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A Variational Bayesian Approximation Approach via a Sparsity Enforcing Prior in Acoustic Imaging

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***Abstract*—Acoustic imaging is an advanced technique for acoustic source localization and power reconstruction from limited noisy measurements at microphone sensors. To solve this ill-posed inverse problem, the Bayesian inference methods using proper prior knowledge have been widely investigated. In this paper, we propose to use a hierarchical Variational Bayesian Approximation for the robust acoustic imaging. And we explore the Student's-t priors with heavy tails to enforce source sparsity and non-Gaussian noises, so that we can achieve the super spatial resolution and wide dynamic range of source powers. In addition, proposed approach is validated by simulations and real data from wind tunnel in automobile industry.**

***Keywords*—Acoustic imaging; Variational Bayesian Approximation; Student's-t prior; non-Gaussian noises.**

I. Introduction

In general, the conventional Beamforming [1] method can give a direct and fast acoustic power imaging, but its spatial resolution is often very coarse at low frequencies. Based on the beamforming, the forward model of acoustic power propagation can be modeled by a determined linear system of equations in frequency domain: $\mathbf{y} = \mathbf{C}\mathbf{x}$, where $\mathbf{y}$ denotes observed beamforming power vector at microphone array; $\mathbf{x}$ denotes unknown acoustic power vector on the source plane; $\mathbf{C}$ denotes the propagation matrix, which usually depends on the geometric distance between the source and sensor for a given frequency. Therefore $\mathbf{C}$ is shift-variant and usually a singular matrix, which causes ill-poses problems. Recently, the Deconvolution Approach for Mapping of Acoustic Source (DAMAS) method [2] has effectively applied in wind tunnel experiments by the NASA. For super resolution imaging in strong Gaussian noises, many methods with sparsity constraints have been effectively developed [3], [4]. However, the sparsity parameters have to be selected empirically. In order to obtain the robust parameter estimations, we have recently the Bayesian inference approaches with the sparsity enforcing priors have been widely investigated [5], [6], [7], [8]. We have proposed to use the Double Exponential (DE) prior, and apply the Maximum A Posteriori (MAP) estimation [9] which gives better results than DAMAS and its extensions. However, the MAP method often confronts the time-consuming non-quadratic optimization.

In this paper, our motivation is to achieve the robust acoustic imaging on the vehicle surface in wind tunnel tests, which is widely used in car manufacture. To overcome the above difficulties, we propose to use a hierarchical Bayesian inference via Variational Bayesian Approximation (VBA). Moreover, the Student's-t prior is used to enforce the sparsity of source power distribution.

This paper is organized as follows: Section II introduces the forward model of acoustic power propagation; Proposed VBA approach is discussed in Section III; And method validations are carried out on simulations in Section IV and real data in Section V; Finally Section VI concludes this paper.

II. Problem statement

We consider M sensors in a non-uniform 2D array and K unknown sources on the source plane. Some assumptions are made as: Acoustic sources are uncorrelated, monopoles [2]; Sensors are omni-directional with unitary gain. After discretization into N identical grids on the source plane, we get N potential sources with K non-zero items (K-sparsity signal if $K << N$). Background noises are usually generated by both the noises at the sensors and the model uncertainty [9] caused by acoustic reverberation, reflection and refraction in wind tunnels. Thus background noises actually have colored frequency spectrum, instead of Gaussian white one in [2], [9]. Therefore we proposed the forward model of acoustic power propagation in colored noises as:

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\xi}\,, \tag{1}$$

where $\boldsymbol{\xi}$ denotes the colored noises; Propagation matrix $\mathbf{C} = \{\|\mathbf{a}_i^H \mathbf{a}_j\|_2^2 / \|\mathbf{a}_i\|_2^2\}_{N\times N}$ is obtained in [9], in which, the beamforming steering vector $\mathbf{a}_i = \left\{\frac{\exp[-j2\pi f\, r_{i,m}]/c_0}{r_{i,m}}\right\}_M$ [2] depends on the geometric distance $r_{i,m}$ between source i and sensor m at a given frequency f, with c_0 being acoustic speed in common air. Thus (1) is an under-determined linear system of equations for $\mathbf{x}$ and $\boldsymbol{\xi}$.

