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Fast Calculation of Electromagnetic Scattering in Anisotropic Multilayers and its Inverse Problem

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ENDE Xi’an June 2014
Electromagnetic modeling of fiber-based composite structures

**Motivation**
- Accurate computational models of complex anisotropic multilayered composite structures
- Robust, fast, end-user’s friendly imaging procedures

**Forward modeling**
\[ \tilde{\varepsilon}_r = \text{diag}[\tilde{\varepsilon}_{r11} \quad \tilde{\varepsilon}_{r22} \quad \tilde{\varepsilon}_{r22}] \]

**Figure:** Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers

**Figure:** The transformation between the local and global coordinates by rotation matrix
EM response of anisotropic multilayers to distributed sources

The state equation

\[
\frac{d}{dz} \tilde{\varphi}(z) = \tilde{A}_n \cdot \tilde{\varphi}(z) + \tilde{f}(z)
\]

based on the 4-component vector:

\[
\tilde{\varphi}(k_x, k_y, z) = \begin{bmatrix}
    k_x \tilde{H}_x (k_x, k_y, z) + k_y \tilde{H}_y (k_x, k_y, z) \\
    k_y \tilde{H}_x (k_x, k_y, z) - k_x \tilde{H}_y (k_x, k_y, z) \\
    k_x \tilde{E}_x (k_x, k_y, z) + k_y \tilde{E}_y (k_x, k_y, z) \\
    k_y \tilde{E}_x (k_x, k_y, z) - k_x \tilde{E}_y (k_x, k_y, z)
\end{bmatrix}
\]

The solution of the state equation

\[
\tilde{\varphi}(d_{n+1}) = e^{\tilde{A}_n(\delta_n)} \cdot \tilde{\varphi}(d_n) + \int_{d_n}^{d_{n+1}} e^{\tilde{A}_n(\delta'_{n})} \cdot \tilde{f}(z') dz'
\]
EM response of anisotropic multilayers to distributed sources

The new recurrence equation:

\[ \varphi(d_n) = \bar{\Omega}_n \cdot \begin{bmatrix} \alpha_n \\ \beta_n \end{bmatrix} \]

New recurrence relations based on the propagator matrix method:

- To efficiently calculate the spectral response of the laminate
- Capable of stably dealing with distributed source along \( z \)
- More efficient compared to the traditional Green’s function method
- To numerically solve the state equation containing the tangential components of the fields
Two methods for calculating the scattered field

1\textsuperscript{st} method
- Induced current integral equation (ICIE)
- Windowing technique
- Padua interpolation-integration technique

2\textsuperscript{nd} method
- Lippman-Schwinger integral formula
- Polarization tensor
- Padua interpolation-integration technique

\textbf{Figure}: Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers
Forward modeling

Discretization of the integral equation - $1^{st}$ method

**Induced current integral equation (ICIE)**

\[
\tilde{\chi}(r) \cdot E^{inc}(r) = \frac{I(r)}{-i\omega\epsilon_0} - \tilde{\chi}(r) \cdot i\omega\mu_0 \int \tilde{G}(r; r') \cdot I(r') \, dr'
\]

**By MoM**

\[
\sum_{v=1}^{3} \chi_{u,v;m,n,p} E^{inc}_{v;m,n,p} = \frac{-1}{i\omega\epsilon_0} \sum_{m',n',p'} f^{(u)}_{m',n',p'} \psi^{(u)}_{m',n',p'}(r_{m,n,p}) \\
- \sum_{v=1}^{3} \chi_{u,v;m,n,p} \sum_{\kappa=1}^{3} \sum_{m',n'} \eta_{v\kappa;p;p'}(m-m') (n-n') f^{(\kappa)}_{m',n',p'}
\]

**By fast Fourier transform (FFT)**

\[
\bar{\xi}_{v\kappa;p;p'} = \sum_{m',n'} \eta_{v\kappa;p;p'}(m-m') (n-n') f^{(\kappa)}_{m',n',p'} = \text{IFFT} \left\{ \text{FFT} \left\{ \bar{\eta}_{v\kappa;p;p'} \right\} \otimes \text{FFT} \left\{ (0) f^{(\kappa)}_{p'} \right\} \right\}
\]
The techniques for calculating the impedance matrix - 1\textsuperscript{st} method

