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Abstract**:** To describefailure propagation dynamics in complex dynamical communication networks, we propose an efficient and compartmental standard-exception-failure propagation dynamics model based on the method of modeling disease propagation in social networks. Mathematical formulas are derived and differential equations are solved to analyze the equilibrium of the propagation dynamics. Stability is evaluated in terms of the balance factor *G* and it is shown that equilibrium where the number of nodes in different states does not change, is globally asymptotically stable if $G \geq 1$. The theoretical results derived are verified by numerical simulations. We also investigate the effect of some network parameters, e.g. node density and node movement speed, on the failure propagation dynamics in complex dynamical communication networks to gain insights for effective measures of control of the scale and duration of the failure propagation in complex dynamical communication networks.

Keywords: dynamics model, fault propagation; equilibrium analysis; standard-exception-failure propagation model; complex dynamical communication network.

1. Introduction

A complex dynamical network (CDN) is modelled by nodes interconnected by edges, in which each node represents a dynamical systemand the edges represent their coupling relationships. Many real systems can be described by CDNs, such as Internet, social relationship networks, metabolic networks, food chains, disease propagation networks[1-3]. Because of the complexityand the coupling between nodes, there are many challenges in CDN modeling and analysis. Synchronization of the network, network performance and reliability assessment, failure propagation dynamics have attracted increasing attention[4-6]. As a special CDN, the complex dynamical communication network (CDCN) is also facing these challenges and failure propagation dynamics has been one of the main topics of interest due to its various applications[7].

Various models of failure propagation dynamics in complex networks have been put forward, such as capacity-load model, binary model, sand pile model and coupled map lattice model[8-12]. However, these models do not consider the node dynamics, which have great effect on the propagation of faults. Reference[13] uses Bayesian networks to build a fault propagation model for a mobile ad hoc network (MANET), in which the dynamics of the topology is taken into consideration. But the number of nodes in the network is only 15, which cannot reflect the general situation of large scale network systems. Since the computation complexity is large and heavily depends on the UAFAReS architecture proposed in[14].

To study the propagation dynamics in CDN, many works have been made on modeling the dynamic rules[15-25]. To study the propagation dynamics of disease, reference[24] formulates a compartmental susceptible – exposed – infectious – susceptible with vaccination (that is, anti-virus treatment) (SEIS-V) epidemic transmission model of worms in a computer network with natural death rate. A threshold point of the modified reproductive number has been found by a mathematical analysis ofthe equilibrium state.A novel epidemic model of computer viruses has been established based on the classical SIRS modeland its dynamical properties have been investigated intensively [25].

It is common sense that the disease is spread by contact in social networks and through edges between the normal node and the infected node in internet or computer networks. On the contrary, the propagation of faults in complex dynamical communication networks is caused by the cumulative effect of the stress resulting from the data stream and the influence of environmental factors. Then, the propagation mechanism is different and the propagation dynamics of computer networks and social networks cannot reflect the actual situation.

In this paper, we propose a novel dynamics model (standard-exception-failure, SEF) for studying the propagation dynamics of faults in CDCNs. The rest of the paper is organized as follows. The

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proposed mathematical model is described in Section 2. Section 3 presents the mathematical analysis of two equilibrium points. The numerical simulations are conducted in Section 4 to verify the theoretical analysis and study the propagation dynamics rules in CDCNs. Finally, in Section 5, we provide some general conclusions on failure propagation dynamics in CDCNs and indicate possible measures that can be used to control the propagation of fault in CDCNs.

2. Modeling failure propagation dynamics in CDCN

To build the model of failure propagation dynamics in CDCNs, the node dynamics model is required. The mobility model that we use is the random walk mobility model with parameters*V* and *θ*[26,27]:the new position $\left(x\left(t+\tau \right), y\left(t+\tau \right) \right)$ of node*i* over time is determined at regular time intervals *τ*as a function of *V* and *θ*, using the following equation:

$$\begin{cases} x\left(t+\tau \right) = \phi\left(x\left(t \right) + V \times \cos\theta\left(\tau \right), L \right) \\ y\left(t+\tau \right) = \phi\left(y\left(t \right) + V \times \sin\theta\left(\tau \right), L \right) \end{cases} \tag{1}$$

where

$$\phi\left(A, B \right) = \begin{cases} -A, & A < 0 \\ A, & 0 \le A \le B \\ 2B - A, & B < A < 2B \end{cases} \tag{2}$$

where*L* is the length of the simulation area, *V* is a constant value determined by the actual situation in the study, *θ*(*τ*) obeys the uniform distribution between 0 and 2π. We assume that the node can only connect with other nodes within the range of *R* around it. Because the node moves in the area according to the mobility model presented above, the neighbors of each node change with time, and the adjacent matrix **M(t)**that determines the network topology at time *t*is decided by the distance *d* between different nodes and the communication radius*R* of nodes.

