

Eddy-current NDE of combustion turbine blade coatings. Determination of conductivity profiles in the presence of a diffusion process

Frédéric Nougier, Marc Lambert, Riadh Zorgati

► **To cite this version:**

Frédéric Nougier, Marc Lambert, Riadh Zorgati. Eddy-current NDE of combustion turbine blade coatings. Determination of conductivity profiles in the presence of a diffusion process. ENDE'08, Jun 2008, Séoul, South Korea. Proceedings of the 13th International Workshop on Electromagnetic Non-Destructive Evaluation, 2008. <hal-01104123>

HAL Id: hal-01104123

<https://hal-supelec.archives-ouvertes.fr/hal-01104123>

Submitted on 16 Jan 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Eddy-current NDE of combustion turbine blade coatings. Determination of conductivity profiles in the presence of a diffusion process

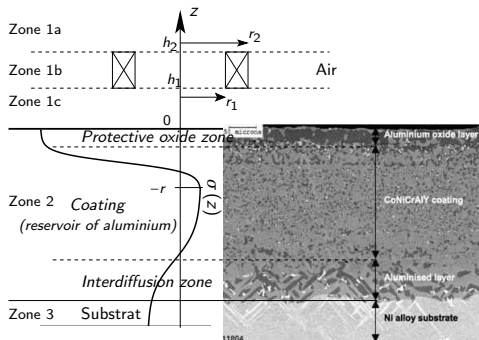
Frédéric Nougier Marc Lambert Riadh Zorgati

Département de Recherche en Électromagnétisme,
Laboratoire des Signaux et Systèmes (UMR8506 CNRS-SUPELEC-Univ. Paris 11),
3, rue Joliot Curie, 91192 Gif-sur-Yvette cedex, France

ENDE 2008

Context and configuration of the study

- Eddy-Current measurements over combustion turbine blade coatings affected by depletion of aluminium;
- Model taking inward and outward depletion of aluminum inside the coating into account;
- Conductivity profile follows a two-hyperbolic-tangent law;
- Analytical formulation of the variation of impedance obtained combining the approaches found in [1, 2, 3]



General formulation

- $\mathbf{A}(r) = A(r, z) \mathbf{u}_\theta + A_{ec}(r, z) = R(r)W(z)$ - lead to

$$\frac{\partial^2}{\partial r^2} R(r) + \frac{1}{r} \frac{\partial}{\partial r} R(r) + \left(a^2 - \frac{1}{r^2} \right) R(r) = 0 \quad (1)$$

$$\frac{\partial^2}{\partial z^2} W(z) = \left[a^2 + j\omega \mu \sigma(z) \right] W(z) \quad (2)$$

- Following [2] the general solution given by

$$W(z) = CF_1(f(z)) + BF_2(f(z)) \quad (3)$$

where F_1 and F_2 typical mathematical functions related to $\sigma(z)$

- Expression of

$$A_{1c}(r, z) = \int_0^{+\infty} \frac{\mu N I I(r_1, r_2) e^{-az} J_1(ar) (e^{-ah_1} - e^{-ah_2})}{2a^3 (h_2 - h_1) (r_2 - r_1)} da + \int_0^{+\infty} C_1 e^{-az} J_1(ar) da, \text{ with } I(r_1, r_2) = \int_{ar_1}^{ar_2} x J_1(x) dx \quad \forall z \in [0; +\infty[$$

$$A_2(r, z) = \int_0^{+\infty} [C_2 F_1(f(z)) + B_2 F_2(f(z))] J_1(ar) da \quad \forall z \in [-r; 0]$$

$$A_3(r, z) = \int_0^{+\infty} B_3 F_3(g(z)) J_1(ar) da \quad \forall z \in]-\infty; -r]$$

- $A_{1c}(r, z)$ known, then Z deduced as

$$Z = K \int_0^{+\infty} \frac{I(r_1, r_2)^2}{a^6} \left[2 \left(e^{-a(h_2 - h_1)} - 1 + a(h_2 - h_1) \right) + \left(e^{-ah_2} - e^{-ah_1} \right)^2 \phi(a) \right] da \quad (4)$$

$$\text{with } \phi(a) = \frac{C_1}{\mathcal{X}}; \mathcal{X} = \frac{\mu N I I(r_1, r_2) (e^{-ah_1} - e^{-ah_2})}{2a^3 (h_2 - h_1) (r_2 - r_1)}; K = \frac{-j\omega\pi\mu N^2}{(h_2 - h_1)^2 (r_2 - r_1)^2}.$$

- Continuity conditions of the quantities and/or their derivatives with respect z and/or their cancellation at $\pm\infty$ express $\phi(a)$ as

$$\phi(a) = \frac{(aM - O)RT - (aL - N)ST + [a(LQ - MP) - NQ + OP]U}{(aM + O)RT - (aL + N)ST + [a(LQ - MP) + NQ - OP]U} \quad (5)$$

where

$$\begin{aligned} L &= F_1(f(z=0)); & M &= F_2(f(z=0)); & N &= F_1'(f(z))|_{z=0}; \\ O &= F_2'(f(z))|_{z=0}; & P &= F_1(f(z=-r)); & Q &= F_2(f(z=-r)); \\ R &= F_1'(f(z))|_{z=-r}; & S &= F_2'(f(z))|_{z=-r}; & T &= F_3(g(z=-r)); \\ U &= F_3'(g(z))|_{z=-r} \end{aligned} \quad (6)$$

