Eddy-current NDE of combustion turbine blade coatings. Determination of conductivity profiles in the presence of a diffusion process

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Frédéric Nougier Marc Lambert Riadh Zorgati

Département de Recherche en Électromagnétisme,
Laboratoire des Signaux et Systèmes (UMR8506 CNRS-SUPELEC-Univ. Paris 11),
3, rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France

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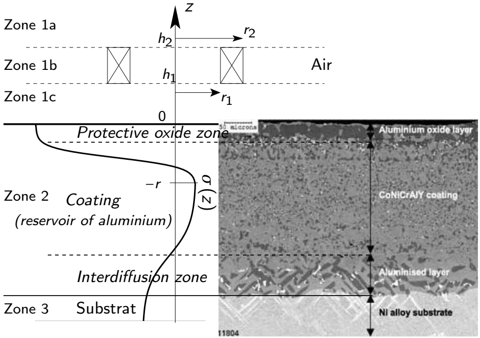
Context and configuration of the study

- Eddy-Current measurements over combustion turbine blade coatings affected by depletion of aluminium;
- Model taking inward and outward depletion of aluminum inside the coating into account;
- Conductivity profile follows a two-hyperbolic-tangent law;
- Analytical formulation of the variation of impedance obtained combining the approaches found in [1, 2, 3]



General formulation

- $\mathbf{A}(\mathbf{r}) = A(r,z)\,\mathbf{u}_\theta + A_{ec}(r,z) = R(r)W(z)$– lead to

$$\frac{\partial^2}{\partial r^2}R(r) + \frac{1}{r}\frac{\partial}{\partial r}R(r) + \left(a^2 - \frac{1}{r^2}\right)R(r) = 0 \quad (1)$$

$$\frac{\partial^2}{\partial z^2}W(z) = \left[a^2 + j\omega\mu\sigma(z)\right]W(z) \quad (2)$$

- Following [2] the general solution given by

$$W(z) = CF_1(f(z)) + BF_2(f(z)) \quad (3)$$

where F_1 and F_2 typical mathematical functions related to $\sigma(z)$
- Expression of

$$A_{1c}(r,z) = \int_0^{+\infty} \frac{\mu N I I(r_1,r_2) e^{-az} J_1(ar)\left(e^{-ah_1} - e^{-ah_2}\right)}{2a^3(h_2-h_1)(r_2-r_1)}da + \int_0^{+\infty} C_1 e^{-az} J_1(ar)da, \text{ with } I(r_1,r_2) = \int_{ar_1}^{ar_2} xJ_1(x)dx \qquad \forall z \in [0;+\infty[$$

$$A_2(r,z) = \int_0^{+\infty} \left[C_2F_1(f(z)) + B_2F_2(f(z))\right] J_1(ar)da \qquad \forall z \in [-r;0]$$

$$A_3(r,z) = \int_0^{+\infty} B_3F_3(g(z))J_1(ar)da \qquad \forall z \in]-\infty;-r]$$

- $A_{1c}(r,z)$ known, then Z deduced as

$$Z = K\int_{0}^{+\infty} \frac{I(r_1,r_2)^2}{a^6}\left[2\left(e^{-a(h_2-h_1)}-1+a(h_2-h_1)\right)+\left(e^{-ah_2}-e^{-ah_1}\right)^2\phi(a)\right]da \tag{4}$$

with $\phi(a)=\dfrac{C_1}{\mathcal{K}}; \mathcal{K}=\dfrac{\mu NII(r_1,r_2)\left(e^{-ah_1}-e^{-ah_2}\right)}{2a^3(h_2-h_1)(r_2-r_1)}; K=\dfrac{-j\omega\pi\mu N^2}{(h_2-h_1)^2(r_2-r_1)^2}$.

- Continuity conditions of the quantities and/or their derivatives with respect z and/or their cancellation at $\pm\infty$ express $\phi(a)$as

$$\phi(a)=\frac{(aM-O)RT-(aL-N)ST+[a(LQ-MP)-NQ+OP]\,U}{(aM+O)RT-(aL+N)ST+[a(LQ-MP)+NQ-OP]\,U} \tag{5}$$

where

$$\begin{aligned}
&L = F_1(f(z=0)); && M = F_2(f(z=0)); && N = F_1'(f(z))\big|_{z=0};\\
&O = F_2'(f(z))\big|_{z=0}; && P = F_1(f(z=-r)); && Q = F_2(f(z=-r));\\
&R = F_1'(f(z))\big|_{z=-r}; && S = F_2'(f(z))\big|_{z=-r}; && T = F_3(g(z=-r));\\
&U = F_3'(g(z))\big|_{z=-r}
\end{aligned} \tag{6}$$

$'$ means derivative with respect to z

Formulation for a two-tanh profile

- Conductivity profile given by

$$\sigma(z) = \begin{cases} \sigma_{12} + \dfrac{\sigma_1 - \sigma_{12}}{2}\left[1 + \tanh\left(\dfrac{z + c_1}{2v_1}\right)\right] & \forall z \in [-r, 0] \\ \sigma_2 + \dfrac{\sigma_{12} - \sigma_2}{2}\left[1 + \tanh\left(\dfrac{z + c_2}{2v_2}\right)\right] & \forall z \in]-\infty, -r] \end{cases} \tag{7}$$