$$\mathbf{M}\left(t \right) = \begin{bmatrix} M_{11}\left(t \right) & M_{12}\left(t \right) & \cdots & M_{1n}\left(t \right) \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1}\left(t \right) & M_{n2}\left(t \right) & \cdots & M_{nn}\left(t \right) \end{bmatrix} \tag{3}$$

where

$$M_{ij}(t) = \begin{cases} 0, & d_{ij}(t) > R \\ 1, & d_{ij}(t) \le R \end{cases} \tag{4}$$

$$d_{ij}(t) = \sqrt{\left(x_i(t) - x_j(t) \right)^2 + \left(y_i(t) - y_j(t) \right)^2} \tag{5}$$

where $d_{ij}(t)$ is the distance between the node *i* and the node *j* at time *t*. The neighbors of a node are determined by the value of $d_{ij}(t)$ and *R*. The application scenario is presented in Fig.1.

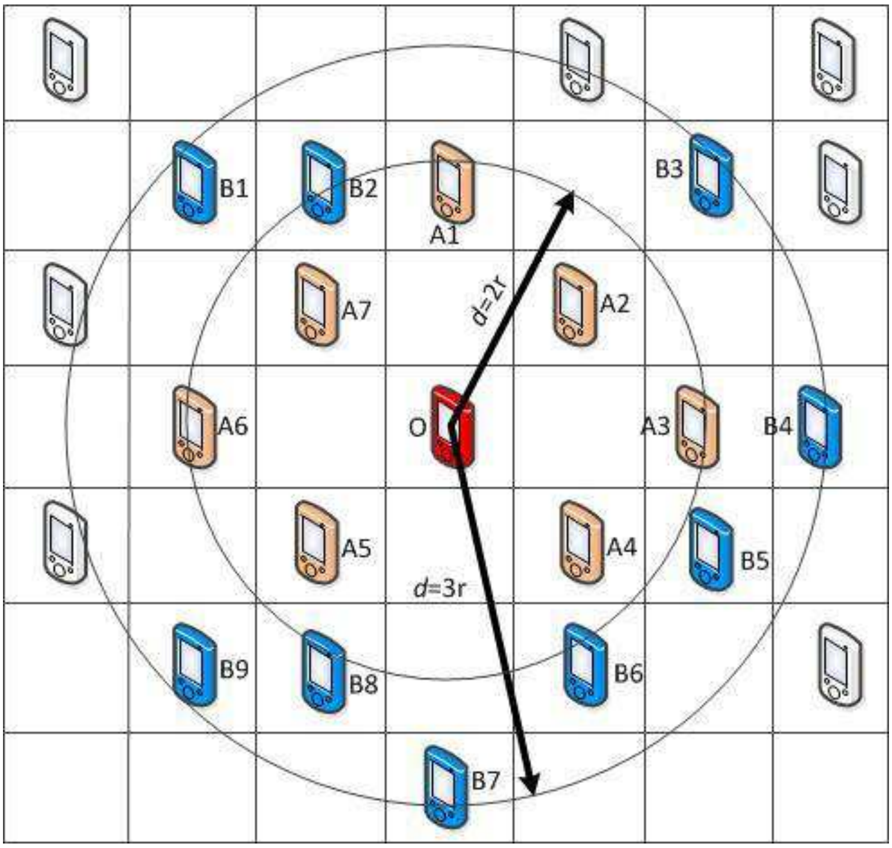

Fig.1. The application scenario. Mobile nodes Ai (i=1,2,…,7) are the neighbors of node O when the communication radius R=d=2*r*. Both mobile nodes Ai and Bj (j=1,2,…,9) are the neighbors of node O when the communication radius R=d=3*r*. *r* is the cell side.

To model the propagation of faults through the CDCN, we assume that the total number of nodes in the network is N. Each node changes over time among three states: Standard (S), Exception (E) and Failure (F) due to the spreading of faults. We describe these three states and the state transitions among them in details as follows.

Standard(S): A node can move and keep contact with neighbors normally. It can be affected by the mobility and number of nodes that in exception state around it. Moreover, it will suffer from the failure of general electronic products.

Exception (E): A node can move normal in the area, but it may drop packets because of the mobility or buffer overflow. It will also suffer from the failure of general electronic products and the failure caused by its neighbor nodes.

Failure (F): A node can move according to the mobility model, but it cannot connect with the neighbor nodes. Since the general equipment will have certain protective measures and control strategies in the design, It can be restored to normal state with a given probability.

Let $S(t)$, $E(t)$, $F(t)$ be the number of nodes in state S, E and F at time t, respectively. Then, at any time t, we have:

$$S(t) + E(t) + F(t) = N \tag{6}$$

From the above description, the state transition relationship of fault propagation in the model can be presented in Fig.2, with the description of the related parameters in Table1.