The impedance matrix:

\[ \eta_{\nu;\kappa;p;p'}(x, y) = i\omega\mu_0 \int_D G_{\nu;\kappa} [(x - x'), (y - y'); z_p; z'] \psi_{0,0,p'}(r) dr' \]

After detouring the integral path to avoid branch cuts or singularities for the integral involving \( \tilde{\eta}_{\nu;\kappa;p;p'}(k_x, k_y) \), the DFT of \( \tilde{\eta}_{\nu;\kappa;p;p'} \) (the windowed version of \( \tilde{\eta}_{\nu;\kappa;p;p'} \)) can be constructed as

\[
\hat{\eta}'_{\nu;\kappa;p;p'}(\alpha, \beta) = \frac{1}{4\pi^2\Delta x\Delta y} \exp \left( i \frac{2\pi\alpha}{M_t\Delta x} x_s + i \frac{2\pi\beta}{N_t\Delta y} y_s \right) \int_{-\infty}^{+\infty} d\sigma_x d\sigma_y \tilde{\eta}_{\nu;\kappa;p;p'} \left[ \sigma_x (\sigma_x, \sigma_y), \sigma_y (\sigma_x, \sigma_y) \right] \frac{\partial (\sigma_x, \sigma_y)}{\partial (\sigma_x, \sigma_y)} \times \\
\left\{ \sum_{l_1, l_2 = -\infty}^{+\infty} e^{i2\pi[l_1(x_s/\Delta x) + l_2(y_s/\Delta y)]} \tilde{w} \left[ \frac{2\pi}{\Delta x} \left( \frac{\alpha}{M_t} + l_1 \right) - \sigma_x (\sigma_x, \sigma_y), \frac{2\pi}{\Delta y} \left( \frac{\beta}{N_t} + l_2 \right) - \sigma_y (\sigma_x, \sigma_y) \right] \right\}
\]

2-D Hamming window

\[ w(x, y) = \begin{cases} 
0.54 + 0.46 \cos \left( \frac{(x - \Delta x/2)\pi}{a} \right) \\
0.54 + 0.46 \cos \left( \frac{(y - \Delta y/2)\pi}{b} \right) 
\end{cases} \]
The goal is to compute the I-FT of fast oscillating spectrum in the $k_x - k_y$ plane

$$G(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{G}_0(k_x, k_y)e^{(ik_xx + ik_yy)} dk_x dk_y$$

Interpolation of the non-oscillating part at the Padua points with Chebyshev’s polynomial interpolant

$$\mathcal{L}_n \tilde{G}_0(k_x, k_y) = \sum_{k=0}^{n} \sum_{j=0}^{k} c_{j, k-j} \hat{T}_j(k_x) \hat{T}_{k-j}(k_y) - \frac{1}{2} c_{n,0} \hat{T}_n(k_x) T_0(k_y)$$

with weights $c_{j, k-j}$ computed using\(^2\)

Fourier transform of Chebyshev polynomials given by

$$\int_{-1}^{1} \hat{T}_n(k_x) \exp(-ik_xx) dk_x$$

are managed using\(^3\)

---

Alternative representation as self intersections and boundary contacts of the generating curve
\[ \gamma(t) = (-\cos((n+1)t), -\cos(nt), t \in [0, \pi]) \]

Figure: The Padua points with their generating curve for \( n = 12 \) (left, 91 points) and \( n = 13 \) (right, 105 points), also as union of two Chebyshev-Lobatto sub-grids (red and blue bullets). Image taken from\(^5\)

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Forward modeling

Fast calculation of EM scattering problems - 2\textsuperscript{nd} method

The Lippman-Schwinger integral formulation

\[ E^{sca}(r) = i\omega\mu_0 \int \tilde{G}^{ee}(r, r') \cdot \tilde{\chi}(r') \cdot E^{tot}(r') \, dr' \]

where the contrast function

\[ \tilde{\chi}(r) = -i\omega\epsilon_0 \cdot (\tilde{\epsilon}_i - \tilde{\epsilon}_b) \]

\( \tilde{\epsilon}_i \) is the permittivity tensor of an inclusion in the composite medium.
\( \tilde{\epsilon}_b \) is the permittivity tensor of the composite medium in the global coordinate system.

