5] B. Li and A. Eryilmaz, "Distributed channel probing for efficient transmission scheduling over wireless fading channels," in *Proc. of IEEE INFOCOM 2012*, Mar. 2012, pp. 3026 –3030.

[6] S. J. Baek and G. de Veciana, "Opportunistic feedback and scheduling to reduce packet delays in heterogeneous wireless systems," *IEEE Trans. Vehic. Technol.*, vol. 61, no. 7, pp. 3282 –3289, Sept. 2012.

[7] M. Ouyang and L. Ying, "On optimal feedback allocation in multichannel wireless downlinks," in *Proc. ACM MobiHoc '10*, 2010, pp. 241–250.

[8] P. Chaporkar, A. Proutiere, H. Asnani, and A. Karandikar, "Scheduling with limited information in wireless systems," in *Proc. ACM Mobihoc '09*.

[9] M. Karaca, Y. Sarikaya, Ö. Erçetin, T. Alpcan, and H. Boche, "Throughput optimal scheduling with dynamic channel feedback," *ArXiv e-prints*, 2012.

[10] A. Eryilmaz, R. Srikant, and J. R. Perkins, "Stable scheduling policies for fading wireless channels," *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 411–424, Apr. 2005.

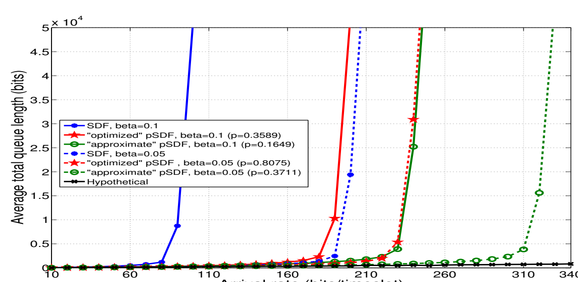

Fig. 2. Average Total Queue Length for Different Mean Arrival Rates for 25 users and different values of β