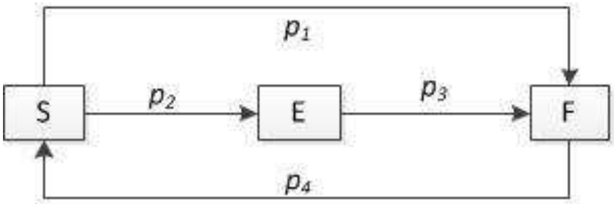

Fig. 2. State transition relationship for fault propagation in SEF model.

Table 1
Parameters description

Parameter	Meaning
p_1	Probability with which a node in state S becomes a node in state F
p_2	Probability with which a node in state S becomes a node in state E
p_3	Probability with which a node in state E becomes a node in state F
p_4	Probability with which a node in state F becomes a node in state S

Ordinary differential equations can be obtained from the state transition relationship in Fig. 2, which govern the state evolution with time:

$$\begin{cases} \frac{dS(t)}{dt} = -\left(p_1 + p_2\right)S(t) + p_4 F(t) \\ \frac{dE(t)}{dt} = p_2 S(t) - p_3 E(t) \\ \frac{dF(t)}{dt} = p_1 S(t) + p_3 E(t) - p_4 F(t) \end{cases} \tag{7}$$

For discrete simulation, the above ordinary differential equations can be transformed into differenceequations in discrete time steps:

$$\begin{cases} S(n+1) = \left(1 - p_1 - p_2\right)S(n) + p_4 F(n) \\ E(n+1) = p_2 S(n) + \left(1 - p_3\right)E(n) \\ F\left(n+1\right) = p_1 S\left(n\right) + p_3 E\left(n\right) + \left(1 - p_4\right)F(n) \end{cases} \tag{8}$$

In the following, we will discuss the value of $p_i\left(i = 1,2,3,4\right)$. To simplify and without loss of generality, we assume that the failure rate of the normal nodes and exception nodes obey exponential distributions with parameters λ and β ($\beta \geq \lambda$). We assume that the probability of the node restoring to the standard state from the fault state also obeys an exponential distribution with parameter μ. Considering the effect of node mobility on the state of node, in order to highlight the influence of node moving speed, we assume that the probability p_2 is determined by the ratio between the number of exception nodes that the node can contact while it is moving and that it can contact when it is stationary in one step and the proportion of exception nodes.Then, the state transition probabilities are,

$$\begin{cases} p_1 = \lambda \\ p_2 = k \times \frac{V \times 2R \times E(n)/L^2}{\pi \times R^2 \times E\left(n\right)/L^2} \times \frac{E(n)}{N} = k \times \frac{2V}{\pi R} \times \frac{E(n)}{N} \\ p_3 = \beta \\ p_4 = \mu \end{cases} \tag{9}$$

where$k$ is an adjustment coefficient that can be determined according to the network parameters. In order to simply the problem. Hence, the equations (8) can be rewritten as follows:

$$\begin{cases} \Delta S(n) = -\lambda S(n) - \frac{2kV}{\pi RN} E(n)S(n) + \mu F(n) \\ \Delta E(n) = \frac{2kV}{\pi RN} E(n)S(n) - \beta E(n) \\ \Delta F(n) = \lambda S(n) + \beta E(n) - \mu \mathrm{F(n)} \end{cases} \tag{10}$$

The system of equation (10) is to be solved with the following initial conditions:

$$\begin{cases} 0 \leq S(n) \leq N \\ 0 \leq E(n) \leq N \\ 0 \leq F(n) \leq N \\ 0 \leq p_i \leq 1, \quad i = 1,2,3,4 \\ p_1 + p_2 \leq 1 \end{cases} \tag{11}$$

3. Mathematical analysis for system stability

Explicit mathematical analysis can provide the theoretical foundation for predicting the propagation dynamics. In this section, we will find the equilibria of equation(10) and investigate their dynamical properties.

In order to isolate the parameters that do not change with time step and study the equilibrium of the system,we define balance factor *G* as follows:

$$G = \frac{2k\mu V}{(\lambda+\mu)\beta\pi R} \tag{12}$$

At equilibrium

$$\begin{cases} S(n+1) = S(n) = S^* \\ E(n+1) = E(n) = E^* \\ F(n+1) = F(n) = F^* \end{cases} \tag{13}$$

Solving (10) with consideration of (13), we find that

$$Q_1^* = (S_1^*, E_1^*, F_1^*) = \left(\frac{\mu}{\lambda+\mu} \mathrm{N}, 0, \frac{\lambda}{\lambda+\mu} N \right) \tag{14}$$

and

$$Q_2^* = (S_2^*, E_2^*, F_2^*) = \left(\frac{\beta\pi R}{2kV} N, \frac{2k\mu V - (\lambda+\mu)\beta\pi R}{2k(\mu+\beta)V} N, \frac{2k\beta V - (\beta-\lambda)\beta\pi R}{2k(\mu+\beta)V} N \right) \tag{15}$$