The incident field is defined as

\[ E^{inc}(r) = i\omega\mu_0 \int \tilde{G}^{ee}(r, r') \cdot J_0(r') \, dr' \]

\[ = i\omega\mu_0 \tilde{G}^{ee}(r, r') \cdot \beta ll \]
Forward modeling

Asymptotic formulation with polarization tensor - 2\textsuperscript{nd} method

The Lippman-Schwinger integral formulation

When permittivity tensor of the inclusion $\tilde{\epsilon}_i = \bar{I}\epsilon_i$ and its size is small enough, the scattered field can be derived into the asymptotic formulation.

$$E^{sca}(r) = i\omega\mu_0 \tilde{G}^{ee}(r, r_m) \cdot \tilde{\varrho} \cdot E^{inc}(r_m)$$

The polarization tensor

The polarization tensor in the local coordinate for the inclusion with volume $V$ is:

$$\tilde{\varrho} = -i\omega\varepsilon_0 V \cdot \begin{bmatrix} \alpha_l & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_t \end{bmatrix}$$

$$\alpha_l = \varepsilon_{11} \frac{\epsilon_i - \varepsilon_{11}}{\varepsilon_{11} + L_l (\epsilon_i - \varepsilon_{11})}$$

$$\alpha_t = \varepsilon_{22} \frac{\epsilon_i - \varepsilon_{22}}{\varepsilon_{22} + L_t (\epsilon_i - \varepsilon_{22})}$$

- $L_l$ and $L_t$ are dependent on the shape of inclusion.
- For a cubic inclusion, $L_l = c \arctan \left( c / \sqrt{1 + 2c} \right)$ and $L_t = (1 - L_l) / 2$, where $c = \varepsilon_{11} / \varepsilon_{22}$

Comparison of the scattered field by MoM and COMSOL - 1\textsuperscript{st} Case

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The Configuration of the 1\textsuperscript{st} Case\textsuperscript{7}}
\end{figure}

- freq=6 GHz
- $\bar{\epsilon}_2 = \epsilon_0 \ \text{diag} \ [3, 2, 2]$, rotation angle $\theta_2 = 30^\circ$
- $\bar{\epsilon}_3 = \epsilon_0 \ \text{diag} \ [4, 2.5, 2.5]$, rotation angle $\theta_3 = 60^\circ$

Comparison of the scattered field by MoM and asymptotic method - 2\textsuperscript{nd} Case

**Figure:** The Configuration of the 2\textsuperscript{nd} Case

- freq=3 GHz
- \( \bar{\epsilon}_1 = \epsilon_0 \text{ diag } [2 + i0.3, 3 + i0.1, 3 + i0.1] \)
- rotation angle \( \theta = 60^\circ \)
- 11 electric dipole sources with x/y/z polarization
- a cubic air scatterer with side length 0.1\( \lambda_0 \)

**Figure:** The scattered electric field with sources in x/y/z-polarization
Comparison of the scattered field by MoM and asymptotic method - 3\textsuperscript{rd} Case

**Figure: The Configuration of the 3\textsuperscript{rd} Case**

- freq = 3 GHz
- $d_1 = 0.5\lambda_0$; $d_2 = 0.2\lambda_0$; $d_3 = 0.35\lambda_0$; $d_4 = 0.2\lambda_0$
- layer 1: $\tilde{\epsilon}_1 = \epsilon_0 \text{diag} [2 + i0.3, 3 + i0.1, 3 + i0.1]$; rotation angle $\theta_1 = 60^\circ$
- layer 2: $\tilde{\epsilon}_2 = \epsilon_0 \text{diag} [4.5 + i0.2, 6 + i0.05, 6 + i0.05]$; rotation angle $\theta_2 = 45^\circ$
- 11 electric dipole sources with x polarization
- a cubic airscatterer with side length $0.1\lambda_0$