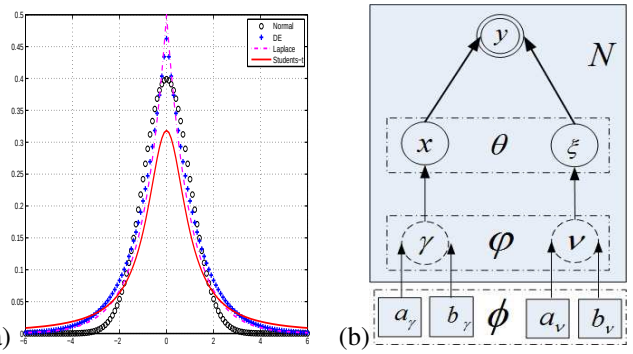
III. Proposed VBA inference approach

For the inverse problem in (1), some prior information (constraints) on both source powers $\boldsymbol{x}$ and colored noises $\boldsymbol{\xi}$ should be investigated in order to reduce the uncertainty of solutions. Let $\boldsymbol{\theta} = [\boldsymbol{x}, \boldsymbol{\xi}]^T$ denote the unknown parameters, where operator $(\cdot)^T$ denotes transpose, and $\mathbf{D}$ denotes observed data and known parameters. The inverse problem via

priors can be effectively solved by the following Bayesian inference approaches [5], [10], [6]: If we assign the specific prior probability $p(\boldsymbol{\xi})$ to noises $\boldsymbol{\xi}$, we can define the likelihood $p(\mathbf{D}|\boldsymbol{\theta})$ which is used classically by the Maximum Likelihood (ML) estimation as $\hat{\boldsymbol{\theta}}_{ML} = \arg\max_{\boldsymbol{\theta}} \{p(\mathbf{D}|\boldsymbol{\theta})\}$; In the Bayesian approach, we also assign the specific prior probabilities $p(\boldsymbol{\theta})$ to all unknown parameters. Then we use the MAP estimation as $\hat{\boldsymbol{\theta}}_{MAP} = \arg\max_{\boldsymbol{\theta}} \{ln\, p(\boldsymbol{\theta}|\mathbf{D})\} \propto \arg\min_{\boldsymbol{\theta}} \{-ln\, p(\mathbf{D}|\boldsymbol{\theta}) - ln\, p(\boldsymbol{\theta})\}$ according to Baye's rule. In fact, MAP can be seen as a regularization of ML, but MAP has the advantage of adaptively estimating the regularization parameter, compared to conventional regularization methods. However in MAP, $ln\, p(\boldsymbol{\theta}|\boldsymbol{D})$ is often hardly to get an analytical form and usually a nonlinear function with respect to $\boldsymbol{\theta}$. Moreover, both ML and MAP are the point estimators.