The characteristic equation at Q_1^* is:

$$\det \begin{pmatrix} -\lambda - x & -\frac{2kV}{\pi RN} S_1^* & \mu \\ 0 & \frac{2kV}{\pi RN} S_1^* - \beta - x & 0 \\ \lambda & \beta & -\mu - x \end{pmatrix} = 0 \tag{16}$$

The characteristic roots of (16) are: $x_1 = 0$, $x_2 = -\lambda - \mu$, $x_3 = \frac{2k\mu V - \beta(\lambda+\mu)\pi R}{(\lambda+\mu)\pi R}$. Combining (12), x_3 can be rewritten as $x_3 = \beta(G-1)$.

Obviously, if $G \le 1$, then $x_3 \le 0$, (10) has no positive real part characteristic roots. According to Routh Hurwitz stability criterion, the equilibrium Q_1^* of (10) is locally asymptotically stable.If G>1, then $x_3 > 0$ and there is a positive real part characteristic root of (10). According to Routh Hurwitz stability criterion, the equilibrium Q_1^* of (10) is unstable.

Similarly, the characteristic equation of (10) at Q_2^* is:

$$\det \begin{pmatrix} -\lambda - \frac{2kV}{\pi RN} E_2^* - x & -\frac{2kV}{\pi RN} S_2^* & \mu \\ \frac{2kV}{\pi RN} E_2^* & \frac{2kV}{\pi RN} S_2^* - \beta - x & 0 \\ \lambda & \beta & -\mu - x \end{pmatrix} = 0 \tag{17}$$

which is equivalent to

$$x\left[x^2 + \frac{2k\mu V + \mu(\lambda+\mu)\pi R}{(\mu+\beta)\pi R} x + \frac{2k\mu V - \beta(\lambda+\mu)\pi R}{\pi R} \right] = 0 \tag{18}$$

Equation (18) has a characteristic root $x_1 = 0$ and the roots of the equation,

$$x^2 + \frac{2k\mu V + \mu(\lambda+\mu)\pi R}{(\mu+\beta)\pi R} x + \frac{2k\mu V - \beta(\lambda+\mu)\pi R}{\pi R} = 0 \tag{19}$$

Obviously, (19) has two real roots x_2 and x_3, and $x_2 + x_3 = -\frac{2k\mu V + \mu(\lambda+\mu)\pi R}{(\mu+\beta)\pi R} < 0$, $x_2 x_3 = \frac{2k\mu V - \beta(\lambda+\mu)\pi R}{\pi R}$. Combining (12), $x_2 x_3$ can be rewritten as $x_2 x_3 = \beta(\lambda+\mu)(G-1)$.

If G≥1, then $x_2 x_3 \ge 0$, so both x_2 and x_3 are negative real roots and (10) has no positive real part characteristic roots. According to Routh Hurwitz stability criterion, the equilibrium Q_2^* of (10) is locally asymptotically stable.On the contrary, if G<1, then $x_2 x_3 < 0$ and there is a positive real part characteristic root of (10). According to Routh Hurwitz stability criterion, the equilibrium Q_2^* of (10) is unstable.

Hence, we can obtain the following conclusions:

(a) If $G \le 1$, then the equilibrium Q_1^* of (10) is locally asymptotically stable.

(b) If G>1, then the equilibrium Q_1^* of (10) is unstable.

(c) If G≥1, then the equilibrium Q_2^* of (10) is locally asymptotically stable.

(d) If G<1, then the equilibrium Q_2^* of (10) is unstable.

4. Simulation results and discussions

We evaluate the feasibility of the proposed scheme using cellular automata to simulate the dynamics of fault propagation in CDCNs, and verify the effectiveness and rationality of the proposed model for describing the failure propagation in CDCNs. To this aim, aMatlab simulator has been implemented. In the simulator, the wireless nodes are deployed into a 30 × 30 regular grid, and the length of each grid is 1. We define the node density as the number of nodes in unite space and the expression is $D = N/L^2$.