- Particular functions F_1, F_2 and F_3 are

$$F_1(y_2(z)) = y_2^{\mu}(z)\,[1 - y_2(z)]^{\nu}\, F(\mu + \nu, \mu + \nu + 1, 2\mu + 1; y_2(z)) \tag{8}$$

$$F_2(y_2(z)) = y_2^{-\mu}(z)\,[1 - y_2(z)]^{\nu}\, F(\nu - \mu + 1, \nu - \mu, -2\mu + 1; y_2(z)) \tag{9}$$

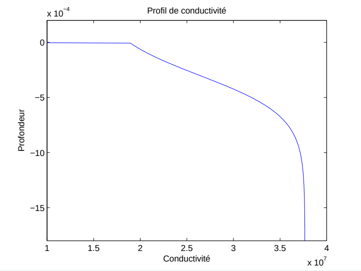
$$F_3(y_3(z)) = y_3^{\lambda}(z)\,[1 - y_3(z)]^{\tau}\, F(\lambda + \tau, \lambda + \tau + 1, 2\lambda + 1; y_3(z)) \tag{10}$$

$$\begin{gathered} \text{with } y_2(z) = \left(1 + e^{-\frac{z + c_1}{v_1}}\right)^{-1}, \quad y_3(z) = \left(1 + e^{-\frac{z + c_2}{v_2}}\right)^{-1} \\ \mu = v_1\sqrt{a^2 + j\omega\mu_0\sigma_{12}}, \quad \nu = v_1\sqrt{a^2 + j\omega\mu_0\sigma_1} \\ \lambda = v_2\sqrt{a^2 + j\omega\mu_0\sigma_2}, \quad \tau = v_2\sqrt{a^2 + j\omega\mu_0\sigma_{12}} \end{gathered} \tag{11}$$

$F(\alpha, \beta, \gamma; x)$ is the hypergeometric function

Description of the configuration

r_1	1.3 mm
r_2	3.3 mm
h_1	0.5 mm
h_2	7.8 mm
N_{turn}	580
σ_1	$1.883\ 10^7\,\mathrm{S\,m^{-1}}$
σ_{12}	$3.766\ 10^7\,\mathrm{S\,m^{-1}}$
c_1	0.3 mm
v_1	0.1857 mm

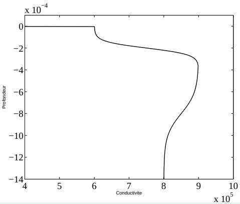


Comparison with the results given in [1]

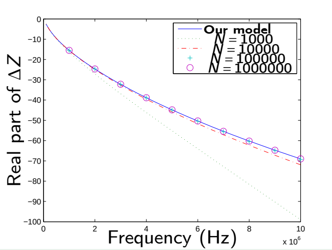
	Real part of Z, $(b = NR_2)$			Imaginary part of Z, $(b = NR_2)$		
Frequency	from [1]	$N = 10$	$N = 20$	from [1]	$N = 10$	$N = 20$
1 kHz	0.00817	0.008169	0.008165	−0.00828	−0.008267	−0.00828
10 kHz	0.02583	0.02585	2.5823	−0.22571	−0.22557	−0.22566
100 kHz	-0.68836	-0.68799	-0.6882	−1.49719	−1.49645	−1.496769

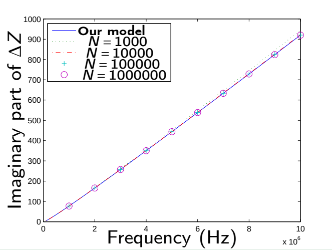
Description of the configuration

$r_1 = 2.0\,\mathrm{mm}$	$\sigma_1 = 6\,10^5\,\mathrm{S\,m^{-1}}$
$r_2 = 4.0\,\mathrm{mm}$	$\sigma_{12} = 9\,10^5\,\mathrm{S\,m^{-1}}$
$h_1 = 0.5\,\mathrm{mm}$	$\sigma_2 = 8\,10^5\,\mathrm{S\,m^{-1}}$
$h_2 = 7.3\,\mathrm{mm}$	$N_{\mathrm{turn}=200}$
$c_1 = 0.2\,\mathrm{mm}$	$v_1 = 0.03\,\mathrm{mm}$
$c_2 = 0.8\,\mathrm{mm}$	$v_2 = 0.1\,\mathrm{mm}$



Comparison with the results obtained from a multi-layer model





- E. Uzal and J. Rose, The impedance of eddy current probes above layered metals whose conductivity and permeability vary continuously, *IEEE Trans. Magn.* **29**, (1993), 1869–1873.
- T. Theodoulidis, T. Tsiboukis and E. Kriezis, Analytical solutions in eddy current testing of layered metals with continuous conductivity profiles, *IEEE Trans. Magn.* **31**, (1995), 2254–2260.
- T. Theodoulidis and E. Kriezis, Series expansions in eddy current nondestructive evaluation models, *J. Mater. Process. Technol.* **161**, (2005), 343–347.