These difficulties can be overcome by the VBA [7], [8] estimation, in which, posterior $p(\boldsymbol{\theta}|\boldsymbol{D})$ is approximated by a family of basic easily handled probability distributions $q(\boldsymbol{\theta})$, namely $p(\boldsymbol{\theta}|\boldsymbol{D}) \approx q(\boldsymbol{\theta})$; and proper $q(\boldsymbol{\theta})$ are estimated by maximizing variational bound $\mathfrak{L}(\boldsymbol{\theta})$ as: $\hat{q}(\boldsymbol{\theta}) = \arg\max_{q(\boldsymbol{\theta})} \{\mathfrak{L}(\boldsymbol{\theta})\}$, where $\mathfrak{L}(\boldsymbol{\theta}) = \int q(\boldsymbol{\theta})\, ln \frac{p(\boldsymbol{D},\boldsymbol{\theta})}{q(\boldsymbol{\theta})}\, d\boldsymbol{\theta}$. Generally, $\boldsymbol{\theta}$ are supposed to be mutually independent: $q(\boldsymbol{\theta}) = \prod_i q(\theta_i)$. Then $\mathfrak{L}(\boldsymbol{\theta})$ is maximized by the mean field approximation as: $\hat{q}(\theta_i) = \frac{exp[I(\theta_i)]}{\int exp[I(\theta_i)]\, d\theta_i}$, where $I(\cdot)$ denotes the partition function, defined as $I(\theta_i) =< ln\, p(\boldsymbol{D},\boldsymbol{\theta}) >_{q(\boldsymbol{\theta}_{-i})} = \int q(\boldsymbol{\theta}_{-i})\, ln\, p(\boldsymbol{D},\boldsymbol{\theta}) d\boldsymbol{\theta}_{-i}$, where $\boldsymbol{\theta}_{-i}$ denotes the parameter vector except item θ_i. In fact, $I(\theta_i)$ could hardly be analytically computed, since it depends on $q(\boldsymbol{\theta}_{-i})$. But VBA inference can still obtain the approximating posterior $\hat{q}(\boldsymbol{\theta})$ owing to the conjugate distributions, in which, $\hat{q}(\boldsymbol{\theta})$ comes from the same family of the prior $p(\boldsymbol{\theta})$ based on the conjugate prior combination [11] of the likelihood and priors.

A. Heavy tail prior on colored noises



(a) (b)

Fig. 1. *(a) Sparse priors on Gaussian normal, Laplace, DE and Student's-t; (b) N dimension hierarchical Bayesian Graphical model; Double circle: observed data; Single: unknown variables; Dash: hidden variables; Square: hyper-parameters; Arrow: dependence.*

In wind tunnel experiments, we model the colored noises $\boldsymbol{\xi}$ by the Student's-t prior $St(\boldsymbol{\xi})$ that has a long heavy tail, instead of Gaussian white ones whose sharp tail excessively penalizes the large errors of forward model. Another attractive superposition property is that $St(\boldsymbol{\xi})$ can be generated by marginalizing hidden variable $\boldsymbol{\nu}$ as $St(\boldsymbol{\xi}) = \int p(\boldsymbol{\xi}|\boldsymbol{\nu}) p(\boldsymbol{\nu})\, d\boldsymbol{\nu}$, in which, conditional prior $p(\boldsymbol{\xi}|\boldsymbol{\nu}) = \mathcal{N}(\boldsymbol{\xi}|0, \boldsymbol{\Sigma}_\xi^{-1})$ is the multivariate Gaussian distribution, with $\boldsymbol{\Sigma}_\xi = Diag\{\boldsymbol{\nu}\}$ being noise covariance matrix; $Diag(\cdot)$ denotes diagonal matrix; $\boldsymbol{\nu} = \{\nu_n\}_N$ denotes the noise precision vector; And $p(\boldsymbol{\nu}) = \prod_{n=1}^N Gama(\nu_n|a_\nu, b_{\boldsymbol{\nu}}) = \prod_{n=1}^N \Gamma(a_\nu)^{-1} (b_\nu)^{a_\nu} \nu_n^{a_\nu - 1} e^{-b_\nu \nu_n}$, with a_ν, b_ν being the hyperparameters of $p(\boldsymbol{\nu})$ and $\Gamma(x) = \int t^{x-1} e^{-t} dt$.

According to proposed forward model of (1), the likelihood is determined by the conditional prior $p(\boldsymbol{\xi}|\boldsymbol{\nu}) = \mathcal{N}(\boldsymbol{\xi}|0, \boldsymbol{\Sigma}_\xi^{-1})$ as:

$$p(\mathbf{y}|\mathbf{x}, \boldsymbol{\nu}) = \frac{|\boldsymbol{\Sigma}_\xi|^{1/2}}{(2\pi)^{N/2}} e^{-\frac{1}{2}(\mathbf{y}-\mathbf{C}\mathbf{x})^H \boldsymbol{\Sigma}_\xi (\mathbf{y}-\mathbf{C}\mathbf{x})}, \qquad (2)$$

where operator $(\cdot)^H$ denotes conjugate transpose.