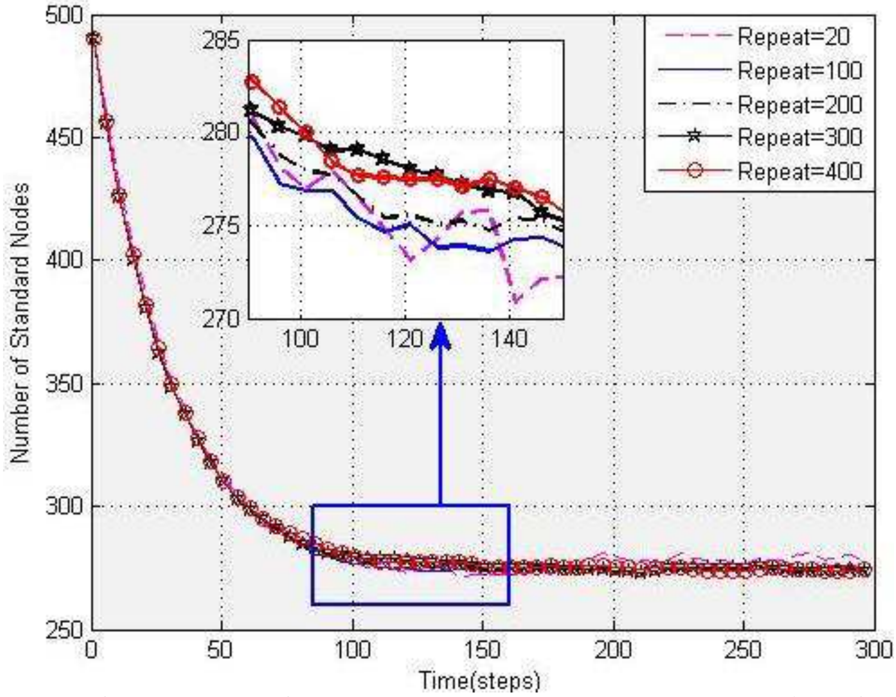

Fig.3. Mean number of standard nodes in a failure propagation dynamics simulation. We use *V*=3,*R*=3,*λ*=0.01,*β*=0.02,*μ*=0.02, and a lattice size *L*=30. The node density is *D*=5/9. Averages over 20, 100, 200, 300 and 400 simulations are shown.

Because of the stochastic nature of the simulation process, each realization of a propagation sequence will be different. For this reason, we take the mean and variance of a large number of simulations and see how these quantities stabilize as the ensemble of simulations increases in number. At each time step we have a mean number of nodes in standard state on the lattice and the associated variance. In Fig. 3,the evolution of the mean number (MN) of standard nodes is shown. The node density is D=5/9. An average over 20,100, 200, 300 and 400 separate simulations was taken to obtain the five profiles in the Figure. Above 300 simulations the mean has stabilized and increasing the number of runs any further add very little to the numerical accuracy of the result.

Fig.4 shows the associated standard deviation (SD) as a function of time for each of the graphs in Fig. 3. Again, this quantity stabilizes above 300 simulations. Whilst we cannot predict the outcome of an individual realization of a simulation, the mean and variance at least give us statistical estimators of the likely standard node quantity as a function of time. From this approach, we can begin to investigate whether a given node density is likely to maintain the fault in a propagation state. From this, we infer that the node density of 5/9 is beyond the critical node threshold. As when using any stochastic model, it is necessary to bear in mind the expected size of variance of the non-fault node when estimating quantities such as the time-to-extinction or the critical community size.

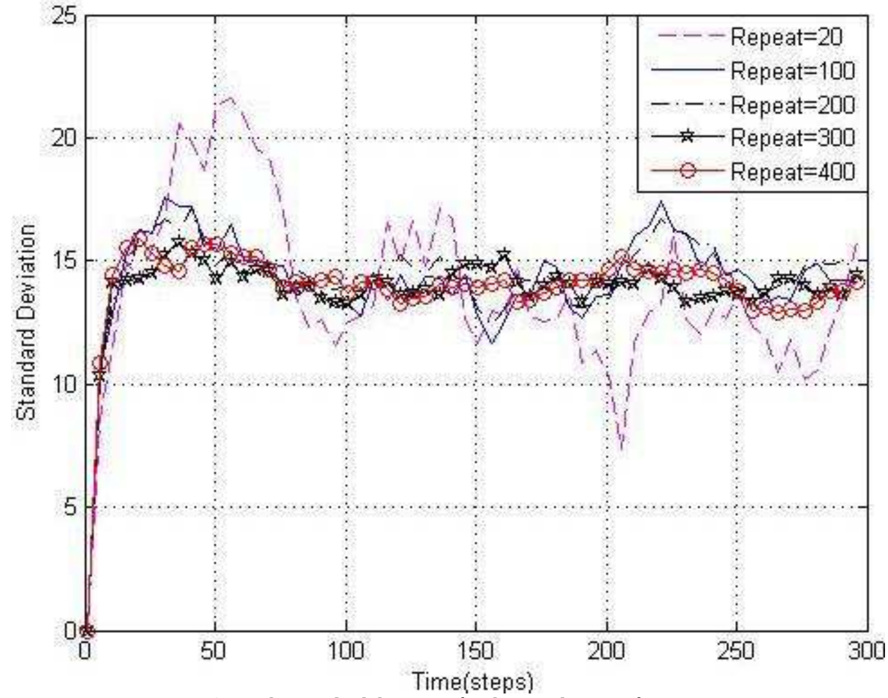

Fig.4. The standard deviation for the simulations of Fig.3.