' means derivative with respect to z

Formulation for a two-tanh profile

- Conductivity profile given by

$$\sigma(z) = \begin{cases} \sigma_{12} + \frac{\sigma_1 - \sigma_{12}}{2} \left[1 + \tanh\left(\frac{z + c_1}{2v_1}\right) \right] & \forall z \in [-r, 0] \\ \sigma_2 + \frac{\sigma_{12} - \sigma_2}{2} \left[1 + \tanh\left(\frac{z + c_2}{2v_2}\right) \right] & \forall z \in]-\infty, -r] \end{cases} \quad (7)$$

- Particular functions F_1, F_2 and F_3 are

$$F_1(y_2(z)) = y_2^\mu(z) [1 - y_2(z)]^\nu F(\mu + \nu, \mu + \nu + 1, 2\mu + 1; y_2(z)) \quad (8)$$

$$F_2(y_2(z)) = y_2^{-\mu}(z) [1 - y_2(z)]^\nu F(\nu - \mu + 1, \nu - \mu, -2\mu + 1; y_2(z)) \quad (9)$$

$$F_3(y_3(z)) = y_3^\lambda(z) [1 - y_3(z)]^\tau F(\lambda + \tau, \lambda + \tau + 1, 2\lambda + 1; y_3(z)) \quad (10)$$

$$\text{with } y_2(z) = \left(1 + e^{-\frac{z+c_1}{v_1}} \right)^{-1}, \quad y_3(z) = \left(1 + e^{-\frac{z+c_2}{v_2}} \right)^{-1} \quad (11)$$

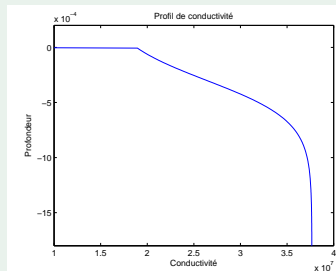
$$\mu = v_1 \sqrt{a^2 + j\omega\mu_0\sigma_{12}}, \quad \nu = v_1 \sqrt{a^2 + j\omega\mu_0\sigma_1}$$

$$\lambda = v_2 \sqrt{a^2 + j\omega\mu_0\sigma_2}, \quad \tau = v_2 \sqrt{a^2 + j\omega\mu_0\sigma_{12}}$$

$F(\alpha, \beta, \gamma; x)$ is the hypergeometric function

Description of the configuration

| | |
|-------------------|------------------------------------|
| r_1 | 1.3 mm |
| r_2 | 3.3 mm |
| h_1 | 0.5 mm |
| h_2 | 7.8 mm |
| N_{turn} | 580 |
| σ_1 | $1.883 \cdot 10^7 \text{ Sm}^{-1}$ |
| σ_{12} | $3.766 \cdot 10^7 \text{ Sm}^{-1}$ |
| c_1 | 0.3 mm |
| v_1 | 0.1857 mm |

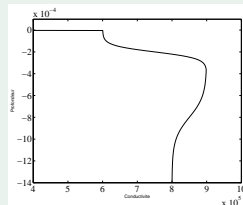


Comparison with the results given in [1]

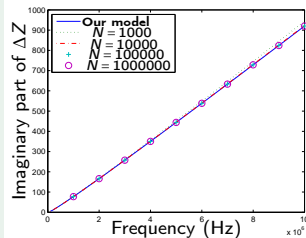
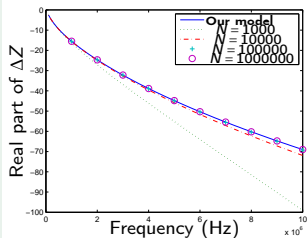
| Frequency | Real part of Z , ($b = NR_2$) | | | Imaginary part of Z , ($b = NR_2$) | | |
|-----------|-----------------------------------|----------|----------|--|-----------|-----------|
| | from [1] | $N = 10$ | $N = 20$ | from [1] | $N = 10$ | $N = 20$ |
| 1 kHz | 0.00817 | 0.008169 | 0.008165 | -0.00828 | -0.008267 | -0.00828 |
| 10 kHz | 0.02583 | 0.02585 | 2.5823 | -0.22571 | -0.22557 | -0.22566 |
| 100 kHz | -0.68836 | -0.68799 | -0.6882 | -1.49719 | -1.49645 | -1.496769 |




Description of the configuration

| | |
|------------------------|---|
| $r_1 = 2.0 \text{ mm}$ | $\sigma_1 = 6 \cdot 10^5 \text{ S m}^{-1}$ |
| $r_2 = 4.0 \text{ mm}$ | $\sigma_{12} = 9 \cdot 10^5 \text{ S m}^{-1}$ |
| $h_1 = 0.5 \text{ mm}$ | $\sigma_2 = 8 \cdot 10^5 \text{ S m}^{-1}$ |
| $h_2 = 7.3 \text{ mm}$ | $N_{\text{turn}} = 200$ |
| $c_1 = 0.2 \text{ mm}$ | $v_1 = 0.03 \text{ mm}$ |
| $c_2 = 0.8 \text{ mm}$ | $v_2 = 0.1 \text{ mm}$ |



Comparison with the results obtained from a multi-layer model



-  E. Uzal and J. Rose, The impedance of eddy current probes above layered metals whose conductivity and permeability vary continuously, *IEEE Trans. Magn.* **29**, (1993), 1869–1873.
-  T. Theodoulidis, T. Tsiboukis and E. Kriezis, Analytical solutions in eddy current testing of layered metals with continuous conductivity profiles, *IEEE Trans. Magn.* **31**, (1995), 2254–2260.
-  T. Theodoulidis and E. Kriezis, Series expansions in eddy current nondestructive evaluation models, *J. Mater. Process. Technol.* **161**, (2005), 343–347.