B. Sparse prior on acoustic power image

Acoustic source in wind tunnel experiments are generated by the wind collision on the specific parts of the vehicle surface. Therefore sources sparsely locate on some particular parts, while on the most of common parts, there are few sources. This is why acoustic powers $\mathbf{x}$ become K-sparsity signals when the source plane is discretized into N grids. Such a sparse distribution can be represented by the distribution that has a very high value around the original zero (sparsity) and a long heavy tail (dynamic range of source powers).

Here we apply the Student's-t priors $St(\boldsymbol{x})$ [7] to enforce the sparsity and wide dynamic range of source power distribution. Owing to the superposition property, hidden variable $\boldsymbol{\gamma}$ is marginalized out for $St(\boldsymbol{x}) = \int p(\boldsymbol{x}|\boldsymbol{\gamma})\, p(\boldsymbol{\gamma}) d\boldsymbol{\gamma}$, where $p(\boldsymbol{x}|\boldsymbol{\gamma}) = \mathcal{N}(\boldsymbol{x}|0, \boldsymbol{\Sigma}_x^{-1})$ is assigned to multivariate Gaussian distribution, in which, $\boldsymbol{\Sigma}_x$ denotes power covariance matrix, defined as $\boldsymbol{\Sigma}_x = Diag\{\boldsymbol{\gamma}\}$ with $\boldsymbol{\gamma} = \{\gamma_n\}_N$ being the power precision vector; and $p(\boldsymbol{\gamma}) = \prod_{n=1}^N Gama(\gamma_n|a_\gamma, b_\gamma)$, where a_γ, b_γ denotes the hyper-parameters of $p(\boldsymbol{\gamma})$. $0 < \gamma_n < 1$ greatly promotes the sparsity, while $\gamma_n \to \infty$ makes $St(x_n)$ to approach Gauss normal distribution, which has no sparsity. Compared to the Double Exponential (DE) prior in [9], $St(x_n)$ involves different hidden variable γ_n for every x_n, while DE prior requires two fix parameters for all $\mathbf{x}$ in order to achieve the same sparsity and heavy tail distribution, as the red curve shown in Fig.1a.

C. VBA parameter estimations

In Fig.1a, the graphical model [11] describes the dependencies between the observed data $\mathbf{y}$, unknown variables $\boldsymbol{\theta} = [\mathbf{x}, \boldsymbol{\xi}]^T$, their hidden variables $\boldsymbol{\varphi} = [\boldsymbol{\gamma}, \boldsymbol{\nu}]^T$ and hyperparameters $\boldsymbol{\phi} = [a_\gamma, b_\gamma, a_\nu, b_\nu]^T$. According to Baye's rule, we have $p(\boldsymbol{\theta}, \boldsymbol{\varphi}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\varphi}) p(\boldsymbol{\theta}|\boldsymbol{\varphi}) p(\boldsymbol{\varphi})$. Owing to the multivariate Gaussian likelihood in (2) and the superpositions of Student's-t priors on $\boldsymbol{\theta}$, we specifically obtain the posterior as

$$\begin{aligned} p(\boldsymbol{\theta}, \boldsymbol{\varphi}|\boldsymbol{y}) \quad &= \mathcal{N}(\boldsymbol{x}|\boldsymbol{y} - \mathbf{C}\boldsymbol{x}, \boldsymbol{\Sigma}_\xi^{-1}) \mathcal{N}(\boldsymbol{x}|0, \boldsymbol{\Sigma}_x^{-1}) \\ &\quad Gama(\boldsymbol{\gamma}|a_\gamma, b_\gamma) \mathcal{N}(\boldsymbol{\xi}|0, \boldsymbol{\Sigma}_\xi^{-1}) Gama(\boldsymbol{\nu}|a_\nu, b_\nu) \end{aligned}. \qquad (3)$$