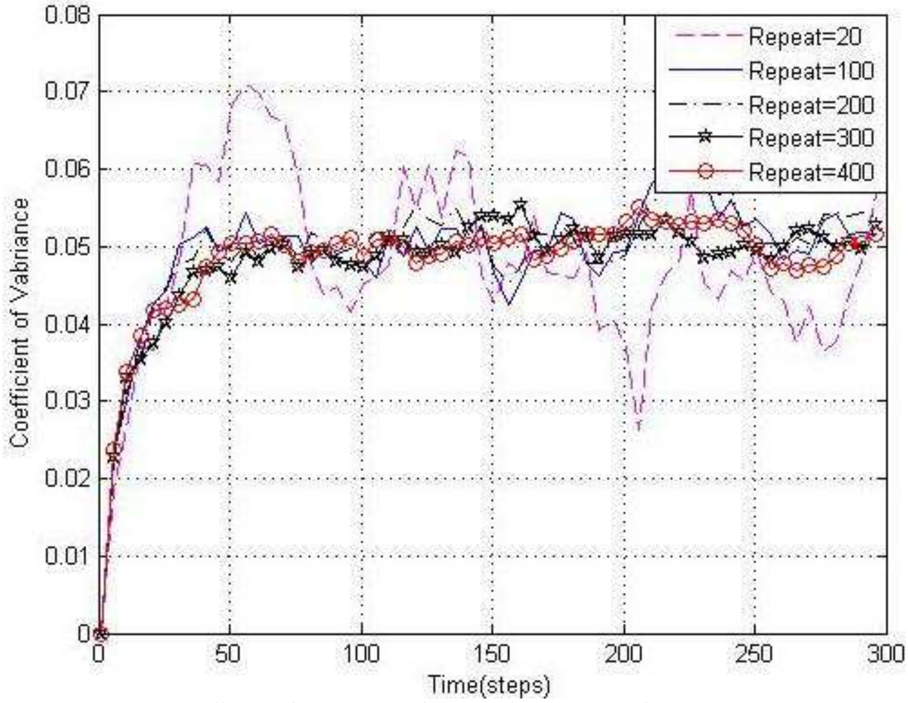

Fig.5. The coefficient of variation as a function of time obtained from Fig.3 and Fig.4.

As the magnitude of the variance is related to the infective population size, it is useful to look at the coefficient of variation, that is $CV(t) = SD(t)/MN(t)$. In Fig. 5,*CV*(*t*) as a function of *t* is plotted. Clearly this quantity converges and is less than 1 $\forall t>0$. The implication of this is that the fault (including exception and failure) will not die out in the CDCN.

The evolutions on the numbers of standard, exception and failure nodes with different simulation parameters are presented in Fig.6 and Fig.7. It can be found that the increase rate of failure nodes is larger than that of exception nodes before t<30, since the number of exception nodes is small, which causes the transition probability from standardto exception to be smaller than the transition probability to failure. Also, it can be calculated that G=33.3is larger than $\pi R/2V$=2.355 with the parameters used in Fig.6, while the value calculated with the parameters used in Fig.7 is G=3.33 smaller than$\pi R/2V$=7.065. According to the conclusions (c) and (d), it is easy to explain the different phenomenon. And it can be implied that it is not enough to enhance the repair ability in order to make the system work at a stable and efficient state, but it is also necessary to improve the reliability of the nodes.

