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Fast Calculation of Electromagnetic Scattering in Anisotropic Multilayers and its Inverse Problem

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Electromagnetic modeling of fiber-based composite structures

Motivation
- Accurate computational models of complex anisotropic multilayered composite structures
- Robust, fast, end-user’s friendly imaging procedures

Forward modeling
\[ \tilde{\epsilon}_r = \text{diag} \left[ \tilde{\epsilon}_{r11} \quad \tilde{\epsilon}_{r22} \right] \]

**Figure**: Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers

**Figure**: The transformation between the local and global coordinates by rotation matrix
**Forward modeling**

**EM response of anisotropic multilayers to distributed sources**

**The state equation**

\[
\frac{d}{dz} \vec{\varphi}(z) = \vec{A}_n \cdot \vec{\varphi}(z) + \vec{f}(z)
\]

based on the 4-component vector:

\[
\vec{\varphi}(k_x, k_y, z) = \begin{bmatrix}
k_x \vec{H}_x(k_x, k_y, z) + k_y \vec{H}_y(k_x, k_y, z) \\
k_y \vec{H}_x(k_x, k_y, z) - k_x \vec{H}_y(k_x, k_y, z) \\
k_x \vec{E}_x(k_x, k_y, z) + k_y \vec{E}_y(k_x, k_y, z) \\
k_y \vec{E}_x(k_x, k_y, z) - k_x \vec{E}_y(k_x, k_y, z)
\end{bmatrix}
\]

**The solution of the state equation**

\[
\vec{\varphi}(d_{n+1}) = e^{\vec{A}_n(\delta_n)} \cdot \vec{\varphi}(d_n) + \int_{d_n}^{d_{n+1}} e^{\vec{A}_n(\delta_n')} \cdot \vec{f}(z') dz'
\]
The new recurrence equation:

\[ \vec{\varphi}(d_n) = \vec{\Omega}_n \cdot \begin{bmatrix} \alpha_n \\ \beta_n \end{bmatrix} \]

New recurrence relations based on the propagator matrix method:

- To efficiently calculate the spectral response of the laminate
- Capable of stably dealing with distributed source along \( z \)
- More efficient compared to the traditional Green’s function method
- To numerically solve the state equation containing the tangential components of the fields
Two methods for calculating the scattered field

1st method
- Induced current integral equation (ICIE)
- Windowing technique
- Padua interpolation-integration technique

2nd method
- Lippman-Schwinger integral formula
- Polarization tensor
- Padua interpolation-integration technique

Figure: Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers
Forward modeling

Discretization of the integral equation - 1\textsuperscript{st} method

\textbf{Induced current integral equation (ICIE)}

\[ \tilde{\chi}(r) \cdot E^{inc}(r) = \frac{I(r)}{-i\omega\varepsilon_0} - \tilde{\chi}(r) \cdot i\omega\mu_0 \int \tilde{G}(r, r') \cdot I(r') \, dr' \]

\textbf{By MoM}

\[ \sum_{v=1}^{3} \chi_{u,v;m,n,p} E^{inc}_{v;m,n,p} = \frac{-1}{i\omega\varepsilon_0} \sum_{m',n',p'} I^{(u)}_{m',n',p'} \psi^{(u)}_{m',n',p'}(r_{m,n,p}) 
- \sum_{v=1}^{3} \chi_{u,v;m,n,p} \sum_{\kappa=1}^{3} \sum_{m',n'} \eta_{v\kappa;p;p'}(m-m'),(n-n') I^{(\kappa)}_{m',n',p'} \]

\textbf{By fast Fourier transform (FFT)}

\[ \tilde{\xi}_{v\kappa;p;p'} = \sum_{m',n'} \eta_{v\kappa;p;p'}(m-m'),(n-n') I^{(\kappa)}_{m',n',p'} = \text{IFFT} \left\{ \text{FFT} \left\{ \tilde{\eta}_{v\kappa;p;p'} \right\} \otimes \text{FFT} \left\{ (0) I^{(\kappa)}_{p'} \right\} \right\} \]
The techniques for calculating the impedance matrix - 1\textsuperscript{st} method