Owing to the conjugate prior, approximating posterior belongs to Student's-t distribution which consists of multivariate Gaussian $\hat{q}(\boldsymbol{x})$ and Gamma distributions $\hat{q}(\boldsymbol{\gamma})$, $\hat{q}(\boldsymbol{\nu})$ as:

$$\begin{cases} \hat{q}(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\hat{\boldsymbol{\mu}}_x, \hat{\boldsymbol{\Sigma}}_x) \\ \hat{q}(\boldsymbol{\gamma}) = \prod_{n=1}^{N} Gama(\gamma_n|\hat{a}_\gamma, \hat{b}_\gamma^n) \\ \hat{q}(\boldsymbol{\nu}) = \prod_{n=1}^{N} Gama(\nu_n|\hat{a}_\nu, \hat{b}_\nu^n), \end{cases} , \tag{4}$$

where $\boldsymbol{x}$ and $\boldsymbol{\varphi}$ are supposed to be mutually independent; and expected variable estimations are as:

$$\begin{cases} \hat{\boldsymbol{\mu}}_x = \hat{\boldsymbol{\Sigma}}_x \mathbf{C}^T < \boldsymbol{\Sigma}_\xi > \boldsymbol{y} \\ \hat{\boldsymbol{\Sigma}}_x = (\mathbf{C}^T < \boldsymbol{\Sigma}_\xi > \mathbf{C} + < \boldsymbol{\Sigma}_x >)^{-1} \\ \hat{a}_\gamma = a_\gamma + \frac{N}{2}\,,\ \hat{b}_\gamma^n = b_\gamma + \frac{1}{2} < \boldsymbol{x}\boldsymbol{x}^T >_{nn} \\ \hat{a}_\nu = a_\nu + \frac{N}{2}\,,\ \hat{b}_\nu^n = b_\nu + \frac{1}{2} < \boldsymbol{\xi}\boldsymbol{\xi}^T >_{nn} \end{cases} , \tag{5}$$

where operator $(\cdot)_{nn}$ denotes the nth diagonal item, and $< \cdot >$ denotes expectation, which are calculated as:

$$\begin{cases} < \boldsymbol{\Sigma}_\xi > = Diag\{< \nu_n >\}_N = Diag\{< \hat{a}_\nu / \hat{b}_\nu^n >\}_N \\ < \boldsymbol{\Sigma}_x > = Diag\{< \gamma_n >\}_N = Diag\{< \hat{a}_\gamma / \hat{b}_\gamma^n >\}_N \\ < \boldsymbol{x}\boldsymbol{x}^T > = \hat{\boldsymbol{\mu}}_x \hat{\boldsymbol{\mu}}_x^T + \hat{\boldsymbol{\Sigma}}_x \\ < \boldsymbol{\xi}\boldsymbol{\xi}^T > = \boldsymbol{y}\boldsymbol{y}^T - 2\mathbf{C}\hat{\boldsymbol{\mu}}_x \boldsymbol{y}^T + \mathbf{C} < \boldsymbol{x}\boldsymbol{x}^T > \mathbf{C}^T \end{cases} , \tag{6}$$

All the solutions in (5, 6) require the values of the hyperparameters $\boldsymbol{\phi} = [a_\gamma, b_\gamma, a_\nu, b_\nu]^T$. During the iterations of parameter estimations, variables $\boldsymbol{\theta} = [\mathbf{x}, \boldsymbol{\varphi}]$ are firstly computed, then the hyperparameters ϕ can be alternatively estimated by making the first partial derivative of variational bound $\mathfrak{L}_{\boldsymbol{\theta}}(\boldsymbol{\phi})$ equal zero ($\frac{\partial \mathfrak{L}}{\partial \boldsymbol{\phi}_i} = 0$) as follows:

$$\begin{cases} \frac{\partial \mathfrak{L}}{\partial a_\nu} = N \ln b_\nu - N\digamma(a_\nu) + \sum_{n=1}^{N} < \ln \nu_n > \\ \frac{\partial \mathfrak{L}}{\partial b_\nu} = N \frac{a_\nu}{b_\nu} - \sum_{n=1}^{N} < \nu_n > \end{cases} , \tag{7}$$

where $< \nu_n >$ and $< \gamma_n >$ are computed in (6), and $\digamma(\cdot)$ denotes the digamma function defined by $\psi(x) = \Gamma'(x)/\Gamma(x)$ where $\Gamma(\cdot)$ denotes the Gamma function. a_ν, b_ν can be simultaneously estimated from the same procedure.