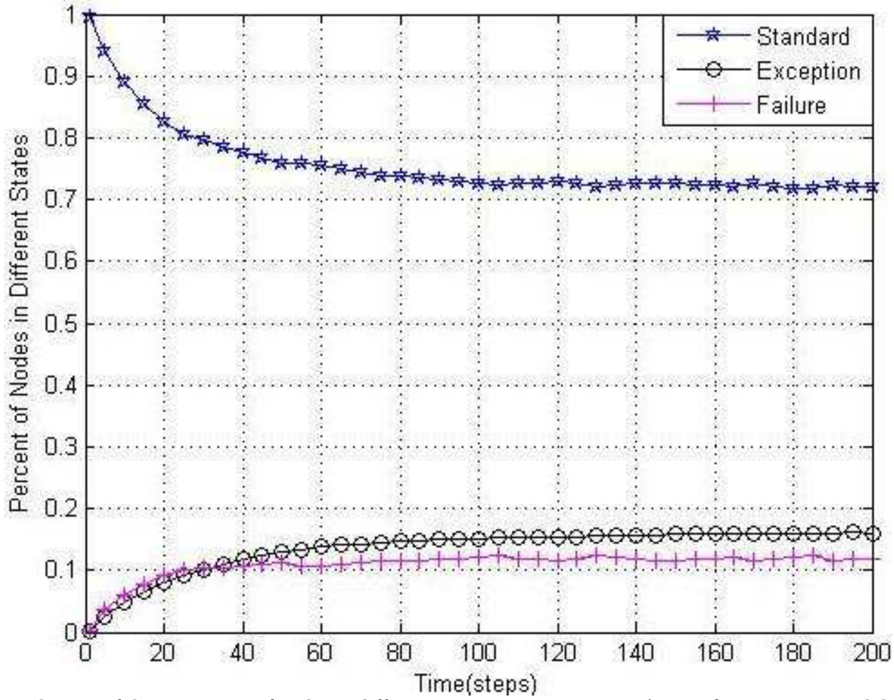

Fig.6. The evolutions of the percentage of nodes in different states. We use V=2,R=3,λ=0.01,β=0.02,μ=0.02 and the length of the area L=30. The node density is D=2/3.

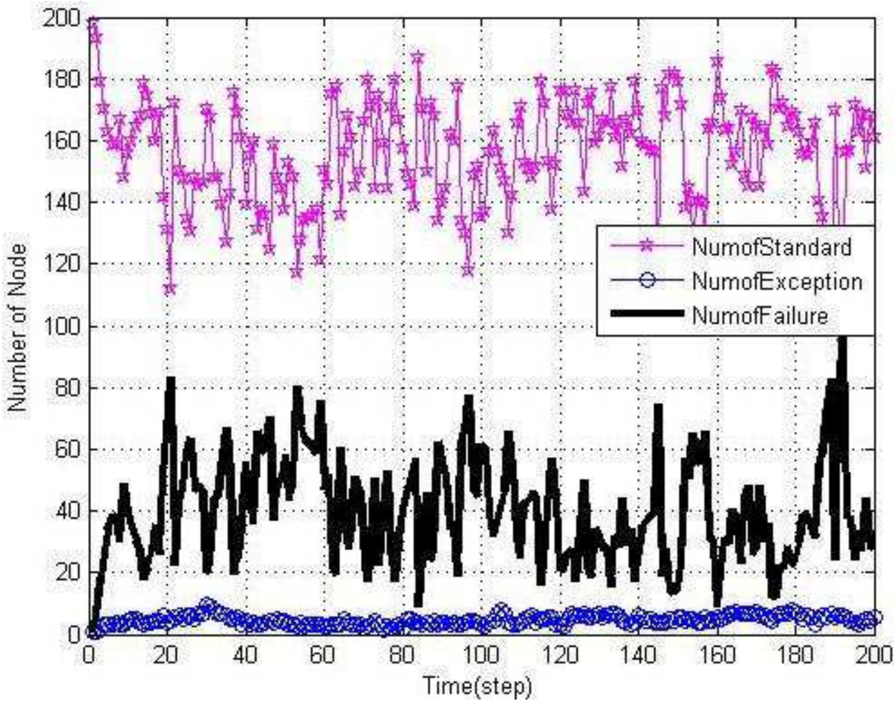

Fig.7. The evolutions of the percentage of nodes in different states. We use V=2,R=9,λ=0.1,β=0.2,μ=0.2 and the length of the area L=30. The node density is D=2/3.

Moreover, we have also explored the influence of some network parameters on the propagation dynamics of faults in CDCNs, such as the node density and the node movement speed.

The evolutions on the number of non-standard (including exception and failure) nodes with different node densities are given in Fig.8. As the parameters used in the simulation meet the stability condition, the percentage of nodes in the non-standard state will approach a stable value alongthe simulation. Because the study is carried out based on the method of statistical physics without considering the packets transmitted in the network and the limit of the node capacity, the node density has nearly no effect on the evolutions of the node number in Fig.8.