The impedance matrix:

\[ \eta_{\nu\kappa;p;p'}(x, y) = i\omega \mu_0 \int_{\Omega} G_{\nu\kappa}[(x - x'), (y - y'); z_p, z'] \psi_0^{(\kappa)}(r') dr' \]

After detouring the integral path to avoid branch cuts or singularities for the integral involving \( \tilde{\eta}_{\nu\kappa;p;p'}(k_x, k_y) \), the DFT of \( \eta_{\nu\kappa;p;p'}'(k_x, k_y) \) (the windowed version of \( \tilde{\eta}_{\nu\kappa;p;p'} \)) can be constructed as

\[
\hat{\eta}_{\nu\kappa;p;p'}'(x, y) = \frac{1}{4\pi^2 \Delta x \Delta y} \exp \left( i \frac{2\pi \alpha}{M_t \Delta x} x_s + i \frac{2\pi \beta}{N_t \Delta y} y_s \right) \times \left\{ \sum_{l_1, l_2 = -\infty}^{+\infty} e^{i2\pi[l_1(x_s/\Delta x) + l_2(y_s/\Delta y)]} \tilde{w} \left[ \frac{2\pi}{\Delta x} \left( \frac{\alpha}{M_t} + l_1 \right) - \sigma_x (\sigma_x^R, \sigma_y^R), \frac{2\pi}{\Delta y} \left( \frac{\beta}{N_t} + l_2 \right) - \sigma_y (\sigma_x^R, \sigma_y^R) \right] \right\}
\]

2-D Hamming window

\[ w(x, y) = \begin{cases} 
0.54 + 0.46 \cos \left( \frac{(x - \Delta x/2)\pi}{a} \right) \\
0.54 + 0.46 \cos \left( \frac{(y - \Delta y/2)\pi}{b} \right)
\end{cases} \]
Padua interpolation-integration technique - calculating fast oscillating integrals

- The goal is to compute the I-FT of fast oscillating spectrum in the $k_x - k_y$ plane

$$G(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_0(k_x, k_y)e^{(ik_x x + ik_y y)} \, dk_x \, dk_y$$

- Interpolation of the non-oscillating part at the Padua points with Chebyshev’s polynomial interpolant

$$\mathcal{L}_n \hat{G}_0(k_x, k_y) = \sum_{k=0}^{n} \sum_{j=0}^{k} c_{j,k-j} \hat{T}_j(k_x) \hat{T}_{k-j}(k_y) - \frac{1}{2} c_{n,0} \hat{T}_n(k_x) T_0(k_y)$$

with weights $c_{j,k-j}$ computed using $^2$

- Fourier transform of Chebyshev polynomials given by

$$\int_{-1}^{1} \hat{T}_n(k_x) \exp(-ik_x x) \, dk_x$$

are managed using $^3$

---

Alternative representation as self intersections and boundary contacts of the generating curve
\[ \gamma(t) = (-\cos((n+1)t), -\cos(nt), t \in [0, \pi]) \]

**Figure:** The Padua points with their generating curve for \( n = 12 \) (left, 91 points) and \( n = 13 \) (right, 105 points), also as union of two Chebyshev-Lobatto sub-grids (red and blue bullets). Image taken from\(^5\)

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Forward modeling

Fast calculation of EM scattering problems - 2\textsuperscript{nd} method

The Lippman-Schwinger integral formulation

\[ E^{\text{ sca}}(r) = i\omega\mu_0 \int \tilde{G}^{ee}(r, r') \cdot \tilde{\chi}(r') \cdot E^{\text{ tot}}(r') \, dr' \]

where the contrast function

\[ \tilde{\chi}(r) = -i\omega\varepsilon_0 \cdot (\tilde{\epsilon}_i - \tilde{\epsilon}_b) \]

\(\tilde{\epsilon}_i\) is the permittivity tensor of an inclusion in the composite medium.

\(\tilde{\epsilon}_b\) is the permittivity tensor of the composite medium in the global coordinate system.