D. Computational analysis

From the solutions in (5), $\hat{\boldsymbol{\Sigma}}_x$ involves the matrix inversion which can not be calculated explicitly nor efficiently. We have to approximate $\hat{\boldsymbol{\Sigma}}_x$ with a circulant matrix as $\hat{\boldsymbol{\Sigma}}_x \approx (< \bar{\nu} > \mathbf{C}^H \mathbf{C} + < \bar{\gamma} >)^{-1}$, where $\bar{\nu} = \sum_{n=1}^{N} \nu_n$, $\bar{\gamma} = \sum_{n=1}^{N} \gamma_n$ denote the arithmetic mean. Then the products of circulant matrices can be efficiently computed in the Discrete Fourier Transform (DFT) domain. In (5), the estimated expectation $\hat{\boldsymbol{\mu}}_x$ of source powers can be analytically expressed as $\hat{\boldsymbol{\Sigma}}_x^{-1} \hat{\boldsymbol{\mu}}_x = \mathbf{C}^H < \boldsymbol{\Sigma}_\xi > \mathbf{y}$. This linear system of equations is solved iteratively with the conjugate gradient algorithm, which requires $O(N \log N)$ computations to treat N dimension vector $\mathbf{x}$. If Q iterations are needed, total computations are of $O(Q\,N \log N)$, which remains moderate burden.

IV. Simulations

The Simulation configurations are based on the wind tunnel experiments in Fig.3a: the distance between sensor and source plane is 4.50m; There are M$=64$ sensors; The source plane is discretized into 5cm$\times$5cm grids. In Fig.2a, source powers $\mathbf{x}$ are generated by 4 monopoles and 5 extended sources within 14dB dynamic range, and image size are of 27$\times$17 pixels. The colored noises are generated by using the Gaussian white noises via low pass filter (cut-off frequency 3000Hz), and the averaged Signal-to-Noise Ratio (SNR) is set as low as 0dB.

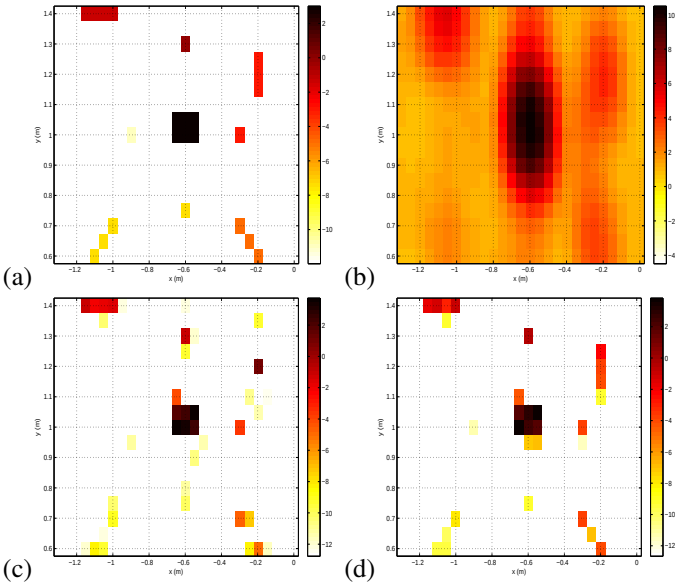


Fig. 2. *Simulation at 2500Hz, 0dB SNR in colored noises, 14dB display: (a) Source powers (b) Beamforming powers (c) Bayesian MAP inversion and (d) Proposed VBA inversion*

In Fig.2b-d, the beamforming merely gives the strong sources and failed to distinguish most of weak sources; our proposed VBA inference outperforms the MAP method [9] for the more precise localization and power estimations, especially for the better colored noise suppression.

