Nondestructive testing of fiber array with multiple missing fibers

Zicheng Liu, Changyou Li, Dominique Lesselier, Yu Zhong

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Z.C. Liu†, C.Y. Li†, D. Lesselier†, Y. Zhong‡
† Laboratoire des Signaux et Systèmes - UMR8506 (CNRS-CentraleSupélec-Univ. Paris-Sud)
3, rue Joliot-Curie, 91192 Gif-sur-Yvette, France
‡ A*STAR, Institute of High Performance Computing, Singapore 138632, Singapore
Contact: zicheng.liu@l2s.centralesupelec.fr

***Abstract*—Our goal is to detect defects in composite materials composed by multilayer planar plates with a periodic set of circular cylindrical fibers embedded in each layer. As a starter, the work presented is electromagnetic (EM) modeling and imaging of missing fibers within a fiber array standing in air. The multiple scattering method is utilized to analyze the electromagnetic behavior, and the corresponding imaging model is established directly from Lippman-Schwinger integral formulation. Standard MUltiple SIgnal Classification (MUSIC) and the proposed joint sparsity which borrows the idea of sparse theory are applied to retrieve the locations of missing fibers. Numerical results are provided to confirm availability and accuracy of EM modeling and defect imaging.**

1. Introduction

Fiber-based laminated composite materials are widely used in aeronautic and automotive industries due to their advantageous characters in stiffness and strength. In the applications, a challenge for safety consideration is to find possible defects in the fiber-based materials using electromagnetic sources and probes. As an example, there may be missing or misplaced fibers, or voids or other damages produced during manufacturing and/or in-service. Two main steps then appear: one needs an accurate enough scattering model (forward problem) and a high-resolution imaging solution (inverse problem).

In the present investigation, one is concerned with the time-harmonic electromagnetic response of fiber-reinforced composite structures taken as stacks of planar layers, and high-resolution imaging approaches to locate the positions of defects inside .

As a first step towards such an investigation, emphasis here is put on a structure made of a finite set of fibers in air. All fibers are identical and arranged with their centers aligned and at same distance from one another except multiple missing fibers, which actually disorganizes the whole structure.

Both the response to a given excitation (plane wave or line source) and the specific Green's function of the original, intact structure (then disorganized) can be handled by means of the multiple scattering method [1], letting the fields inside and outside the fibers being properly multipole expanded. The background field is perturbed by the fact that fibers have been removed and their scattering contribution can be evaluated via a Lippman-Schwinger integral formulation [1]. The integral in explicit form leads to the imaging model which establishes a link between background field and locations of missing fibers.

Since defects as missing fibers are the only ones in consideration, all that one needs is to retrieve their indices (or labels) in the array. To get them, the classical MUltiple SIgnal Classification (MUSIC) imaging method [2] is implemented to test the EM modeling and the defects localization. Then, to enhance the discrimination ability, since rare defects exist in the defectuous material, a sparsity-tailored reconstruction method, called joint sparsity, is put forth. Borrowing ideas from pioneering works about Directions of Arrival (DOA) searches [3], the proposed method utilizes the fact that sparsity is invariant when different illuminations are considered.

2. Methodology

The main idea of the multiple scattering method is sketched here. Assuming that the number of fibers (including the missing ones) is N, incoming waves inside and around the n-th fiber, $n = 1, 2, \ldots, N$, can be multipole expanded into Bessel functions of the first kind with coefficients A_m^n and C_m^n, while outgoing waves can be expanded into Hankel functions of the first kind with coefficients B_m^n and Q_m^n, respectively. Considering mutual scattering among fibers, Rayleigh identities for each one can be written, while their combination yields a linear relationship between the vector of A_m^n (denoted by $\mathbf{A}$) and the vector of B_m^n (denoted by $\mathbf{B}$), in short form $\mathbf{A} = \mathbf{SB} + \mathbf{K}$ with known matrix $\mathbf{S}$ and vector $\mathbf{K}$. $\mathbf{K}$ indicates the strength of the incident field. Obtaining another linear relation from fiber boundary conditions, $\mathbf{B} = \mathbf{RA} + \mathbf{TQ}$, one gets the solution of $\mathbf{B}$ by $\mathbf{B} = (\mathbf{I} - \mathbf{RS})^{-1}(\mathbf{RK} + \mathbf{TQ})$, which plays a key to the calculation of fields in the whole space.

The imaging model is directly derived from a Lippman-Schwinger integral formulation: $\widetilde{E}_y^n(\mathbf{r}) - E_y^n(\mathbf{r}) = \sum_{p=1}^{P} \int_{D_p} G(\mathbf{r}, \mathbf{r}') k^2 (1 - \epsilon_r) \widetilde{E}_y^n(\mathbf{r}') \, \mathrm{d}\mathbf{r}'$, within which $\widetilde{E}_y^n(\mathbf{r})$ and $E_y^n(\mathbf{r})$ are the electric field strength at point $\mathbf{r}$ of disorganized and well-organized structure, respectively, P is the number of missing fibers, D_p is the surface of the p-th missing fiber, k is the wave number in air, and ϵ_r is the relative dielectric permittivity of the fiber material. The integral involved in this formulation can be interpreted as the scattered field by the 'fiber' source as illustrated in Fig. 1, in which $E^d(\mathbf{r}) = \widetilde{E}_y^n(\mathbf{r}) - E_y^n(\mathbf{r})$. Mathematically, taking the collected data of $E^d(\mathbf{r})$ as a column vector $\mathbf{g}$, $\mathbf{g}$ can be represented by a product of matrix $\mathbf{Z}$ and vector $\mathbf{b}$, *i.e.* $\mathbf{g} = \mathbf{Zb}$, within which $\mathbf{Z}$ is known and independent of possible defects, and positions of nonzero elements in $\mathbf{b}$ indicate the indices of the missing fibers. In practice, the scattered fields due to M emitting sources are collected in order to improve the imaging resolution and stability.