The incident field is defined as

\[ E^{\text{ inc}}(r) = i\omega\mu_0 \int \tilde{G}^{ee}(r, r') \cdot J_0 (r') \, dr' \]

\[ = i\omega\mu_0 \tilde{G}^{ee}(r, r') \cdot \beta ll \]
Asymptotic formulation with polarization tensor - 2nd method

The Lippman-Schwinger integral formulation

When permittivity tensor of the inclusion $\bar{\bar{\epsilon}}_i = \bar{\bar{I}}\epsilon_i$ and its size is small enough, the scattered field can be derived into the asymptotic formulation.

$$E^{\text{sca}}(\mathbf{r}) = i\omega \mu_0 \bar{G}^{ee}(\mathbf{r}, \mathbf{r}_m) \cdot \bar{\varrho} \cdot E^{\text{inc}}(\mathbf{r}_m)$$

The polarization tensor

The polarization tensor in the local coordinate for the inclusion with volume $V$ is:

$$\bar{\varrho} = -i\omega \epsilon_0 V \cdot \begin{bmatrix} \alpha_l & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_t \end{bmatrix}$$

$$\alpha_l = \epsilon_{11} \frac{\epsilon_i - \epsilon_{11}}{\epsilon_{11} + L_l (\epsilon_i - \epsilon_{11})}$$

$$\alpha_t = \epsilon_{22} \frac{\epsilon_i - \epsilon_{22}}{\epsilon_{22} + L_t (\epsilon_i - \epsilon_{22})}$$

- $L_l$ and $L_t$ are dependent on the shape of inclusion.
- For a cubic inclusion, $L_l = c \arctan \left( c / \sqrt{1 + 2c} \right)$ and $L_t = (1 - L_l) / 2$, where $c = \epsilon_{11} / \epsilon_{22}$.  

**Comparison of the scattered field by MoM and COMSOL - 1st Case**

**Figure:** The Configuration of the 1st Case

- freq=6 GHz
- $\varepsilon_2 = \varepsilon_0 \text{ diag } [3, 2, 2]$, rotation angle $\theta_2 = 30^\circ$
- $\varepsilon_3 = \varepsilon_0 \text{ diag } [4, 2.5, 2.5]$, rotation angle $\theta_3 = 60^\circ$

---

Comparison of the scattered field by MoM and asymptotic method - 2\textsuperscript{nd} Case

Figure: The Configuration of the 2\textsuperscript{nd} Case

- freq=3 GHz
- $\bar{\epsilon}_1 = \epsilon_0 \text{ diag } [2 + i0.3, 3 + i0.1, 3 + i0.1, 3 + i0.1]$
- rotation angle $\theta = 60^\circ$
- 11 electric dipole sources with x/y/z polarization
- a cubic air scatterer with side length 0.1$\lambda_0$

Figure: The scattered electric field with sources in x/y/z-polarization
Numerical examples

Comparison of the scattered field by MoM and asymptotic method - 3rd Case

Figure: The Configuration of the 3rd Case

- freq = 3 GHz
- $d_1 = 0.5\lambda_0$; $d_2 = 0.2\lambda_0$; $d_3 = 0.35\lambda_0$; $d_4 = 0.2\lambda_0$
- layer 1 : $\bar{\varepsilon}_1 = \varepsilon_0 \text{diag} [2 + i0.3, 3 + i0.1, 3 + i0.1]$; rotation angle $\theta_1 = 60^\circ$
- layer 2 : $\bar{\varepsilon}_2 = \varepsilon_0 \text{diag} [4.5 + i0.2, 6 + i0.05, 6 + i0.05]$; rotation angle $\theta_2 = 45^\circ$
- 11 electric dipole sources with x polarization
- a cubic air scatterer with side length $0.1\lambda_0$

Figure: The scattered electric field with the x-polarization source located at $(0, 0, 0.5\lambda_0)$
Inverse problem

MUSIC (MUltiple SIgnal Classification) imaging method

**Singular value decomposition**

\[ K = U S V^* \]

- Multistatic response matrix \( K \) maps the currents at the source locations to the scattered fields measured at the detectors.
- Subspace is spanned by the singular vectors corresponding to different singular values.