V. Wind tunnel experiments

Wind tunnel experiments are designed to reconstruct the positions and acoustic powers on the traveling car surface. The grid is of $5cm$, and source plane is thus of 31$\times$101 pixels. The wind speed is $160km/h$; there are 524288 samplings with the sampling frequency f_s=2.56$\times 10^4$ Hz. Total samplings are separated into I=204 blocks with 2560 samplings in each bloc. The working frequency is 2500Hz, which is sensitive to human being. The image results are obtained in frequency domain shown by normalized dB images with 10dB span. Propagation matrix $\mathbf{C}$ in (1) is rectified for the wind refraction and ground reflection respectively as discussed in [9].

Fig.3 illustrates the estimated powers of mentioned methods. Due to the high side-lobe effect, beamforming merely gives a fuzzy image of strong sources in Fig.3b; DAMAS well deconvolves the beamforming image and discovers sources around the wheels and rearview mirror, however, many false targets are also detected on the air in Fig.3c; In Fig.3d and e, MAP inference via DE prior [9] and proposed VBA inference not only manages to distinguish the strong sources around the two wheels and rearview mirror, but also successfully

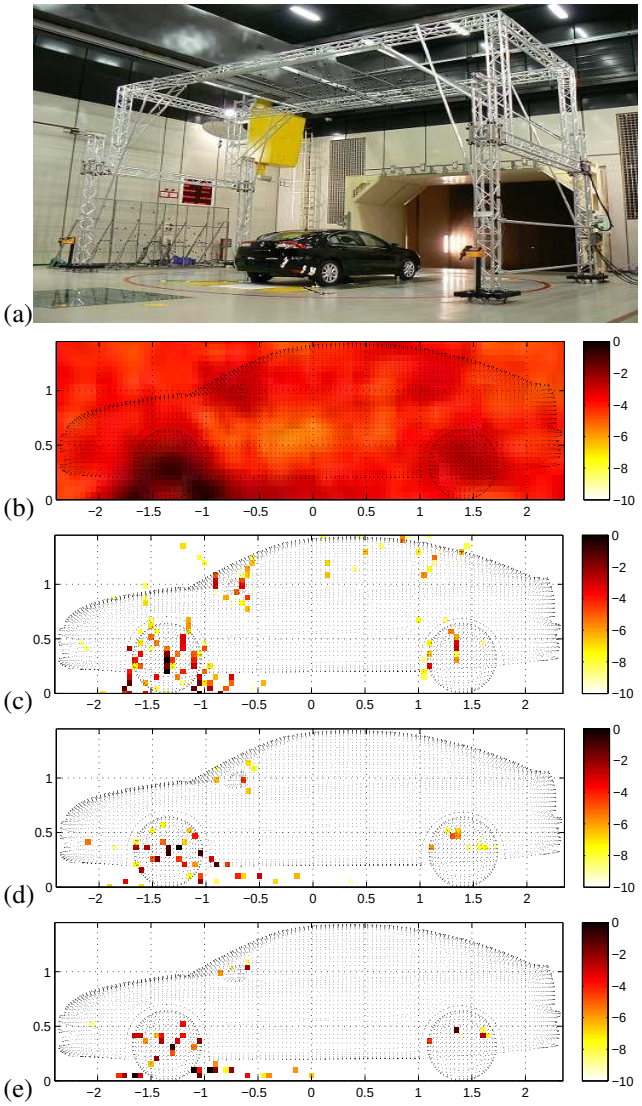


Fig. 3. *Vehicle acoustic imaging at 2500Hz: (a) Wind tunnel S2A Renault France (b) Beamforming (c) DAMAS (d) MAP inference and (e) Proposed VBA inference.*

reconstructs the week ones on the front cover and light. Furthermore, the background noise suppression in proposed VBA are much better achieved than others thanks to the Student's-t prior on the colored noise.

VI. Conclusion

We proposed a VBA inference via Student's-t priors on source powers and colored noises for super spatial resolution, wide dynamic range and robust parameter estimations. Through simulations and the real data in wind tunnel, proposed approach has been validated comparing to classical methods. But proposed VBA arises computations as $O(Q\,N\,log\,N)$. For real-time processing, it should be employing the Graphical Processing Unit (GPU) for accelerations.

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