Characterization of a 3D defect using the Expected Improvement algorithm

Sandor Bilicz, Emmanuel Vazquez, Marc Lambert, Szabolcs Gyimothy, Jozsef Pavo

Characterization of a 3D defect using the Expected Improvement algorithm

S. Bilicz [*‡], E. Vazquez [†], M. Lambert [*], S. Gyimothy [‡] and J. Pavo [‡]
[*] Département de Recherche en Électromagnétisme, Laboratoire des Signaux et Systèmes UMR8506
(CNRS-Supélec-Univ Paris Sud 11), 91192 Gif-sur-Yvette cedex, France
[†] SUPELEC, 91192 Gif-sur-Yvette cedex, France
[‡] Budapest University of Technology and Economics, Egry J. u. 18, H-1521 Budapest, Hungary
E-mail: biliczs@sch.bme.hu

***Abstract*: This paper provides a new methodology for the characterization of a defect embedded in a conductive non-magnetic plate from the measurement of the impedance variations of an air-cored pancake coil at eddy current frequencies. The inversion problem is dealt with using the *Expected Improvement* (EI) global optimization algorithm. The efficiency of the approach is discussed in the light of preliminary numerical examples obtained using synthetic data.**

***Keywords*: ENDE, global optimization, inverse problem, response surface**

I. Introduction

This paper deals with the characterization of a 3D defect inside a flat isotropic non-magnetic metal plate, from measured variations of the impedance of an ECT probe coil driven by low-frequency time-harmonic currents. The probe is an air-cored, pancake-type coil, moved above a damaged zone. The data consist of a map of the variations of the coil impedance due to the presence of the defect, at discrete locations in a planar surface above and parallel to the plate.

II. Inversion by the EI global optimization algorithm

The response observed is modeled by a function $\underline{Z}(\underline{P}) = (z_1(\underline{P}), \ldots, z_M(\underline{P}))$, where the vector $\underline{P} = (p_1, \ldots, p_N)$ characterizes the (unknown) defect. The objective is to find the best $\underline{P}^\star$ in the sense of a cost function $f(\underline{P})$ measuring the discrepancy between the data and the model output $\underline{Z}(\underline{P})$. This optimization problem can be formally written as

$$\underline{P}^\star = \arg\min_{\underline{P}\in\mathbb{P}} f(\underline{P}).$$

We solve this problem using the expected improvement global optimization algorithm [1]. This popular algorithm makes it possible to find a global minimizer of a smooth non-convex function by constructing an approximation of the function to be optimized via a kriging method [2]. The algorithm consists of the following steps.

1) Choose a set of n initialization points $\underline{P}$ and compute the corresponding values of the cost function.
2) Predict the values of the cost function at non-observed $\underline{P}$ over $\mathbb{P}$ and estimate a confidence interval for the prediction *via* kriging, using the past observations $f(\underline{P}_1), \ldots, f(\underline{P}_n)$.
3) Compute the *expected improvement* for all $\underline{P}$.
4) Choose $\underline{P}_{n+1}$ that achieves the highest *expected improvement* and compute $f(\underline{P}_{n+1})$.
5) Add this new point to the set of the past observations;
6) Go to Step 2 until a stopping criterion is reached.

III. Numerical illustrations

To illustrate the EI algorithm, numerical examples are presented. A metal plate is affected by a parallelepiped defect characterized by a maximum of seven parameters: its size along the three directions (L_x, L_y, L_z), the center of the defect in the x-y plane and the position of the top with respect to the upper surface (x_c, y_c, z_t) and its conductivity σ, respectively. An example with four unknown parameters to be retrieved (L_x, L_y, L_z, z_t) whereas the three others are fixed at their exact values $(x_c = 0, y_c = 0, \sigma = 0)$ is shown.

In Figure 1 the evolution of $\log[f(\underline{P})]$ (top) and of $\log[\max(EI)]$ over $\mathbb{P}$ (bottom) are presented, both with respect to the iteration number. As expected the zero of the cost function is reached and the EI function decreases when the number of iterations increases. However, the stopping criterion (chosen such as no more improvement is expected: $\max(\mathrm{EI}) < 10^{-4}$) has to be improved. As a matter of fact the best solution is obtained at iteration 53 whereas optimization process is stopped at iteration 90 so 37 iterations are used to decrease the EI even if the best solution has already been reached.

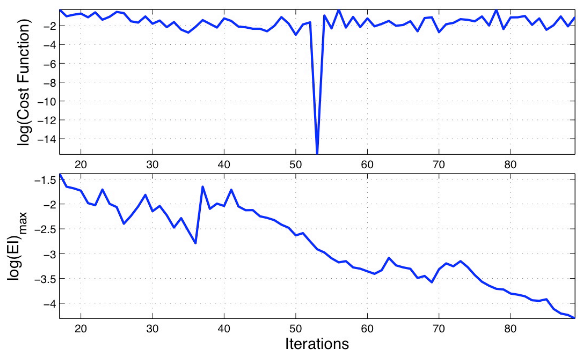

Figure 1. Evolution of the cost function (top) and the maximum of the "Expected Improvement" (bottom) with respect to the iteration's number.

IV. Conclusion

Some preliminary results concerning the application of EI onto an ENDE problem of 3D-defect characterization are presented. The numerical study of the efficiency of such an algorithm is still under development in order (i) to increase the number of unknown parameters, (ii) to test its stability with respect to the use of noisy data.

References

[1] D. R. Jones, "A taxonomy of global optimization methods based on response surfaces," *J. of Global Optimization*, vol. 21, no. 4, pp. 345–383, 2001.
[2] J.-P. Chiles and P. Delfiner, *Geostatistics: Modeling Spatial Uncertainty*, ser. Wiley series in probability and statistics, J. Wiley and Sons, Eds., New York, 1999.