**Figure:** Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers
**Inverse problem**

**MUSIC (MUltiple SIgnal Classification) imaging method**

### Formulations

#### Standard MUSIC imaging method

$\phi(r) = \frac{1}{\sum_{\sigma_j < \sigma_L} \sum_{\nu=1}^3 |\bar{u}_j^* \cdot \bar{G}_\nu(r)|^2}$

#### Enhanced MUSIC imaging method

$\phi(r) = \frac{1}{1 - \sum_{\sigma_j > \sigma_L} |\bar{u}_j^* \cdot \bar{G}(r) \cdot \bar{a}_{\text{test}}|^2}$

with

$\bar{a}_{\text{test}} = \arg \max_{\bar{a}} \frac{\sum_{\sigma_j > \sigma_L} |\bar{u}_j^* \cdot \bar{G}(r) \cdot \bar{a}|^2}{|\bar{G}(r) \cdot \bar{a}|^2}$

### Choice of $\sigma_L$

- $\sigma_i$ being ordered from the largest to the lowest value
- A threshold $T$ arbitrarily chosen

$\sigma_L = T \times \max_i (\sigma_i)$  \hspace{1cm} (1)

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**Exemple for two scatterers $T = 10^3$**

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Configuration of interest

(a) Top view

(b) Side view

(c) 3D view

Figure: Testing configuration

Description of the configuration

Measurement configuration
- location of sources = location of receivers
  (x/y/z polarization)
- $N_{\text{receivers}} = 13 \times 13$
- z position of receivers: $0.5\lambda_0$
- range of receivers in x-y plane: $[-3\lambda_0, 3\lambda_0]$

Region of interest (ROI)
- $N_{\text{cells}} = 21 \times 21 \times 21$
- range of ROI along z: $[-2.35\lambda_0, -0.35\lambda_0]$
- range of ROI in x-y plane: $[-\lambda_0, \lambda_0]$
The 1st case - one scatterer

plane cut representation

- $z = -0.35\lambda_0$
- $z = -0.45\lambda_0$
- $z = -0.55\lambda_0$
- $z = -0.65\lambda_0$
- $z = -0.75\lambda_0$

- $z = -0.85\lambda_0$
- $z = -0.95\lambda_0$
- $z = -1.05\lambda_0$
- $z = -1.15\lambda_0$
- $z = -1.25\lambda_0$

- $z = -1.35\lambda_0$
- $z = -1.45\lambda_0$
- $z = -1.55\lambda_0$
- $z = -1.65\lambda_0$
- $z = -1.75\lambda_0$

- $z = -1.85\lambda_0$
- $z = -1.95\lambda_0$
- $z = -2.05\lambda_0$
- $z = -2.15\lambda_0$
- $z = -2.25\lambda_0$
- $z = -2.35\lambda_0$
The 1\textsuperscript{st} case - one scatterer

**Figure:** Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

**Figure:** Influence of the choice of the isosurface
Inverse problem

The 2\textsuperscript{nd} case - two scatterers

Isosurface = 0.5

\textbf{Figure}: Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

\textbf{Figure}: Influence of the choice of the isosurface
The 3\textsuperscript{rd} case - three scatterers

\textbf{Figure:} Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

\textbf{Figure:} Influence of the choice of the isosurface
The 4th case - four scatterers

Figure: Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

Figure: Influence of the choice of the isosurface

Isosurface = 0.1
Conclusions

Achievements
- Generalized and complete formulation of EM response & Green dyads for undamaged anisotropic multilayers (work at the whole range of frequency for all materials)
- Asymptotic formula-based calculation of EM response for 3-D damaged uniaxial multilayers

Challenges
- To speed up the interpolation and integration for large source-receiver arrays
- To handle delaminations at the interfaces as thin planar defects
- To extend NdT to detecting larger defects by using MUSIC as the first localization tool for non-linearized procedures
- To check the size range of defects possibly detected by the first-order modeling
- To put more endeavor on experiments for practical applications
Thank you & Questions?