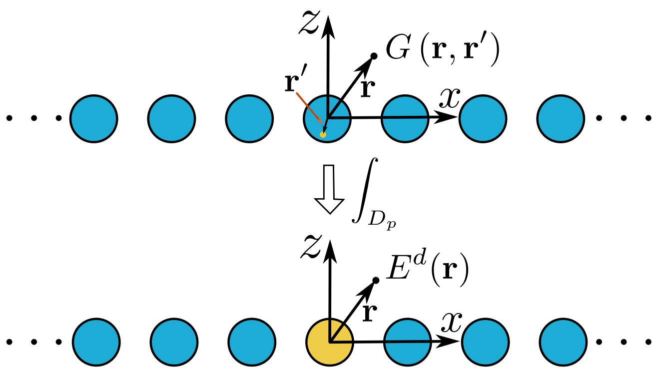

Fig. 1. Sketch for Lippman-Schwinger's integral formulation.

Let $\mathbf{g}_m$, $m = 1, 2, \ldots, M$, be the data vector when the m-th source is operated. Due to the invariance of $\mathbf{Z}$ with emitting sources, equation $\mathbf{g}_m = \mathbf{Z}\mathbf{b}_m$ can be built for each m. Integrating $\mathbf{g}_m$ into a matrix $\mathbf{Y} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_M]$, the imaging model for multiple sources is developed as $\mathbf{Y} = \mathbf{Z}\mathbf{S}$, $\mathbf{S} = [\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_M]$.

In general applications, only a few fibers are with fault, *i.e.* $P \ll N$. As exhibited from the composition of $\mathbf{b}_m$, a few elements are nonzero, *i.e.*, $\mathbf{b}_m$ is sparse. Furthermore, since the indices of missing fibers are invariant with the emitting sources, different $\mathbf{b}_m$ share common positions of nonzero elements, *i.e.*, different $\mathbf{b}_m$ enjoy common sparsity. To take advantage of this prior knowledge, a specific optimization problem is designed to retrieve $\mathbf{S}$.

$$\begin{aligned} &\min \ ||\mathbf{s}||_1 \\ &\text{subject to } ||\mathbf{Y} - \mathbf{Z}\mathbf{S}||_2^2 \le \tau^2 \\ &\quad \mathbf{s}^n = ||\mathbf{S}^n||_2, n = 1, 2, \ldots, N \end{aligned} \tag{1}$$

within which $\mathbf{s} = [s^1, s^2, \ldots, s^N]$, s^n is l_2-norm of n-th row of $\mathbf{S}$, and τ is the parameter constraining the energy of residual. Converting the optimization problem into the second order cone programming (SOCP) form [4], $\mathbf{S}$ can be solved by a standard free package SeDuMi [5].

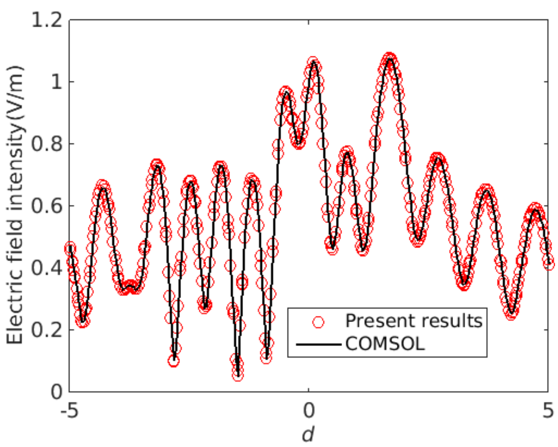

Fig. 2. Validation by COMSOL with carbon fibers illuminated by plane wave, incident angle $\pi/4$, wavelength $\lambda = 0.1$mm, $d = \lambda$, $c = 0.2d$

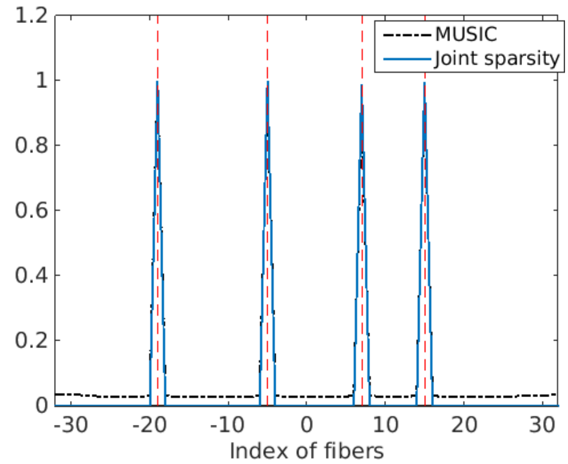

Fig. 3. Imaging results of carbon fiber array with multiple missing ones. $f = 60$GHz, $d = 0.1$mm, $c = 0.2d$

3. Numerical results

Simulations have been conducted with 65 carbon fibers $(\epsilon_r = 12, \text{conductivity}\ \sigma = 3.3 \times 10^2 \text{S/m})$, with index running from -32 to 32. The electromagnetic modeling approach is validated by comparing results with the finite-element COMSOL code, as shown in Fig. 2. Fig. 3 gives imaging results of the carbon fiber array with both MUSIC and the joint sparsity method, in which red dashed line denotes the accurate indices of missing fibers.

References

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