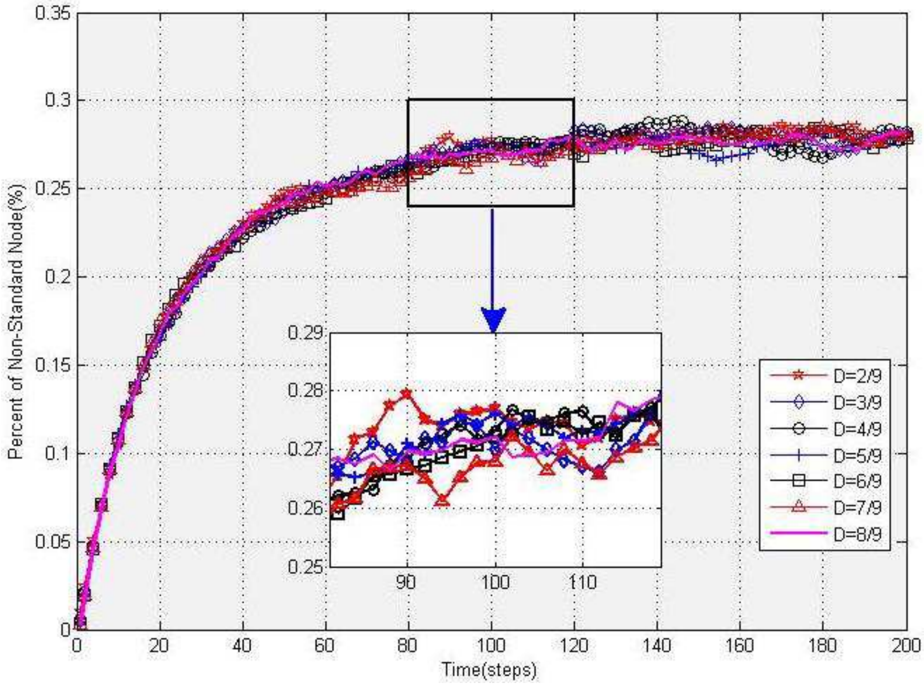

Fig.8. Percentage of non-standard nodes with different node densities. We use V=2,R=3,λ=0.01,β=0.02,μ=0.02, and a lattice size L=30. The number of nodes is 200, 300, 400, 500, 600, 700 and 800, respectively.

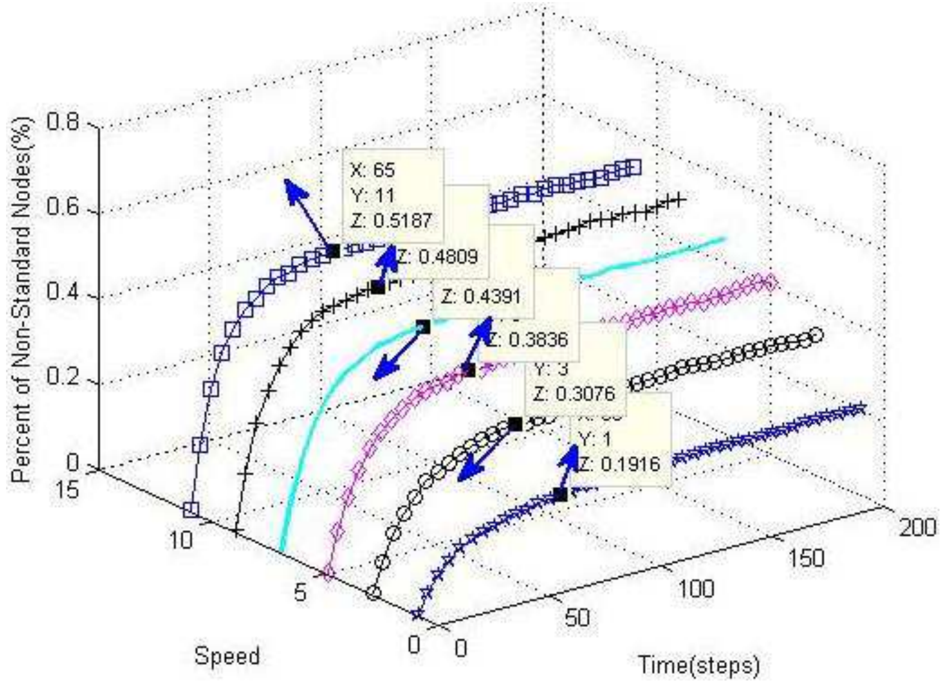

Fig.9. The percentage of non-standard nodes with different speed value V. The parameters are V=2,R=3,λ=0.01,β=0.02,μ=0.02,L=30. The node movement speed is 1, 3, 5, 7, 9 and 11, respectively.

The influence of node movement speed on the number of nodes is obtained in Fig.9. It can be seen that the change rate and the number of non-standard nodes are different for different speed values. The larger the speed is, the larger the percentage of non-standard nodes is at the same time step. Because the node movement speed determines the transition probability of the node from standard state to exception state, and the increase of number of nodes in exception state will further accelerate the transition from standard to non-standard according to (9), which is in accordance with the theoretical analysis results above.

5. Conclusions

The objective of this work is to model the propagation dynamics of faults in CDCNs, and study the underpinning rules by mathematical formula derivation and simulation. To this aim, we introduce the SEF model with variable transition probability from standard state to exception state and investigate long-term failure propagation in CDCN. Within such model, we define the balance factor$G$ that completely determines the global dynamics of failure propagation and it is obtained by explicit mathematical analysis. It can be found that the exception and fault state will stop to propagate and the number of nodes in different states will be stationary in the network when $G>1$, while propagation will continue in the opposite case. The increase of node movement speed enlarges the scale of failure propagation and acceleratesits speed,as the results obtained from the numerical simulations.

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