**Figure: The scattered electric field with the x-polarization source located at $(0, 0, 0.5\lambda_0)$**
**Inverse problem**

MUSIC (MUltiple SIgnal Classification) imaging method

**Singular value decomposition**

\[ K = U S V^* \]

- Multistatic response matrix \( K \) maps the currents at the source locations to the scattered fields measured at the detectors.
- Subspace is spanned by the singular vectors corresponding to different singular values.

**Figure:** Damaged structure with uniaxial dielectric (glass-based) or conductive (graphite-based) multilayers.
Inverse problem

MUSIC (MUltiple SIgnal Classification) imaging method

Formulations

Standard MUSIC imaging method

\[ \phi(r) = \frac{1}{\sum_{j < \sigma_L} \sum_{v=1}^{3} \left| \bar{u}^*_j \cdot \bar{G}_v(r) \right|^2} \]

Enhanced MUSIC imaging method

\[ \phi(r) = \frac{1}{1 - \sum_{j > \sigma_L} \left| \bar{u}^*_j \cdot \bar{G}(r) \cdot \bar{a}_{test} \right|^2} \]

with

\[ \bar{a}_{test} = \arg \max_{\bar{a}} \frac{\sum_{j > \sigma_L} \left| \bar{u}^*_j \cdot \bar{G}(r) \cdot \bar{a} \right|^2}{\left| \bar{G}(r) \cdot \bar{a} \right|^2} \]

Choice of \( \sigma_L \)

- \( \sigma_i \) being ordered from the largest to the lowest value
- A threshold \( T \) arbitrarily chosen

\[ \sigma_L = T \times \max_{i} (\sigma_i) \] (1)

Exemple for two scatterers \( T = 10^3 \)

![Graph showing the comparison between Noiseless and 30 dB noise scenarios]
**Inverse problem**

**Configuration of interest**

![Top view](a) ![Side view](b) ![3D view](c)

**Figure:** Testing configuration

**Description of the configuration**

**Measurement configuration**
- location of sources = location of receivers (x/y/z polarization)
- \( N_{\text{receivers}} = 13 \times 13 \)
- z position of receivers : \( 0.5\lambda_0 \)
- range of receivers in x-y plane : \([-3\lambda_0, 3\lambda_0]\)

**Region of interest (ROI)**
- \( N_{\text{cells}} = 21 \times 21 \times 21 \)
- range of ROI along z : \([-2.35\lambda_0, -0.35\lambda_0]\)
- range of ROI in x-y plane : \([-\lambda_0, \lambda_0]\)
The 1\textsuperscript{st} case - one scatterer

plane cut representation
**The 1\textsuperscript{st} case - one scatterer**

**Figure:** Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

**Figure:** Influence of the choice of the isosurface
The $2^{nd}$ case - two scatterers

Figure: Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

Figure: Influence of the choice of the isosurface
The 3\textsuperscript{rd} case - three scatterers

\textbf{Figure}: Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

\textbf{Figure}: Influence of the choice of the isosurface
The $4^{th}$ case - four scatterers

Figure: Imaging results by MUSIC for noiseless case (left figure) and 30 dB noisy case (right figure)

Figure: Influence of the choice of the isosurface
Conclusions

**Achievements**

- Generalized and complete formulation of EM response & Green dyads for undamaged anisotropic multilayers (work at the whole range of frequency for all materials)
- Asymptotic formula-based calculation of EM response for 3-D damaged uniaxial multilayers

**Challenges**

- To speed up the interpolation and integration for large source-receiver arrays
- To handle delaminations at the interfaces as thin planar defects
- To extend NdT to detecting larger defects by using MUSIC as the first localization tool for non-linearized procedures
- To check the size range of defects possibly detected by the first-order modeling
- To put more endeavor on experiments for practical applications
Thank you & Questions?